Wave Pattern Analysis by a Higher-order Boundary Element Method

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1. INTRODUCTION

These days, Seakeeping analysis is obtained more attention in ship and ocean engineering by scholars and engineers. Such as, Nakos [1)], Kring [2)], Huang [3)], Kara *et al.*[4)], Kim *et al.* [5)], and so forth. The objective of the present research is to develop a newly robust time-domain code for simulating ship advancing in nonlinear waves. As the begging step of this study, a 3D linear NWT (Numerical Wave Tank) is first constructed, and a steady ship wave problem is then calculated using a higher-order boundary element method.

In the present paper, a 3-D linear time-domain Rankine panel method using a Higher-order Boundary Element Method (BEM) is newly developed. To prevent waves reflection from outside of the tank, an artificial damping beach is installed. A 7-point Chebyshev scheme is employed to remove the saw-tooth instability. An iterative time marching scheme is used for numerical accuracy and stability. First, the time-domain higher-order BEM method is described. After the convergence of the mesh and time step is confirmed, wave patterns caused by a Wigley hull is calculated based on both Neumann-Kelvin and double body basis flows. In this study, the Wigley hull is assumed to advancing in still water with a steady forward speed without oscillation.

2. MATHEMATICAL FORMULATION

2. 1 Governing Equation

As mentioned above, the time domain analysis of ship motions by a Rakine Panel Method (element method) has already been studied by many scholars and engineers. We follow Kring [2)], Huang [3)], and Kim *et al.* [5)] closely, but the higher-order BEM is introduced for numerical accuracy and less time-consuming. A reference Cartesian coordinate system, (x, y, z), fixed to the steady motion of the ship is introduced. The reference frame is set with xoy on the still free surface and its origin on the center of the body; z is positive to upward. A surface-piercing vessel is travelling at constant forward speed, U, with respect to a space-fixed frame (x_0, y_0, z_0). The fluid is assumed to be incompressible and inviscid, and the motion is irrotational. The potential is divided into basis, Φ, unsteady, ϕ_d, and incident , ϕ_I, potentials,

$$\Psi = \Phi + \phi_d + \phi_I \tag{1}$$

$$\zeta = \zeta_d + \zeta_I \tag{2}$$

subscripts, I and d denote the incident flow and the disturbed flow, respectively.

For the basis flow, the Neumann-Kelvin linearization and double-body linearization are used. Only the double-body basis flow is used in the expression, which is governed by Laplace equation and satisfied the following boundary conditions,

$$\begin{cases} \Phi_z = 0 & \text{on the free surface} \\ \dfrac{\partial \Phi}{\partial n} = \vec{U} \cdot \vec{n} & \text{on the body surface} \end{cases} \tag{3}$$

The disturbed potential, ϕ_d, is also governed by Laplace equation, and satisfied the following boundary conditions and initial conditions,

$$\frac{\partial \phi_d}{\partial n} = \sum_{j=1}^{6} \left(\frac{\partial \xi_j}{\partial t} \cdot n_j + \xi_j \cdot m_j \right) - \frac{\partial \phi_I}{\partial n} \qquad \text{on the body surface} \tag{4}$$

$$\frac{\partial \phi_d}{\partial n} = 0 \qquad \text{on the water bottom} \qquad (5)$$

$$\left[\frac{\partial}{\partial t} - (\vec{U} - \nabla\Phi)\nabla\right]\zeta_d = \frac{\partial^2 \Phi}{\partial z^2}\zeta_d + \frac{\partial \phi_d}{\partial z} + (\vec{U} - \nabla\Phi)\nabla\zeta_I \qquad \text{on the free surface} \qquad (6)$$

$$\left[\frac{\partial}{\partial t} - (\vec{U} - \nabla\Phi)\nabla\right]\phi_d = \vec{U}\cdot\nabla\Phi - g\zeta_d - \frac{1}{2}\nabla\Phi\cdot\nabla\Phi + (\vec{U} - \nabla\Phi)\nabla\phi_I \quad \text{on the free surface} \qquad (7)$$

$$\phi_d\big|_{t=0} = 0\,, \qquad \frac{\partial \phi_d}{\partial n}\Big|_{t=0} = 0 \qquad \text{initial condition} \qquad (8)$$

where n is the normal vectors, pointed out from the computational domain; ξ denotes the displacement of ship in the six degrees of freedom about the reference frame; *m* is the m-term, which can be expressed as follows,

$$\begin{aligned} (m_1, m_2, m_3) &= (\vec{n}\cdot\nabla)(\vec{U} - \nabla\Phi) \\ (m_4, m_5, m_6) &= (\vec{n}\cdot\nabla)\left(\vec{x}\times(\vec{U} - \nabla\Phi)\right) \end{aligned} \qquad (9)$$

The m-terms, m_j, provide a couple between the basis flow and unsteady flow. These terms tend to be largest at the ends of the ship. In the present study, the Wigley hull is fixed and advanced with a forward speed. As the ξ_j = 0, it is no need to calculate m-terms in Eq. (4) in ship-fixed case.

2. 2 Boundary Integral Equation

The higher-order boundary element method is used for solving the mixed boundary value problem in this numerical simulation. A boundary integral equation for the potential components, Φ and ϕ_d, over the whole boundaries *S* can be derived through Green's second identity,

$$C(P)\phi(P) = \frac{1}{2\pi}\iint_S \left[\phi\frac{\partial G(P;Q)}{\partial n} - \partial G(P;Q)\frac{\partial \phi}{\partial n}\right]ds \qquad (10)$$

where *P* is source point; *Q* is field point; $C(P)$ is the solid angle. A treatment of solid angle is simplified by assuming that a uniform potential is applied over a closed domain, which produces no flux, in such a case leads to,

$$C(P) = \frac{1}{2\pi}\iint_S \frac{\partial G(P;Q)}{\partial n}ds \qquad (11)$$

A Rankine source is adopted as the Green function, with water bottom surface satisfied.

$$G(P;Q) = \frac{1}{r_1} + \frac{1}{r_2} \qquad (12)$$

where

$$\begin{aligned} r_1 &= \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \\ r_2 &= \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z+z_0+2h)^2} \end{aligned}$$

The finial discretized boundary integral equation can be written as,

$$\begin{bmatrix} A_1 & A_2 \end{bmatrix}\begin{Bmatrix} \phi\big|_{S_B} \\ \frac{\partial \phi}{\partial n}\Big|_{S_F} \end{Bmatrix} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{Bmatrix} \frac{\partial \phi}{\partial n}\Big|_{S_B} \\ \phi\big|_{S_F} \end{Bmatrix} \qquad (13)$$

here, S_B and S_F denotes body surface and free surface, respectively.

3. NUMERICAL RESULTS

3.1 Wigley hull

The standard Wigley hull is of mathematical hull form with its geometric surface defined as

$$y(x,z) = \pm \frac{B}{2}[1-(2x/L)^2][1-(z/T)^2], \tag{14}$$

where L is the hull length, B the full hull beam and T the hull draft. For the standard Wigley hull used in this computation, the ratio of length-to-beam, L/B is 10, and the ratio of beam-to-draft, B/T is 1.6.

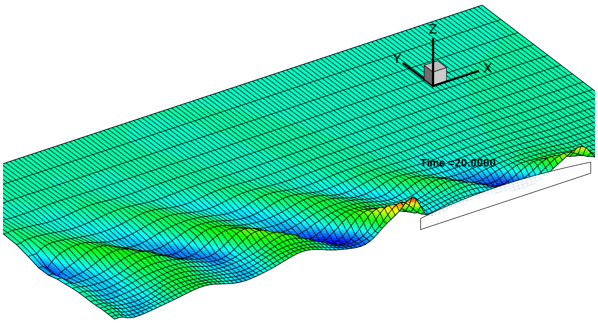

Fig. 1 Wave patterns generated by a Wigley hull with forward speed *Fn* = 0.316

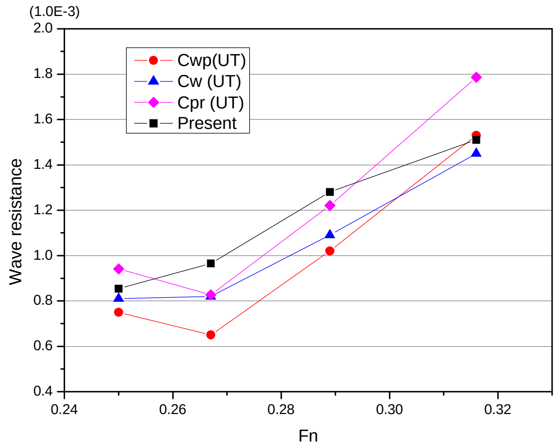

Fig. 3 Comparison of wave resistance for diffident forward speed *Fn* = 0.250, 0.267, 0.289 and 0.316, respectively. (Experimental data from Kajitani and Miyata [6].)

3.2 Wave patterns

In the present study, the Wigley hull has been fixed in position and never changed the wetted hull surface. So the results given here are based on a model-fixed Wigley hull at the mean hull surface. A rectangular computational domain is employed in the present study. In the following numerical simulation, the grid-size is $L/40$ along the x-direction. 41×7 nodes are used on the hull wetted surface, and the grid-size is enlarged with a scale, $q = 1.1$, in y-direction. The computational domain extends 1.8 L ship-length downstream, 0.4 L ship-length upstream, and 1.2 L ship-length in y-direction.

First, the computed wave patterns generated by the standard Wigley hull at Froude number 0.316 is shown in Fig. 1. The wave patterns are calculated based on both Neumann-Kelvin basis flow and double body basis flow, and the comparison between two basis flows is shown in Fig. 2. From Fig. 2, it can be found that they agree with each other well.

3.3 Wave resistance

A comparison of wave resistance coefficients with the published results is given in Fig. 3 with diffident forward speed *Fn* = 0.250, 0.267, 0.289 and 0.316, respectively. The wave resistance coefficients are of the form $C_f = F_x / (0.5 \rho U^2 S_A)$, where S_A is the area of the wetted surface. The measurement results are from Kajitani and Miyata [6] for model-fixed hull. C_{wp} is form wave pattern analysis, and C_w is wave resistance coefficient derived from towing test, C_{pr} is wave resistance coefficient derived from integrating hull surface pressure. From Fig. 3, it can be found that a reasonable agreement is obtained between the current result and experimental measurement from Kajitani and Miyata [6] marked as UT(University of Tokyo).

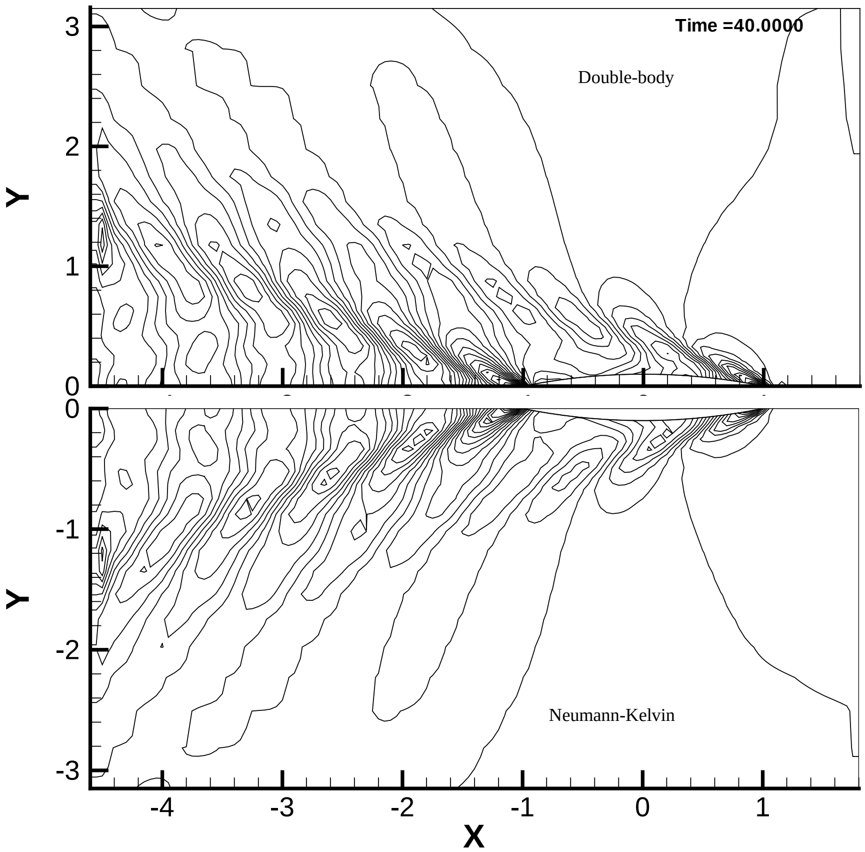

Fig. 2 Comparison of wave patterns between Neumann-Kelvin and double body basis flows, when ship advancing at Fn = 0.316 in the still water.

4. CONCLUSIONS

A 3D linear time domain numerical wave tank was newly developed based on potential flow by using the higher-order BEM. The wave patterns caused by a Wigley hull was calculated based on both Neumann-Kelvin and double body basis flows. The wave resistance was calculated and compared with the experiment measurement in reasonable agreement. As the final objective of the present study is that time domain simulation of ship advancing in nonlinear waves, the unsteady ship wave problem in the nonlinear wave should be developed and comparison study with other methods and experiments should also be carried out.

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