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Laboratory experiments on waves in moderate and shallow water and their kinematics

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1 Motivation and background

Frequent requests from industry regard the properties of shallow water waves and the wave induced kinematics. This problem focus has become a very actual one in relation to new plans for offshore wind farms in shallow water. Examples include the development on Sheringham Shoal, north of Norfolk, East England. The water depth there ranges from 12 to 24 m. Plans for Dogger Bank are similar but the area is larger. Dogger Bank is in the southern part of the North Sea and has a trapeziodally shaped area of 17 thousand km^2 being 260 km long and 97 km broad. The water depth ranges from 15 to 36 m. The longest waves that are of concern have a period of about $T_0 = 12$ seconds, according to information from the design engineers. This means that $\omega_0^2 h/g$ ranges from 0.36 to 1 where h denotes the water depth, g acceleration of gravity and $\omega_0 = 2\pi/T_0$. The corresponding wavenumber range is $kh > 0.6$ (k wavenumber) and $\lambda < 10.5h$ (λ wave length) which may be characterised as moderately shallow.

2 Wave theories

Nonlinear periodic waves (wave trains) in very shallow water are described by cnoidal wave theory. A fifth-order cnoidal mathematical model was developed and tested out by Fenton (1979), and compared to the classical fifth order Stokes wave solution. Fully nonlinear simulations are relevant (e.g. Grue et al. 2008; wave height up to 0.41 times water depth) but the method is restricted when the breaking limit is approached. A computational challenge is precisely the factors that limit the wave amplitude. In the very long wave regime, it is well known that the maximum relative height of a solitary wave is $H/h = 0.83...$ (H wave height). For moderate to long waves one may question what are the limitations of the wave height (or amplitude). In the theoretical paper by Fenton (1979) comparisons were made to shallow water wave experiments by Iwagaki and Sakai (1970). They measured the elevation and fluid velocities for the wave number range with $T\sqrt{g/h}$ between 5.3 and 26.6 and nonlinearity H/h in the range 0.229–0.448. Relevant to our range of interest with $T\sqrt{g/h} \sim 8-10$ (see below) their H/h ranged 0.239–0.429. In terms of the usual steepness for deep water waves this level of nonlinearity corresponds to $\frac{1}{2}Hk \simeq 0.14$, so the waves are not very steep compared to the strong waves that may occur in deep water where the wave slope

ak may well be 0.3 for storm waves, 0.4 for extreme events like the Draupner and Hurricane Camille waves, and $ak = 0.44$ is the theoretical maximum of Stokes waves.

3 Experiments, saturation and velocity profiles

Wave tank experiments in the long wave regime reveal that it is difficult to generate waves of appreciable nonlinearity. We present here one such case, in the intermediate range, with physical still water depth of 20 cm and paddle period of 1/0.8 sec. corresponding to $\omega^2 h/g = 0.515$, $kh = 0.785$ and $T\sqrt{g/h} = 8.76$. This is towards the shallower end of the wave range at sea, but to the deeper wave range in the experiments by Iwagaki and Sakai. While the response of the wave maker increases linearly with input voltage, the wave height reaches a saturation caused by wave breaking a few meters downstream of the wave maker. Thus, with input voltage of 1 V, wave height becomes 6.1 cm ($H/h = 0.31$), voltage of 1.2 V gives wave height 6.9 cm ($H/h = 0.35$), and voltages of 1.4 V and 1.8 V both give wave height of approximately $H = 6.4$ cm ($H/h = 0.32$), see figure 1. An estimate of the wave slope in case 2 is $\frac{1}{2}Hk = 0.135$ corresponding to the experiments by Iwagaki and Sakai.

Experimental wave induced fluid velocities measured by Particle Image Velocimetry have high accuracy. Velocities are scaled by the shallow water speed $c_0 = \sqrt{gh}$. Figure 2 presents the wave induced velocity during one cycle, for the wave series in figure 1 upper panel. The horizontal velocity reaches a maximal value below crest of 0.14 times the shallow water speed and corresponds to what can be extracted from linear theory, see figure 2, plot at frame 0. Some effects in the bottom boundary layer are evident during the cycle.

Conclusions from a larger set of experimental data is that wave trains on shallow water are limited in amplitude because of breaking. Practical wave slopes are up to about $\frac{1}{2}Hk \simeq 0.14$ but may locally attain stronger slopes prior to breaking events where the wave height and amplitude become reduced. Wave induced velocities seem to be well estimated by linear theory in this range, but breaking events currently under investigation show larger velocities. Such results will be presented at the Workshop.

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4 References

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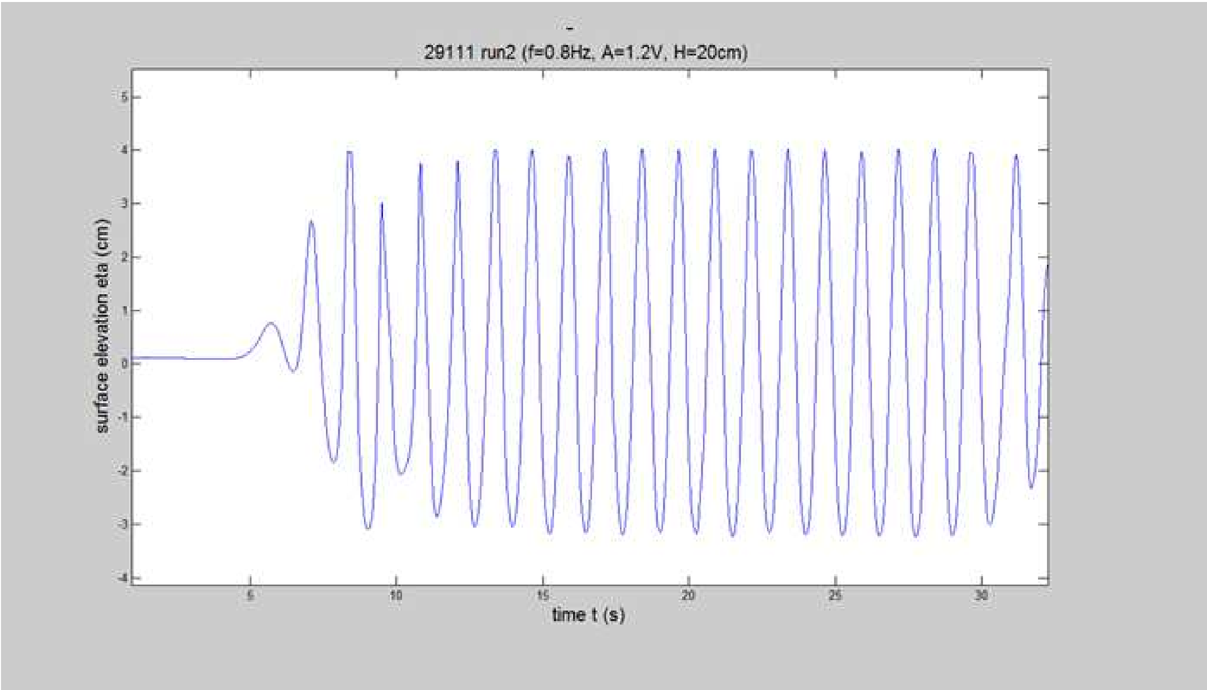

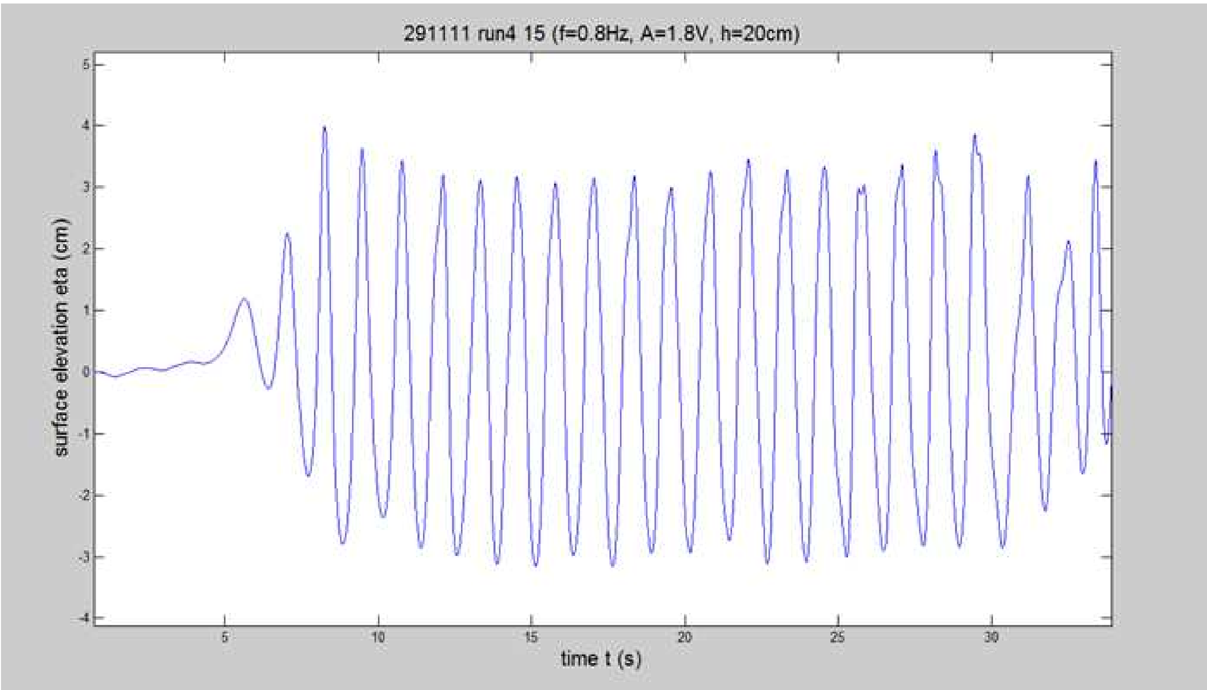

Figure 1: Elevation in wave tank experiments. Still water depth: 20 cm, paddle period 1/0.8 sec. corresponding to $\omega^2 h/g = 0.515$, $kh = 0.785$ and $T\sqrt{g/h} = 8.76$. Input voltage of 1.2 V (upper) – wave height 6.9 cm ($H/h = 0.35$), and 1.8 V (lower) – wave height approx. $H = 6.4$ cm ($H/h = 0.32$).

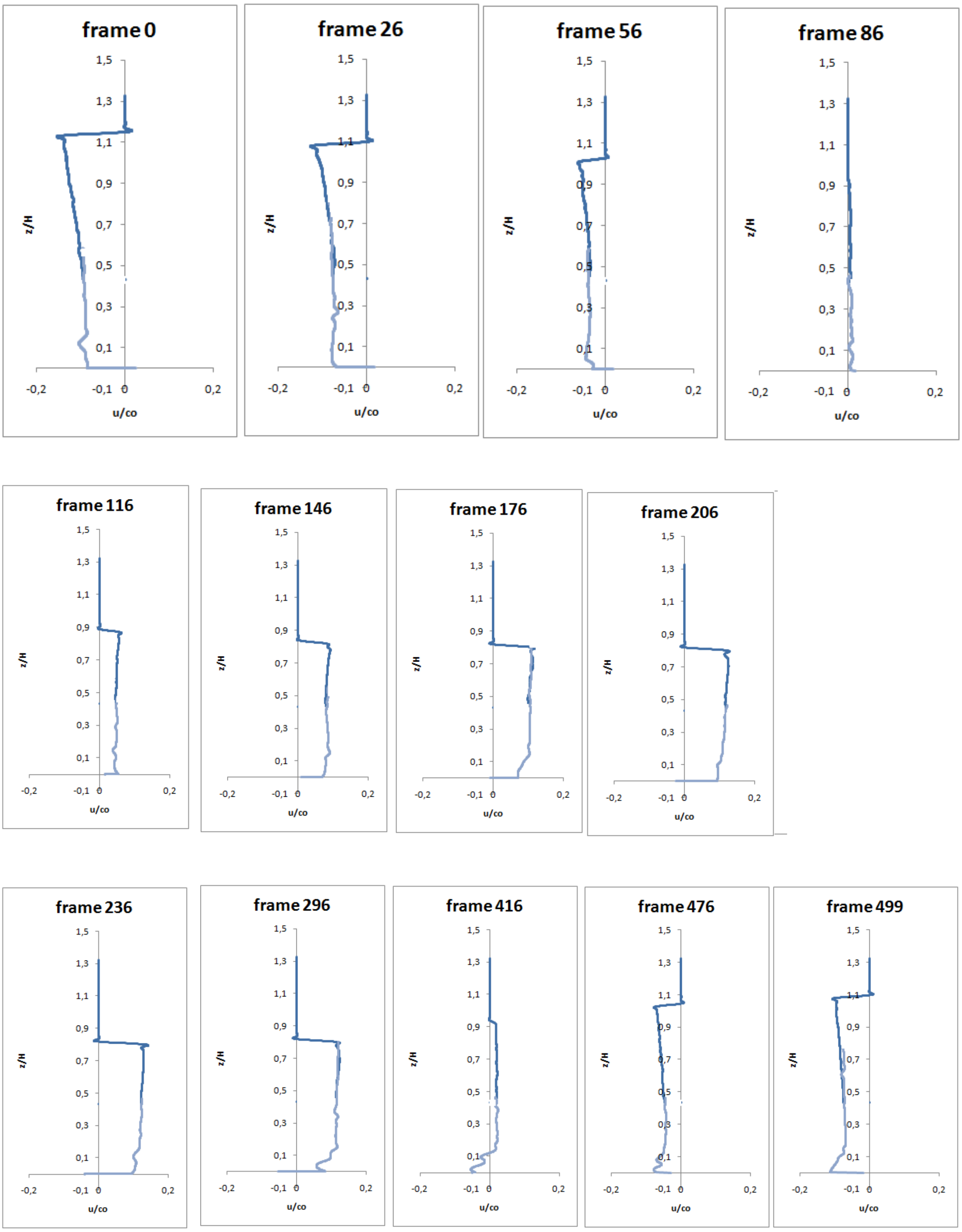

Figure 2: Horizontal velocity u/c_0 by PIV vs. vertical coordinate z/h at frames 0, 26, 56,...,476, 499 (one cycle). Same wave parameters as in figure 1 upper panel.