Taylor Expansion Boundary Element Method
for floating body hydrodynamics

Duan WenYang
Harbin Engineering University,150001, Harbin,China
duanwenyang@hrbeu.edu.cn

Introduction

Boundary Element Method(BEM) is widely used in solving the Boundary Integral Equations(BIE) derived from the corresponding mathematical physical equations. For hydrodynamics analysis of free surface wave interaction with arbitrary form floating bodies, potential flow and the corresponding Laplace equation is most solved numerically through BEM, or named panel method. Mixed distribution of sources and dipoles integral equation and sources only distribution integral equation are two kinds most widely used boundary integral equations in floating body hydrodynamics, where the former is often named direct BEM as it origins from the Green formulas and the later is often named indirect BEM as it comes from the singularity analysis. From the view point of numerical discretization of the corresponding integral equations, there are two kinds of BEM, namely, low order BEM and high order BEM. The early work of panel method by Hess and Smith[1] is based on lower order indirect BEM.

For lower order BEM, direct BEM which is based on mixed sources and dipoles distribution BIE is superior to the indirect BEM which is based on sources only distribution BIE from the view points of accuracy and efficiency for solving the velocity potentials on the boundary. Nevertheless, numerical differentiation have to be used to obtain tangential velocity followed from the direct BEM , where indirect BEM is more robust for obtain the induced velocity. So there are two common routines adopted for solving first order and second order wave hydrodynamics. The first way is using direct BEM for solving potentials and using indirect BEM for obtaining velocity. The second way is using indirect BEM for solving both potential and velocity. For smoothed boundary without corners like a sphere boundary both of the two ways may give satisfied numerical solutions, although the first way converge better than the second way for obtaining the potential. For no-smoothed boundary like a vertical finite draft circular cylinder or box like floating bodies, the velocity is not easy to converge near the corners by the two ways. Especially for solving the tangential velocity around the corner region, direct BEM with numerical differentiation has lower accuracy than that on other smoothed part region panels, while much bad tangential velocity in the corner region relatively to the smoothed region results from indirect BEM no matter how fine panel discretization is used. These characteristics of lower order BEM or constant panel method leads to lower convergences of prediction for second order wave forces and second order potential obtained from the results of the corresponding tangential velocity calculations.

For high order BEM, direct BEM is most used to represent the potential with different order interpolation scheme, for example, Lagrange polynomial interpolation or spline interpolation. Like the work by Hsin et al[3]. For the high order direct BEM, the potential and its tangential difference on the boundary has different level accuracy and can not satisfy the

mixed boundary condition with consistent manner, moreover, it is shown that although the accuracy improved much than the lower order BEM with same number of panels, but the relative accuracy of results is still not satisfied comparing with the corner regions and smoothed part regions on the boundary. For high order indirect BEM, Hess[2] was also the early user, who described the source strength with high order Lagrange polynomials. modified from the indirect BEM, the desingularized source integration method or source point method, like that by Webster[4] ,Cao et al[5] can improve the accuracy much for smoothed boundary value problems. The situations for non-smoothed boundary with sharp corner are observed in similar characteristics as described for direct BEM. Furthermore the robustness of the high order scheme is not so good comparing with lower order BEM especially for dealing with the corner contribution in the numerical treatments of singular boundary integrals.

To improve the accuracy of BEM for the solution of no-smoothed boundary problem, a new numerical scheme named TEBEM is presented by Duan[6]. TEBEM is the abbreviation of Taylor Expansion Boundary Element Method. In TEBEM, the direct BIE is used and the boundary is discretized into straight line elements for 2D problem or triangular or quadrilateral plane elements for 3D problem, the velocity potential on each element is expressed as, the one variable for 2D problem and two variables for 3D problem, Taylor expansion about the control point to different order, where the first order derivatives on each element are exactly the tangential velocity components. If the expansion is truncated after first order, it is named first order TEBEM, if the expansion is truncated after zero order, TEBEM is reduced to the general lower order direct BEM. The unknowns for the first order TEBEM are velocity potentials and tangential velocities components on each element. So if element number is N, then there are 2N unknowns for 2D problem and 3N unknowns for 3D problem. Satisfying boundary condition of direct BIE on the control point of each element can formula N equations. So more equations are needed apart from the direct BIE. This is achieved through utilizing tangential difference of the direct BIE on each control point to establish new BIE to give more N equations for 2D problem and 2N equations for 3D problem. The new BIE have same principle value as the direct BIE with replacement of velocity potential just by the corresponding tangential velocity components.

It is found that TEBEM can achieve high accuracy for both smoothed and no-smoothed boundary 2D or 3D problems. Here we show the application of TEBEM for 2D rectangle body.

Formulations of the first order TEBEM for 2D problem

We use coordinate system (x,y) to formulate the problem. The origin is on the calm water surface and y axis is positive upwards. A floating rectangle box periodically oscillates with small amplitude in heave and sway mode and we seek the corresponding frequency domain solutions using potential flow assumption for infinite water depth.

The Green function satisfied the linear free surface boundary condition is:

$$G(p,q) = \ln \frac{r_{pq}}{r_{p\bar{q}}} + p.v. \int_0^\infty \frac{2e^{k(y_p+y_q)}\cos k(x_p - x_q)}{v-k} dk + 2\pi i e^{v(y_p+y_q)}\cos v(x_p - x_q) \qquad (1)$$

The direct boundary integral equationfor the velocity potential ϕ is:

$$-\pi\phi(p)+\int_{L}\phi(q)\frac{\partial G(p,q)}{\partial n_q}dl_q=\int_{L}\frac{\partial\phi(q)}{\partial n}G(p,q)dl_q \tag{2}$$

The indirect boundary integral equation for source only distribution is:

$$-\pi\sigma(p)+\int_{L}\sigma(q)\frac{\partial G(p,q)}{\partial n_p}dl_q=\frac{\partial\phi(p)}{\partial n} \tag{3}$$

Where σ is the source strength. The mean wetted body surface L is divided into N line elements, the velocity potential, tangential velocity, normal velocity, source strength at the mid-point of each element are denoted as ϕ_j, $\left(\frac{\partial\phi}{\partial l}\right)_j$, $\left(\frac{\partial\phi}{\partial n}\right)_j$, σ_j respectively.

Lower order direct BEM scheme is :

$$\sum_{j=1}^{N}A_{ij}\phi_j=\sum_{j=1}^{N}B_{ij}\left(\frac{\partial\phi}{\partial n}\right)_j \tag{4}$$

Lower order indirect BEM scheme is:

$$\sum_{j=1}^{N}\overline{A}_{ij}\sigma_j=\left(\frac{\partial\phi}{\partial n}\right)_i$$
$$\phi_i=\sum_{j=1}^{N}B_{ij}\sigma_j \quad,\left(\frac{\partial\phi}{\partial l}\right)_i=\sum_{j=1}^{N}C_{ij}\sigma_j \tag{5}$$

First order TEBEM is scheme is:

$$\sum_{j=1}^{N}A_{ij}\phi_j+\sum_{j=1}^{N}A_{ij}^{l}\left(\frac{\partial\phi}{\partial l}\right)_j=\sum_{j=1}^{N}B_{ij}\left(\frac{\partial\phi}{\partial n}\right)_j$$
$$\sum_{j=1}^{N}D_{ij}\phi_j+\sum_{j=1}^{N}D_{ij}^{l}\left(\frac{\partial\phi}{\partial l}\right)_j=\sum_{j=1}^{N}C_{ij}\left(\frac{\partial\phi}{\partial n}\right)_j \tag{6}$$

Where $A_{ij}=\int_{\Delta L_j}\left(\frac{\partial G_{ij}}{\partial n_j}\right)dl$, $B_{ij}=\int_{\Delta L_j}\left(G_{ij}\right)dl$, $\overline{A}_{ij}=\int_{\Delta L_j}\left(\frac{\partial G_{ij}}{\partial n_i}\right)dl$,

$C_{ij}=\int_{\Delta L_j}\left(\frac{\partial G_{ij}}{\partial l_i}\right)dl$, $A_{ij}^{l}=\int_{\Delta L_j}\left(\frac{\partial G_{ij}}{\partial n_j}\right)(\delta l)\,dl$, $D_{ij}=\int_{\Delta L_j}\left(\frac{\partial^2 G_{ij}}{\partial l_i\partial n_j}\right)dl$, $D_{ij}^{l}=\int_{\Delta L_j}\left(\frac{\partial^2 G_{ij}}{\partial l_i\partial n_j}\right)(\delta l)\,dl$

Here δl means local coordinate value corresponding to the mid-point of the element.

Numerical results and discussion

It is shown for example the results for sway motion of rectangle box with breadth/draft=B/T=4 represented by 30 equal length line elements. Figure1 is the added mass and figure 2 is the second order presure distribution. Comparing the results by indirect BEM (5) (IBEM), direct BEM (4)(DBEM) and TEBEM(6), it is found the accuracy of three methods has noticeable difference. The oscillation characteristic of curves is due to the irregular frequencies. It is well known the irregular frequencies are located at series frequency points. Due to numerical error of solving method. The irregular frequencies are shown at a

series frequency band. The narrow the band is, the more accurate the method should be. From this judgments, TEBEM is most accurate than the general IDBEM and DBEM. More interested velocity prediction results will be presented at the workshop.
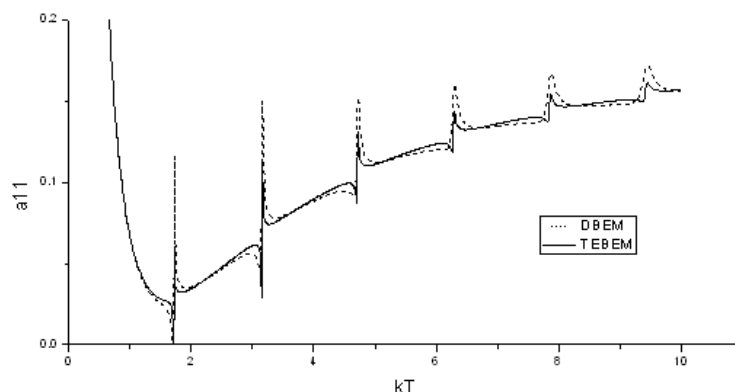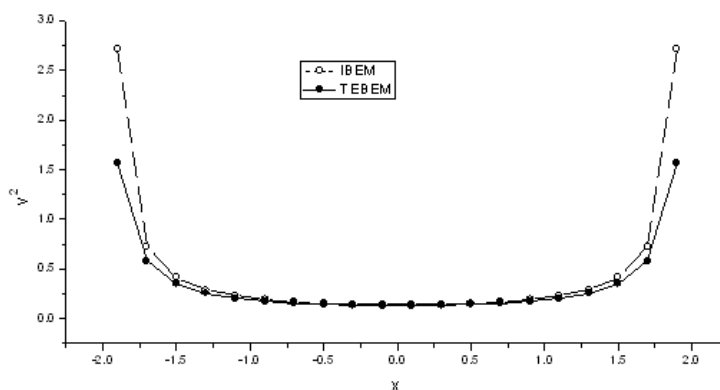

Figure1, Added mass for sway by different BEM

Figure2, 2nd order presure distribution for sway (kT=0.3)by different BEM

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