The New Bristol Cylinder: A submerged cylinder wave energy converter

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1. Introduction and description of the device

We present a novel design for a wave energy converter (WEC) based on the idea of coupled resonances (see Evans & Newman (2011), Evans & Porter (2012)) which aims to provide a theoretical basis for wave power extraction whilst addressing engineering design issues.

The device bears some similarities to the original Bristol Cylinder WEC (Evans *et al.* (1979), Clare *et al.* (1982)) being comprised of a circular cylinder in motion beneath the waves. The original Bristol cylinder device was conceived on the principle that a phase-related combination of forced heave and surge motion of a two-dimensional cylinder produces waves in only one direction. By reversing the direction of outgoing waves, incoming waves were shown to be fully absorbed by the cylinder when constrained to move in a reversal of that forced motion, being that of a circular motion of the cylinder axis. In practice, a complicated arrangement of springs and dampers needed to be attached to splayed mooring legs to extract energy through this motion. Evidently, this complication has deterred engineers from developing this idea. In contrast, the two WEC devices that have emerged recently as front-runners in the UK – the Pelamis and Oyster – are both designed fundamentally on principles of survivability, simplicity of power take off (PTO) and ease of maintainence, arguably at the expense of performance.

We revisit the Bristol Cylinder idea, exploiting the principle that a submerged device is protected from the harsh environment of the ocean surface and appealing to theoretical principles that a device need not be on the surface to capture significant energy from the waves. Our device differs from the original Bristol Cylinder idea in that it is constrained to move in surge motion only, whilst the PTO mechanism is simpler and placed within the cylinder itself. A cross-section of the device is shown in Fig. 1.

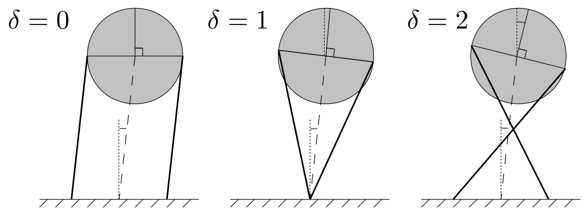

Fig. 1(a): Three possible mooring configurations.

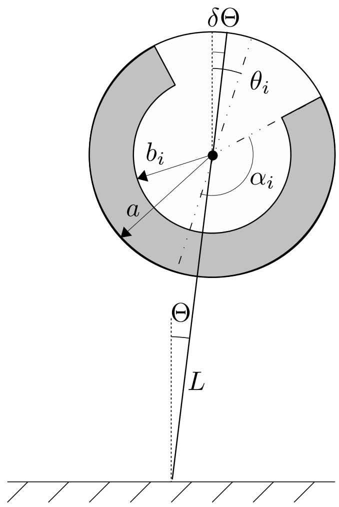

Fig. 1(b): Internal pendulum configuration

Thus, the mooring acts as a passive component in the PTO, holding the buoyant cylinder below the surface and allowing it to move as an upturned pendulum (predominantly surge on small-amplitude theory). The particular configuration of the mooring system can allow the cylinder to roll in proportion to its surge displacement by attaching the lower ends of pairs of equal-length tethers to arbitrary points on the sea bed. This feature is encapsulated in a factor δ, which relates the roll, $\delta\Theta$, of the cylinder about its axis to the pitch, Θ, of the cylinder about a point on the bed. Examples of three different mooring systems with $\delta = 0$, 1 and 2 are shown in Fig. 1(a). This mooring system is a key part of the success of the device. Within the cylinder, a system of N solid-body pendulums are allowed to roll about the cylinder axis (see Fig. 1(b)). Each pendulum is given the same density, ρ_s, and forms an annular sector in cross-section with its own internal radius b_i, a common external radius a (equal to the radius of the cylinder) and extending through an angle $2\alpha_i$, $(i = 1, \dots, N)$. The pendulum rotates through an angle θ_i with respect to the vertical and power is taken off via a linear damper proportional (with constant of proportionality γ_i) to the relative angular velocity of the ith internal pendulum with respect to that of the cylinder.

The cylinder (excluding internal pendulums) is given a mass M and displaces a mass M_w of water. Its radius is a, its length is D and its axis is submerged a distance f below the surface. The mooring length is L and the water depth is h. Mooring lines need not necessarily be connected to the sea bed. In motion, the cylinder rolls about its axis with a moment of inertia of MK^2 where K is its radius of gyration.

Each pendulum has mass $m_i = \rho_s \alpha_i (a^2 - b_i^2) D/N$, and effective length l_i (point of rotation to centre of mass) and radius of gyration k_i given by

$$l_i = \frac{2a \sin\alpha_i}{3\alpha_i} \frac{(1 + \hat{b}_i + \hat{b}_i^2)}{1 + \hat{b}_i}, \quad k_i^2 = \tfrac{1}{2} a^2 (1 + \hat{b}_i^2) - l_i^2,$$

where $\hat{b}_i = b_i/a$. In the absence of damping each pendulum has a resonant period given, for small amplitudes, by

$$T_i^* = 2\pi \sqrt{l_i (1 + k_i^2/l_i^2)/g}, \tag{1}$$

which can be tuned to any desired period, by varying the free parameters associated with each pendulum.

2. Equations of motion

Euler-Lagrange equations are used to derive the equations of motion of the cylinder and internal pendulums as a function of the total hydrodynamic surge force X_w on the cylinder. We linearise the resulting equations, assuming small amplitudes, and consider time-harmonic motion of angular frequency ω. Finally, we replace (small) angles by linear velocities with

$$U = -\mathrm{i}\omega L \Theta, \qquad u_i = -\mathrm{i}\omega l_i \theta_i, \tag{2}$$

for $i = 1, \dots, N$. The complicated details of the derivation of the equations of motion are ommitted. The resulting motion of the cylinder is governed by

$$-\mathrm{i}\omega M (1 + \hat{K}^2) U = X_w - \frac{\mathrm{i}}{\omega} C U + X_e, \tag{3}$$

and the motion of each internal pendulum is related to that of the cylinder by

$$u_i = \frac{1 + \mathrm{i}\hat{\gamma}_i \hat{l}_i \delta}{1 + \hat{k}_i^2 + \hat{\beta}_i} U. \tag{4}$$

In the above, we have introduced dimensionless variables

$$\hat{l}_i = \frac{l_i}{L}, \quad \hat{k}_i = \frac{k_i}{l_i}, \quad \hat{m}_i = \frac{m_i}{M_w}, \quad \hat{\gamma}_i = \frac{\gamma_i}{m_i \omega},$$
$$\hat{\omega}^2 = \frac{\omega^2 L}{g}, \quad \hat{M} = \frac{M}{M_w}, \quad \hat{K} = \frac{\delta K}{L},$$

and written

$$\hat{\beta}_i = \mathrm{i}\hat{\gamma}_i - \frac{1}{\hat{\omega}^2 \hat{l}_i}, \quad C = \frac{M_w g}{L} \left(1 - \hat{M} - \sum_{i=1}^{N} \hat{m}_i \right),$$

whilst

$$\frac{X_e}{M_w \omega} = -\sum_{i=1}^{N} \hat{\gamma}_i \hat{m}_i \left(1 - \delta \hat{l}_i\right) \left(u_i - \delta \hat{l}_i U\right) - \mathrm{i} \sum_{i=1}^{N} \frac{\hat{m}_i}{\hat{\omega}^2 \hat{l}_i} u_i + \mathrm{i} \sum_{i=1}^{N} \hat{m}_i \hat{k}_i^2 u_i. \tag{5}$$

Equation (3) has been arranged into a form indicative of Newton's Law. Thus, mass (including rotational inertia) times acceleration of the cylinder is balanced by three forces: (i) the wave forces on the cylinder; (ii) hydrostatic buoyancy restoring forces; (iii) externally-induced forces from the system of internal pendulums.

It is less clear how to decipher (4), although it too emerges from Newton's Law, each pendulum rotating independently of all others but in proportion (in this linear framework) to the motion of the cylinder.

The external forces, X_e, are also arranged into three separate terms suggestive of the forces the pendulums impart upon the cylinder. The first term represents the effect of the damping; the second is the effect of gravitational restoring forces; and the third is associated with the rotational inertia of the solid-body rotation of the pendulum.

By eliminating u_i from (5) we may write

$$X_e = -\lambda U, \tag{6}$$

where

$$\lambda = M_w\omega \sum_{i=1}^{N} \frac{\hat{m}_i}{1+\hat{k}_i^2+\hat{\beta}_i}\Big(\hat{\gamma}_i\left(1-\delta\hat{l}_i\right)^2 \\ -\mathrm{i}\left(\hat{k}_i^2 - \frac{1}{\hat{\omega}^2\hat{l}_i}\right)\left(\mathrm{i}\hat{\gamma}_i\delta^2\hat{l}_i^2+1\right)\Big). \qquad (7)$$

whose real part, responsible for damping, can be shown to be positive.

3. Hydrodynamic coupling and the calculation of power

Under the assumptions of linearised wave theory, the total single-frequency surge component of the wave exciting force can be written as

$$X_w = (\mathrm{i}\omega A - B)\,U + X_s, \qquad (8)$$

in terms of the surge added mass and radiation damping, $A(\omega)$ and $B(\omega)$, and the surge component of the exciting force on a fixed cylinder, $X_s(\beta)$, which depends on the incident wave angle, β ($\beta = 0$ corresponds to normally-incident from $x = -\infty$).

Combining (8) with (3) gives

$$UZ = X_e + X_s, \qquad (9)$$

where

$$Z \equiv B - \mathrm{i}\omega(A + M(1+\hat{K}^2) - C/\omega^2), \qquad (10)$$

and then, using (6), gives

$$U\,(Z+\lambda) = X_s. \qquad (11)$$

The mean power (time averaged over a period) generated by the device is given by

$$W = \tfrac{1}{2}\Re\{X_w\overline{U}\}, \qquad (12)$$

the overbar denoting complex conjugation. We first note that using (3) in (12) gives

$$W = -\tfrac{1}{2}\Re\{X_e\overline{U}\} = \tfrac{1}{2}\Re\{\lambda\}|U|^2,$$

once (6) has been used. Then from (11) we have

$$W = \frac{\Re\{\lambda\}|X_s|^2}{2|Z+\lambda|^2}. \qquad (13)$$

An explicit calculation of $\Re\{\lambda\}$ is not required since the identity $4\Re\{\lambda\}\Re\{Z\} = |Z+\lambda|^2 - |Z-\overline{\lambda}|^2$ allows us to write

$$W = \frac{|X_s|^2}{8B}\left(1 - \frac{|Z-\overline{\lambda}|^2}{|Z+\lambda|^2}\right), \qquad (14)$$

(noting from (10) that $\Re\{Z\} = B$). We remark that an alternative calculation of mean power resulting in the same expression (14) can be made by summing the contribution from the mean power generated by each pendulum.

We arrive at the well-known result that the maximum achievable mean power is given by

$$W_{max} = \frac{|X_s|^2}{8B}. \qquad (15)$$

For a cylinder spanning a narrow wave tank with waves incident from $x = -\infty$, $X_s(\beta) \equiv X_s$ and all quantities previously depending on the length of the cylinder, D, are redefined by dividing by D (all now per unit length of the cylinder). In this two dimensional setting, the quantities X_s and B are connected by the well-known formula $|X_s|^2/8B = \gamma W_{inc}$ where W_{inc} is the mean incident wave power per unit length of wave crest and $\gamma = |A_+|^2/(|A_-|^2 + |A_+|^2)$ where $A_\pm$ are the radiated wave amplitudes towards $\pm\infty$ due to the the forced surge motion of the body. For our symmetric device, $A_- = A_+$ so $\gamma = \frac{1}{2}$.

Thus, in two dimensions, we can characterise the efficiency of the device by the ratio of the power absorbed per unit crest to the power incident per unit crest with

$$E \equiv \frac{W}{W_{inc}} = \frac{1}{2}\left(1 - \frac{|Z-\overline{\lambda}|^2}{|Z+\lambda|^2}\right),$$

and the maximum efficiency is $\frac{1}{2}$ or 50%.

In three dimensions efficiency is replaced by

$$l(\beta) = W/W_{inc} \qquad (16)$$

which defines the capture width of the device, being the equivalent length of wave crest of incident wave power absorbed by the device.

4. Results

We first consider a two-dimensional device measuring efficiency, $E \leq \frac{1}{2}$, across a range of wave periods. The aim is to make the efficiency response broad-banded. Thus, in each case presented, we optimise efficiency over a number of variables by minimising the integral of $|Z-\overline{\lambda}|^2/|Z+\lambda|^2$ over wave periods from 5s to 11s. Throughout we have taken the cylinder mass M to be $0.15M_w$ and pendulum density ρ_s as 2.4 times sea water density. In Fig. 2 we present a particular case with $a = 7$m, $h = 50$m, $f = 10$m, $L = 14$m and $N = 3$ pendulums. Then b_i and α_i

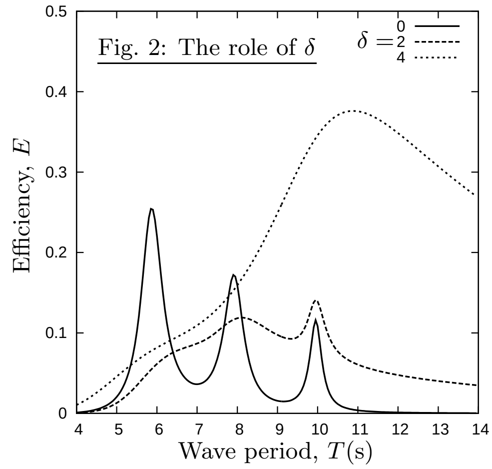

Fig. 2: The role of δ

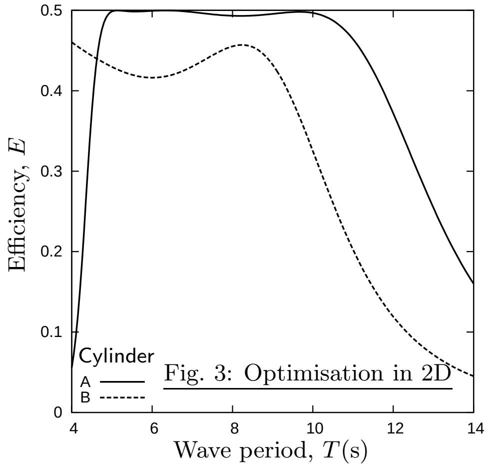

Fig. 3: Optimisation in 2D

are prescribed to give undamped resonant pendulum periods, T_i^*, at 6, 8 and 10s. The only free parameters here are the damping coefficients, γ_i which are tuned to produce optimum efficiency. Fig. 2 illustrates the role that the mooring system (or δ) has in both broadening resonant peaks and raising the overall efficiency of the system. If the mooring lines are not attached directly to the sea bed it makes most sense to choose $\delta = 1$ which relates to a cylinder pivoted about a point a distance L from the cylinder axis which is held under tension by static mooring lines. We adopt this hereafter.

In Fig. 3 we present two configurations of fixed cylinder radius and water depth and optimise efficiency over all remaining free variables. In both cases, the number of pendulums selected as optimal is $N = 1$. For case A, the radius and depth are fixed at $a = 7$m, $h = 50$m. The optimal submergence is $f = 10.1$m, mooring length $L/a = 0.84$ and the optimal pendulum tuning is $T_i^* = 5.34$s. For case B, we fix $a = 3.0$m in 20m depth and find optimal efficiency for $f = 3.75$m, $L/a = 1.06$ and $T_i^* = 3.44$s. The results highlight that smaller cylinders are not tuned as well to extract as much power as larger cylinders.

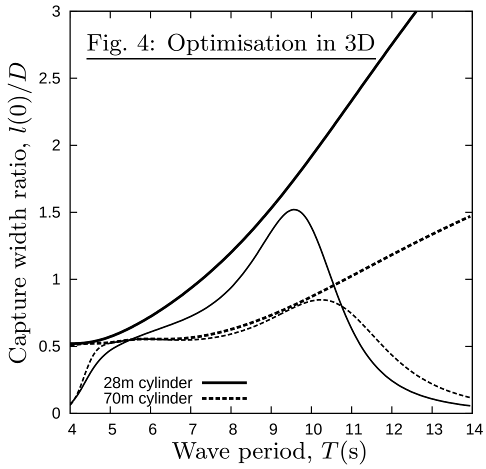

Fig. 4: Optimisation in 3D

We next consider the three-dimensional device fixing $a = 7$m and $h = 50$m and adopt $N = 1$ and $f = 10.0$m from the optimal two-dimensional results (case A) but optimise over the remaining parameters, b_i, α_i,γ_i and L. Two cylinder lengths are considered: a long ($D = 70$m) cylinder and a shorter ($D = 28$m) cylinder. The capture width per cylinder length and their respective maximum values are shown in Fig. 4. Notice how although the 70m cylinder is able to take out more than half its length in equivalent incident wave power, the 28m cylinder takes more than its length in incident wave power over wave periods of $8 - 10$s.

In a model sea state with an average annual power of 30kW/m the 28m cylinder is predicted (assuming perfect power conversion and no other losses) to output roughly 740kW, well in excess of the 100kW produced by the Oyster device (of similar length) and the 300kW produced by the 150m-long Pelamis attentuator. Our WEC does not need 'latching' to achieve resonance and does not suffer from 'end stop problems'.

5. References

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