Abstract for The 26th International Workshop on Water Waves and Floating Bodies, 17-20 April, 2011, Athens, Greece.

On Wave Elevations Under a Moving Pressure Distribution In Minimum-Resistance Conditions

Ronald W. Yeung
University of California at Berkeley
Berkeley, CA 94720-1740, USA,
Correspondence author: E-mail: rwyeung@berkeley.edu

Michael V. Makasyeyev
Institute of Hydromechanics of Ukrainian National Academy of sciences,
8/4, Gelyabova str. 03056, Kyiv, Ukraine, M_Makasyeyev@ukr.net

Chloé Matte
Department of Mechanical Engineering
University of California at Berkeley
Berkeley, CA 94720-1740, USA, chloe.matte@berkeley.edu

In tribute to our good friend and colleague Professor Odd Magnus Faltinsen at the 26IWWWFB, held in his honor.
R. W. Yeung

1. Background

In flows with a rigid surface at a free surface, such as that imposed by the bottom of a planing body, the surface geometry is given and the pressure is unknown. This leads to an integral equation with a "wavy" kernel. The kernel is difficult to treat, at least in the case of three dimensions, but methods of bypassing it using the simple-source or a finite-element hybrid solutions [1-3] have been developed in two and three dimensions. However, for free surfaces generated by cavities and under pressure distributions, the opposite is true; the pressure is known and surface deformation is unknown. Arguably, the surface information is normally not important, other than that it characterizes the volume of the space to be controlled so to achieve some desirable motion, say, for a hovercraft [4], or the occurrence of cavity drag, with the cavity induced by high-speed flow over a hull step or other similar mechanism [5,6]. The sign of the cavitation number in this latter case depends on whether or not there is air injection. However, in steady flow, since the free surface is a streamline, the surface shape under a prescribed pressure distribution can also be replaceable by a rigid plate of the same shape, with the same pressure loading. The spray jet would not take place if the streaming flow indeed can be tangent to the leading edge of the rigid surface. With such a consideration, one may be interested in knowing the local surface shapes, particularly those that are associated with the vanishing of the far-field trailing waves. Such shapes would have at least the desirable property corresponding to a planing surface with minimal wave drag, at least to first-order approximation. Other possibilities of wave-less forms, particularly in three dimensions, have been studied by Tuck [7].

Computations of wave resistance is a classical subject with many references available from [8, 9]. Of contemporary interest, for pressure distributions, are the recent works in [10,11], in which, advantage is taken of an interference resistance formula, which is applicable to a collection of pressure units, as well as surface-effects ships, the latter being modeled as a pressure distribution trapped between thin side hulls. In this paper, we focus only on the linear theory of a two-dimensional moving pressure distribution that is smooth [12], allowing primarily analytical treatment.

2. Governing Equations

A pressure distribution is assumed to be moving to the right at constant speed c in the +x direction (see Fig. 1). The y-axis points up with y=0 corresponds to the calm-water line. The prescribed (gauge) pressure function $p(x)$, is assumed to be given by:

$$p(x) = p_0 f(x),\ |x| < a;\quad p(x) \to 0 \text{ for } |x| > a, \qquad (1)$$

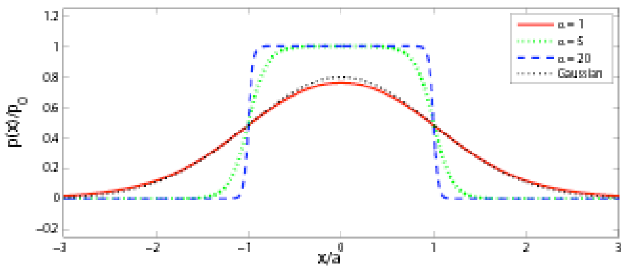


Fig. 1: Pressure distribution $p(x)/p_o$ vs. $\bar{x} = x/a$.

where the effective "craft length" is $2a$, and the load it carries is $2p_0a$. $f(x)$ is the distribution (spatial) shape of the pressure, which drops to zero outside of $|x| > a$. We assume an ideal and infinitely deep fluid of density ρ. With the use of linear wave theory [8, 9], the velocity potential can be shown to satisfy:

$$\varphi_{xx} + \varphi_{yy} = 0\,,\ y < 0\,, \tag{2}$$

$$\varphi_{xx}(x,-0) + k_0\varphi_y(x,-0) = p_x(x)/\rho c, \quad k_0 = g/c^2, \tag{3}$$

$$\varphi_x,\ \varphi_y = 0, \quad y \to -\infty, \tag{4}$$

$$Y(x) = \frac{c}{g}\varphi_x(x,-0) \to 0, \ \ x \to +\infty \tag{5}$$

where $Y(x)$ is the free-surface elevation. Eqn. (5) indicates that there are no upstream waves, a radiation condition that determines uniqueness.

The horizontal force (wave-drag) is given by

$$R_x = -\int_{-\infty}^{\infty} p(x)Y'(x)dx \tag{6}$$

$$p(x,-0)/\rho = c\varphi_x(x,-0) - gY(x), \tag{7}$$

The solution of $\varphi(x,y)$ is characterized by the Froude number: $F_n \equiv c/\sqrt{2ga}$ and the distribution shape function f(x) in (1). Figure 1 depicts the hyperbolic tangent shape [12], which is differentiable, as well as a Gaussian shape that is uniformly smooth:

$$f_{th}(\bar{x}) = \frac{1}{2}\left[\tanh\alpha(\bar{x}+1) - \tanh\alpha(\bar{x}-1)\right], \tag{8}$$

$$f_G(\bar{x}) = \frac{2}{\beta\sqrt{2\pi}}\exp\left(-\frac{\bar{x}^2}{2\beta^2}\right), \tag{9}$$

where α and β are shaping parameters. In order to have $f_G(\pm 1) = 0.5$ we set $\beta=1$. Note that as $\alpha \to \infty$, (17) becomes the constant step distribution

$$f_{th}(\bar{x})\Big|_{\alpha\to\infty} = \theta(\bar{x}+1) - \theta(\bar{x}-1),$$

where $\theta(x)$ is the Heaviside function. At $\alpha = 1$ the function $f_{th}(x)$ is similar in form to the Gaussian distribution function $f_G(x)$.

3. Particular and Homogeneous Solutions.

The problem (2)–(4) can be solved relatively easily by standard Fourier transformation method. The solution is given by:

$$\varphi(x,y) = \frac{1}{\pi\rho c}\,\text{p.v.}\int_0^{\infty}\frac{e^{ky}}{k-k_0}\int_{-\infty}^{\infty}p(\xi)\sin k(x-\xi)d\xi dk + A_\varphi\cos k_0x + B_\varphi\sin k_0x, \tag{10}$$

The first term is the particular solution, and the "steady-wave terms" of wave number k_o are homogeneous solutions, which can be interpreted as the residue coming from the inversion path of k. With the use of the far-field condition (5), it is straight-forward to establish:

$$Y(x) = \frac{k_0}{\rho g}\int_{-\infty}^{\infty}p(\xi)Q(x-\xi)d\xi + A\sin k_0x + B\cos k_0x \tag{11}$$

where

$$Q(x) = \frac{1}{\pi}\int_0^{\infty}\frac{\cos kx}{k-k_0}dk =$$

$$= -\frac{1}{\pi}\left[\cos k_0xCi(k_0|x|) + \sin(k_0|x|)\left(\frac{\pi}{2} + Si(k_0|x|)\right)\right].$$

Here, Si and Ci are the standard sine and cosine integrals, respectively. γ is the Euler constant. Since

$$Ci(x) = \ln\gamma x + \int_0^x \frac{\cos t - 1}{t}dt,$$

a principal-value interpretation of the singular Cauchy integral in (10) is appropriate. In essence, we can obtain, with condition (5) applied,

$$Y(x)\Big|_{x\to+\infty} = 0, \tag{13}$$

$$Y(x)\Big|_{x\to-\infty} = \frac{p_o}{\rho g}k_0\int_{-\infty}^{\infty}f(\xi)\sin k_0(\xi - x)d\xi$$

$$= \frac{p_o}{\rho g}[A\sin k_0x + B\cos k_0x] \tag{14}$$

where A & B are simply the cosine and sine transforms of $p(x)$:

$$A = -k_0\int_{-\infty}^{\infty}f(\xi)\cos k_0\xi\, d\xi, \ \ B = k_0\int_{-\infty}^{\infty}f(\xi)\sin k_0\xi\, d\xi. \tag{15}$$

Then, the normalized wave elevation for all x is :

$$\frac{Y(x)}{(p_o/\rho g)} = k_0\int_{-\infty}^{\infty}f(\xi)\left[Q(x-\xi) + \sin k_0(x-\xi)\right]d\xi, \quad -\infty < x < \infty. \tag{16}$$

The resistance formula (6) can be simplified significantly by considering the energy propagation of the *dimensional* wave amplitudes of A and B:

$$R_w = \frac{1}{4}\rho g\left(\frac{p_o}{\rho g}\right)^2\{A^2 + B^2\} \tag{17}$$

or

$$C_w \equiv \frac{R_w}{p_o^2/\rho g} = \frac{1}{4}\{A^2 + B^2\} \tag{18}$$

where C_w is the non-dimensional resistance coefficient of the pressure system.

4. Computational Results

For a *symmetric* $p(x)$ function about x=0, only A in (15) is required. Thus, a zero-drag system only requires A to vanish. Evaluation of the sine transform of $f_{th}(\bar{x})$ defined by (8) requires a path integral in the complex plane and the result is given by [12]:

$$A = -\frac{2\pi k_o a \sin k_o a}{\alpha \sinh[(\pi k_o a)/2\alpha]} \qquad (19)$$

The associated C_w is given by:

$$2C_w = \frac{2\pi^2 (k_o a)^2 \sin^2 k_o a}{\alpha^2 \sinh^2(\frac{\pi k_o a}{2\alpha})} \to 8\sin^2 k_o a, \text{ as } \alpha \to \infty \qquad (20)$$

Imbedded in this result is the special case of the step distribution $f(\bar{x}) = 1$, which has a sine-square behavior with un-realistic non-decaying resistance at small Froude number.

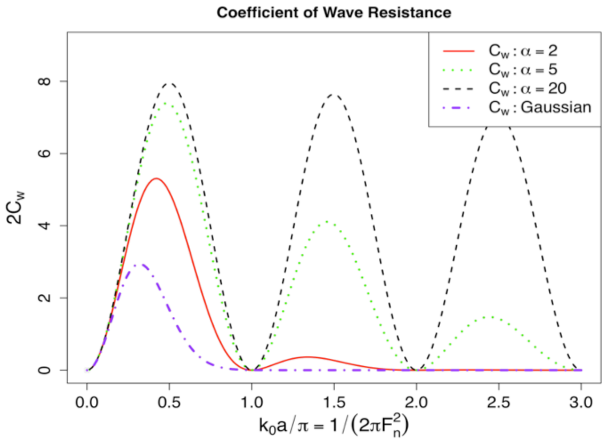


Fig. 2*: Wave-Resistance Coefficient C_w vs F_n.

The expression for C_w corresponding to the Gaussian distribution f_G (Eq. 9) is given by:

$$2C_w = 8k_o a e^{-(\beta k_o a)^2} \qquad (21)$$

These resistance functions (20-21) are plotted in Fig. 2. The Gaussian shape, being uniformly smooth, has no ability to provide "interference" within the distribution patch to generate a zero-resistance condition. On the other hand (20) does. As is well known for this hyperbolic-tangent shape (see e.g., [11]), the effect of α is to damp out the unrealistic cancellation of (large $k_0 a$) short waves. The zero-resistance points correspond to the vanishing of *A*, which is *independent* of the shape factor α. Thus, regardless of α, minimum resistance occurs at

$$k_o a = q\pi, \; q = 1,2,... \quad \text{or}$$
$$F_n = \frac{1}{\sqrt{2q\pi}} = 0.3989, 0.2821, 0.2303, \; ... \qquad (22)$$

For the case of *f(x)*=1, the step distribution, we compute the profile using (16) in the neighborhood of the first zero $F_n = 0.399$, beyond the trivial case of infinitely high speed ($k_0 a = 0$). The Cauchy integral can be treated by numerical quadrature. Figure 3 shows the behavior of the free surface as the Froude number approaches the zero-resistance point from above (3a) and from below (3b). Off the zero-resistance point, the profile is non-symmetric about x=0. When the trailing waves vanishes, the profile under pressure distribution becomes symmetric about x=0. This behavior follows from (11) as one observes that the only contribution to *Y(x)* will then come only from *Q(x)*, which is symmetric in *x*.

The wave shapes at the first three zero-resistance points are shown in Fig. 4. For the higher modes, *q=2, 3*, the elevation are sometimes above y=0, but all elevations have a negative mean value in $|\bar{x}| \le 1$, a depression.

Wave elevations for the hyperbolic-tangent distribution $f_{th}(\bar{x})$ with $\alpha = 5$, at the same zero-resistance F_n's, are shown in Fig. 5. These, as expected, are smaller in magnitude than those of the constant pressure, especially for the higher-order modes. Of worthy note is that the surface elevation has continuous slopes at $\bar{x} = \pm 1$ when α is finite, which is not the case for the step case $f(\bar{x}) = 1$.. The wave elevation is, however, always continuous.

In the Workshop, more details will be given on the behavior of the solution for the f_{th} and f_G functions, which cannot be covered completely in this abstract, as well as implications of the solutions in cavity-flows problems.

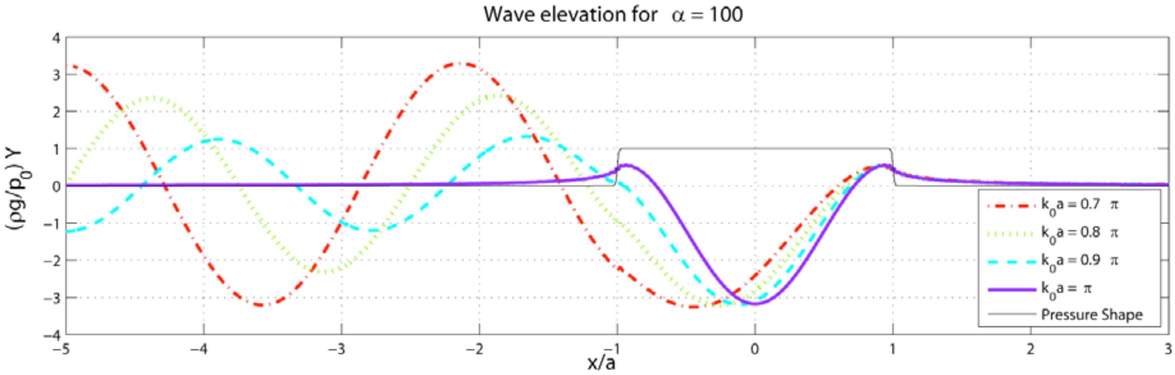
References

1. Yeung, R. W. and Bouger, Y. C., *Int'l Journal for Numerical Meth. in Engrg.*, vol. 14, pp. 317-336, 1979.
2. Bai, K. J. *Journal of Ship Res.*, 22, 4, 216-230, 1978.
3. Madurasinghe, M.A., *Journal Ship Res.* 32, 194-20,1988,
4. Yeung, R. W., *Journal of Ship Res.*, vol. 19, no. 4, pp. 581-607, 1975.
5. Makasyeyev M.V., *Proceedings of 7th International Symposium on Cavitation CAV2009.* Ann Arbor, Michigan, USA – Paper No. 116. – 9 p., 2009.
6. Makasyeyev M.V. In Coll.: Supercavitation*: progress and perspectives.* Kiev: Institute of hydrodynamics of NAS of Ukr. pp. 65–76 (In Russian), 2010.
7. Tuck, E. O., *Journal of Ship Res.* 35, 227-287, 1991
8. Kostyukov A.A., *Theory of ship waves and wave resistance.* – Leningrad: Sudpromgiz,312 p. (In Russian) 1959,
9. Wehasuen, J. V. "The Wave Resistance of Ships", *Advances- In Applied Mechanics*, vol 13, Academic Press, New York, pp. 131-174, 1973.
10. Yeung, R. W., Wan, H., Banumurthy, S. P., Ham, W. L., and Lew, J.-M., *Journal of Marine Science & Ocean Technology*, vol. 4, no.2, pp. 53-62, 2008/2009.

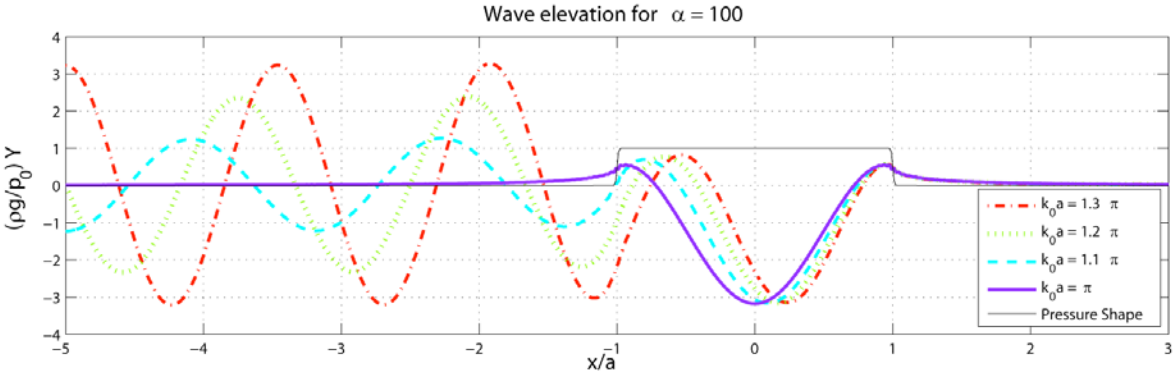
* Revised version of Aug.15, 2001 with typos corrected.

11. Yeung, R. W. and Wan, H., *Journal of Offshore Mechanics and Arctic Engineering*, vol. 130, - Issue 3, 031005 (9 p). 2008.

12. Doctors, L. J. and Sharma, S. D., *Journal Ship Research*, vol. 16, pp. 248-264.1972.



(a)



(b)

Fig. 3: Free-surface elevations near the first zero-resistance point for f(x)=1, using $\alpha = 100$.

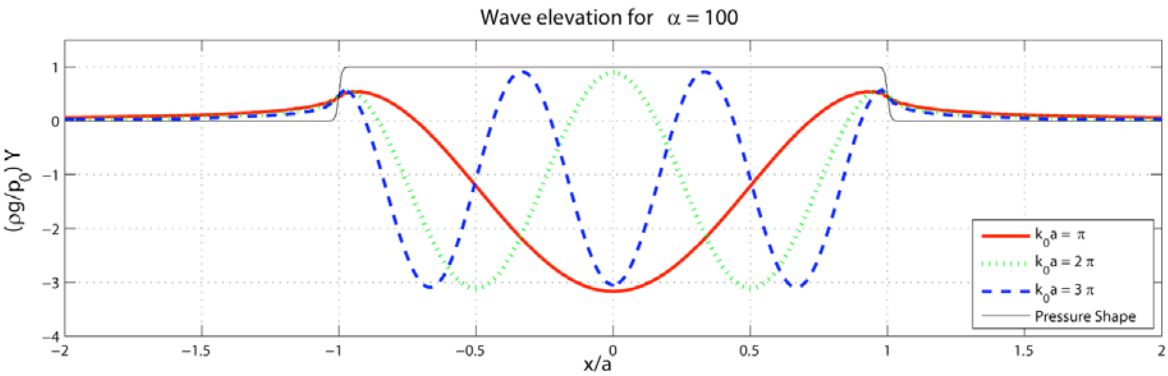


Fig. 4: First three surface shapes at zero wave-resistance points of step distribution, f(x)=1.

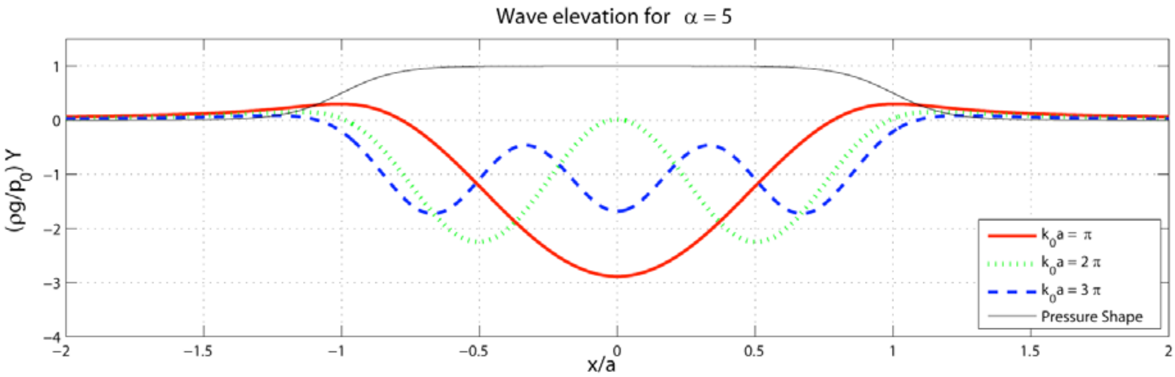


Fig. 5: First three surface shapes at zero wave-resistance points of hyperbolic-tangent distribution, $\alpha = 5$.