

The interaction factor for wave power in arrays

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1 Introduction

The interaction factor with which we are concerned may be defined as the ratio of the maximum power which may be absorbed by an array of identical wave energy devices to the maximum power available from the same number of individual members of the array acting independently of their neighbours. This has been the focus of investigation going back to work such as Budal [1977] and Evans [1980]. Much of the early work made use of the point absorber approximation, which is equivalent to assuming that the devices are small enough relative to the incident wavelength to be weak scatterers (i.e. $ka \ll 1$, where k is the wavenumber and a is the characteristic dimension of the device, e.g. radius). Hence in the calculation of interactions, scattered waves are neglected. McIver [1994] explains the assumption behind the approximation as follows: “The exciting forces on the fixed devices are what they would be if the devices were in isolation. As the devices are assumed to be identical, the only differences in exciting forces throughout the array arise from the different phase of the incident wave at each device. Hydrodynamic interactions arise through the waves radiated by the moving devices so that the off-diagonal terms in the damping matrix are not zero in general. However, the scattering of the radiated waves within the array is neglected so that the far-field of the waves radiated by a particular device is unaffected by the presence of the remaining devices.”

The point absorber approximation was also used by Fitzgerald and Thomas [2007] in their study of the interaction factors for a range of configurations. They established that the average of the interaction factor over all incident wave directions 0 to 2π is equal to unity, also illustrating this with a range of numerical results. In the present

work we are revisiting this matter, still within the context of linear hydrodynamics, but without making the point absorber assumption. We show below that the integral relation still holds more generally (we are not aware that this analysis has previously been published); we also provide numerical results which confirm this, and illustrate various influences of constructive and destructive interference on optimum power in the general case of an array of scattering devices.

2 Optimum power and the interaction factor

The time-averaged power that can be absorbed by an array of N wave energy devices, each oscillating in M modes, may be expressed

$$P = \frac{1}{4}(\mathbf{u}^* \mathbf{F}_e + \mathbf{F}_e^* \mathbf{u}) - \frac{1}{2} \mathbf{u}^* \mathbf{B} \mathbf{u} \quad (1)$$

where \mathbf{F}_e , \mathbf{u} are respectively the $NM \times 1$ vectors of complex amplitudes of the exciting forces and body velocities, \mathbf{B} is the $NM \times NM$ radiation damping matrix and $*$ denotes complex conjugate transpose. From (1) the maximum power, originally presented by Evans [1980] and Falnes [1980], may be derived as

$$P_{N,max} = \frac{1}{8} \mathbf{F}_e^* \mathbf{B}^{-1} \mathbf{F}_e \quad (2)$$

which occurs when the optimum condition

$$\mathbf{u}_{opt} = \frac{1}{2} \mathbf{B}^{-1} \mathbf{F}_e \quad (3)$$

is satisfied (provided the inverse \mathbf{B}^{-1} exists).

As shown by Newman [1976] and others, it is possible to express the radiation damping coefficients in terms of the exciting forces in waves at an angle of incidence β

$$B_{ij} = \frac{k}{16\pi J} \int_0^{2\pi} F_{e,i}(\beta) F_{e,j}(\beta) d\beta \quad (4)$$

where J is the wave energy transport defined in the standard way and k is the wavenumber.

For a single axisymmetric device well-known solutions for the maximum power absorbed due to motion in different modes are known and in particular, for heave the result is (e.g. Evans [1980])

$$P_{1,max} = \frac{J}{k} \quad (5)$$

which occurs when the body is at resonance.

The interaction factor, q , for an array of N heaving devices, may therefore be expressed as

$$q(\beta) = \frac{\frac{1}{8}\mathbf{F}_e^*(\beta)\mathbf{B}^{-1}\mathbf{F}_e(\beta)}{NJ/k} \quad (6)$$

The inverse of the matrix \mathbf{B} may be expressed

$$\mathbf{B}^{-1} = \frac{1}{\det(\mathbf{B})} \begin{bmatrix} c_{ij} \end{bmatrix} \quad (7)$$

where the cofactors c_{ij} are defined in terms of the corresponding minors M_{ij}

$$c_{ij} = (-1)^{i+j} M_{ij} \quad (8)$$

(noting that $c_{ij} = c_{ji}$ since \mathbf{B} is symmetric). The determinant may similarly be expanded along a row or column in terms of minors, where e.g. the first row expansion may be written,

$$\det(\mathbf{B}) = \sum_{j=1}^N (-1)^{1+j} B_{1j} M_{1j} \quad (9)$$

Hence, using (6) and (7) we may write, for an array of heaving axisymmetric devices,

$$\frac{1}{2\pi} \int_0^{2\pi} q(\beta) d\beta = \frac{1}{2\pi} \frac{k}{8NJ} \frac{1}{\det(\mathbf{B})} I \quad (10)$$

where

$$I = \int_0^{2\pi} \sum_{j=1}^N \sum_{i=1}^N F_i^*(\beta) (-1)^{i+j} M_{ij} F_j(\beta) d\beta \quad (11)$$

using the property that \mathbf{B} , and therefore $\det(\mathbf{B})$, is independent of the incoming wave direction.

Integrating the summation term-by-term, and noting again that \mathbf{B} , and therefore M_{ij} , is independent of β we obtain

$$I = \sum_{j=1}^N \sum_{i=1}^N (-1)^{i+j} M_{ij} \int_0^{2\pi} F_i^*(\beta) F_j(\beta) d\beta \quad (12)$$

Using (4) it is then possible to write this as

$$I = \sum_{j=1}^N \sum_{i=1}^N (-1)^{i+j} M_{ij} B_{ij} \frac{16\pi J}{k} \quad (13)$$

Now, considering (9), and recalling that it is possible to expand the determinant along any row or column, it becomes apparent that

$$I = N \det(\mathbf{B}) \frac{16\pi J}{k} \quad (14)$$

and therefore that

$$\frac{1}{2\pi} \int_0^{2\pi} q(\beta) d\beta = 1 \quad (15)$$

Simple expressions for the maximum power absorbed by an axisymmetric device moving in surge and sway, or heave, surge and sway will lead to the same result.

3 Results for simple arrays

In order to demonstrate the validity of the identity (15), the boundary element hydrodynamics program DIFFRACT was used to conduct an investigation of an array of 3 heaving hemispherical devices. Four different configurations were analysed; three triangular layouts as shown in Figure 1 and a fourth layout, with hemispheres spaced $2a$ apart in a straight line. In all cases the water depth was $10a$. The submerged body surface

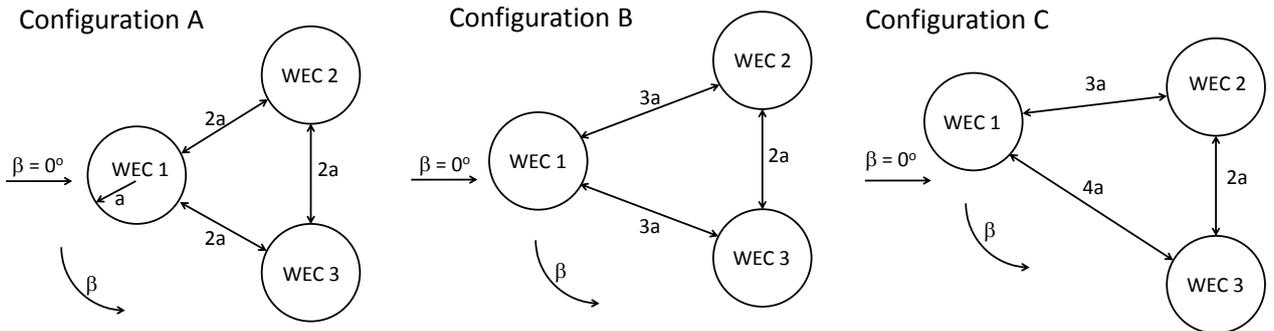


Figure 1: Triangular configurations of wave energy converters.

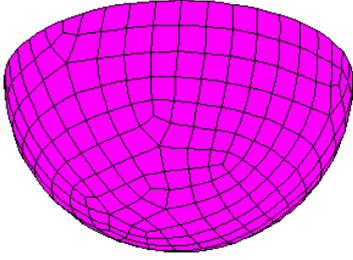


Figure 2: Body mesh for single hemisphere.

of each hemisphere was idealised by 306 quadratic elements as shown in Figure 2.

The variation of q with incident wave direction for configuration A (an equilateral triangle) is shown in Figure 3. Taking advantage of symmetry, the interaction factor is shown only for the range 0 to 60 degrees. Results are given for three non-dimensional wavenumbers $ka=0.5, 1.0$ and 1.5 . It may be noted that results obtained by Mavrakos and McIver [1997], using a full diffraction solution, suggest that the point absorber assumption should give reasonable accuracy up to $ka=0.8$ for a configuration such as this.

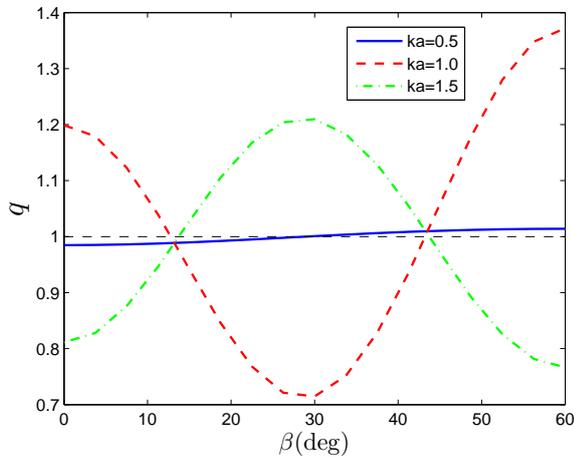


Figure 3: q factor for configuration A (equilateral triangle).

It may be seen that, except at the lowest wavenumber, the interaction factor q oscillates around unity as the angle of wave incidence is varied from 0 to 60° . Numerical integration of q for these wavenumbers leads to the results shown in Table 1. The difference from unity for the highest wavenumber is 0.06% for configuration A (based on a discretisation of 3.75°), and this is thought to be symptomatic of a lack of mesh convergence rather than an indication that the identity given in

(15) is not satisfied. Table 1 also gives equivalent results for the interaction factor integrals obtained for the two other configurations of three heaving hemispheres, and for the linear array of three devices. Similar conclusions may be drawn.

Config.	ka	$\frac{1}{2\pi} \int_0^{2\pi} q(\beta) d\beta$
A	0.5	1.0001
	1.0	1.0000
	1.5	0.9994
B	0.5	1.0001
	1.0	0.9999
	1.5	0.9987
C	0.5	1.0001
	1.0	0.9999
	1.5	0.9994
Line	0.5	1.0001
	1.0	0.9999
	1.5	0.9994

Table 1: Directional averages of the interaction factor obtained from DIFFRACT analyses.

The heave excursions of the three devices are plotted in Figure 4 over the same range of directions, non-dimensionalised by the wave amplitude A . For the wavenumbers $ka=1.0$ and 1.5 , the maximum heave remains less than about twice the wave amplitude, suggesting that for these conditions the assumption of linear dynamics is not unreasonable.

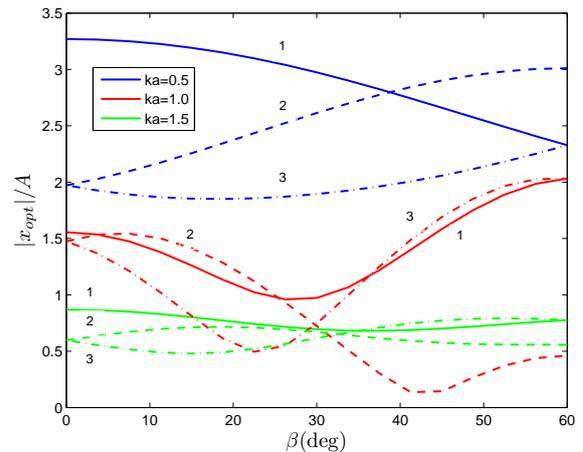


Figure 4: Excursion amplitudes of each body in configuration A.

Figure 5 shows for these wave numbers the fraction of total power absorbed by each hemisphere in configuration A, over the same range of wave

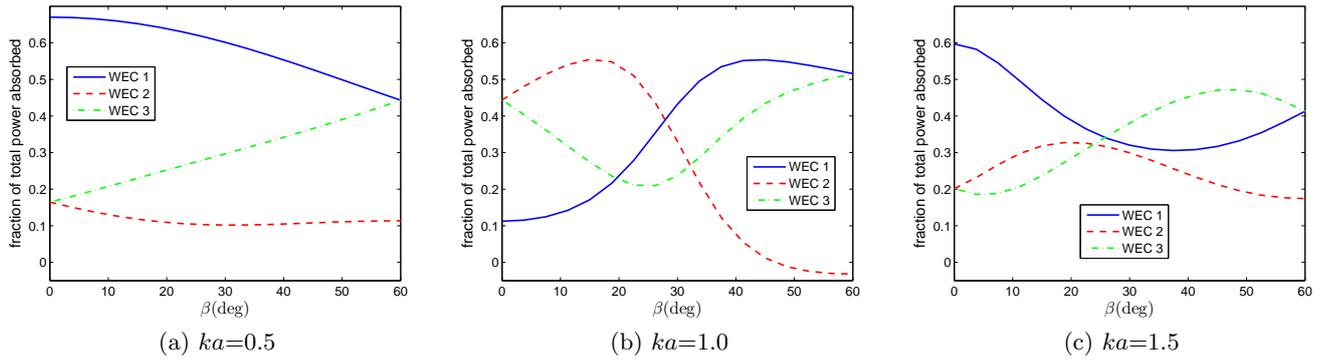


Figure 5: Partition of maximum power amongst wave energy converters.

directions. The expected symmetries may be observed. For the case $ka=1.0$ values of q for one device are negative as the incident wave angle approaches 60° . This of course indicates that the device is supplying power into the wavefield rather than absorbing it.

Figure 6 shows the angular variation of the interaction factor for configuration C. In the absence of symmetry the plot is now shown over the range 0 to 360° .

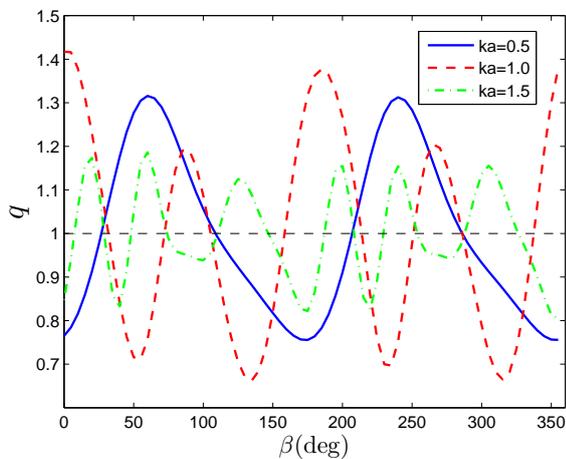


Figure 6: q factor for configuration C (scalene triangle).

The identity (15) indicates that for any wavenumber a peak in the interaction factor at a certain wave heading must be associated with unfavourable values of q at other wave headings. This is significant for array design, particularly in directionally spread seas. One should note, however, that this conclusion is based on the assumption here that each device is axisymmetric (as is the case for many contemporary wave energy converter designs).

Acknowledgement

H. Wolgamot was supported by a University of Oxford Clarendon Fund Scholarship.

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