Tank Green function with partial reflections from side walls

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The so-called tank Green function (TGF) is formulated as a formal sum of open-sea Green functions representing the infinite images between two parallel side walls of the source in the wave tank. Viscous dissipation is considered within the theory of visco-potential flow presented in Chen & Dias (2010), which gives rise to decay factor in the TGF that is not singular any more at the wave numbers associated with the transversal resonances. A constant partial reflection factor is introduced to represent the deficiency of wave energy due to the contact of the waves against the side walls along which special dampers might be installed to reduce the reflection from the walls. Furthermore, new analytical formulations involving the complementary error function is obtained to represent the truncated infinite series of the wave components of the open-sea Green function.

1. Tank Green function with partial reflections from side walls

A Cartesian coordinate system is defined by placing the xoy plane coincided with the undisturbed free surface and the oz axis is oriented positively upwards. The ox axis is coincident with the center plane of the tank whose width is denoted by b. Under the assumption of fairly perfect fluid (Chen & Dias,2010), the TGF $G(M,M')$ representing the velocity potential at a field point $M(x,y,z)$ in the wave tank due to a pulsating source of unit strength located at the point $M'(x',y',z')$, satisfies the following set of equations:

$$\nabla^2 G(M,M') = \delta(M - M') \qquad P \subset D \qquad (1a)$$

$$G_z - \bar{k}G - i4\alpha G_{zz} = 0 \qquad z = 0 \qquad (1b)$$

$$G_z = 0 \qquad z = -h \qquad (1c)$$

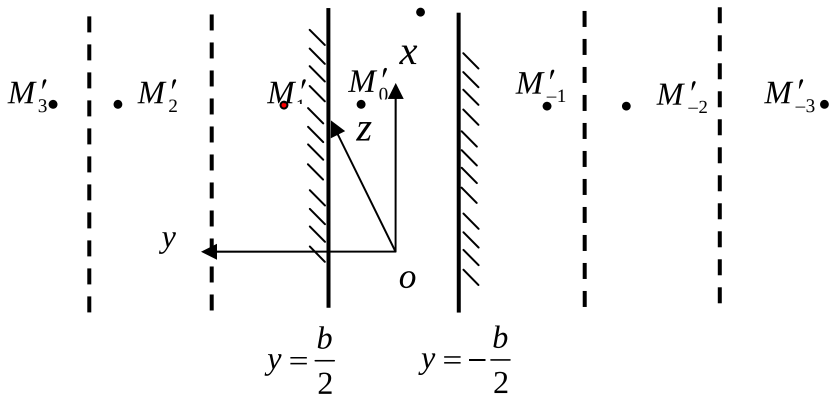
$$\left.\frac{\partial G(M,M_0')}{\partial y}\right|_{y=\frac{b}{2}} = \beta_L \frac{\partial G_0^0(M,M_0')}{\partial y} \qquad y = b/2 \qquad (1d)$$

$$\left.\frac{\partial G(M,M_0')}{\partial y}\right|_{y=-\frac{b}{2}} = \beta_R \frac{\partial G_0^0(M,M_0')}{\partial y} \qquad y = -b/2 \qquad (1e)$$

where $\delta(\bullet)$ is the Dirac function and the parameter above are defined as

$$\bar{k} = \omega^2 / g \quad \text{and} \quad \alpha = \mu\omega/(\rho g), \beta_L = 1 - a_L, \beta_R = 1 - a_R$$

with ω the frequency of pulsating source, μ the fluid viscosity and ρ the fluid density, a_L and a_R the partial reflection factor.



The solution of (1) can be obtained by considering an infinite number of images of the source between two parallel side-walls, that is

$$G\left(M,M_0'\right)=G_0^0\left(M,M_0'\right)+\sum_{n=1}^{\infty}a_L^n a_R^n\begin{bmatrix}G_{2n}^0\left(M,M_{2n}'\right)+G_{-2n}^0\left(M,M_{-2n}'\right)\\+a_R^{-1}G_{2n-1}^0\left(M,M_{2n-1}'\right)+a_L^{-1}G_{-2n+1}^0\left(M,M_{-2n+1}'\right)\end{bmatrix} \tag{2}$$

where G_n^0 is the open-sea Green function with viscous dissipation satisfying the first three equations in (1) (see Qin & Shen,2010), representing the velocity potential at the point due to the image of the source located at with the coordinate defined by :

$$y_n'=\left(-1\right)^n y'+nb \tag{3}$$

The direct computation of the infinite series is slowly convergent especially when the partial reflection factor is close to 1, which means totally reflection of the waves against side walls. The tank Green function can be regrouped into two parts which has been proved to be more computationally efficient:

$$G=G^F+G^H \tag{4}$$

with G^F a finite series:

$$G^F\left(M,M_0'\right)=G_0^0\left(M,M_0'\right)+\sum_{n=1}^{N}a_L^n a_R^n\begin{bmatrix}G_{2n}^0\left(M,M_{2n}'\right)+G_{-2n}^0\left(M,M_{-2n}'\right)\\+a_R^{-1}G_{2n-1}^0\left(M,M_{2n-1}'\right)+a_L^{-1}G_{-2n+1}^0\left(M,M_{-2n+1}'\right)\end{bmatrix} \tag{5}$$

and the remaining by the truncated infinite series

$$G^H=\sum_{n=N+1}^{\infty}a_L^n a_R^n\begin{bmatrix}G_{2n}^0\left(M,M_{2n}'\right)+G_{-2n}^0\left(M,M_{-2n}'\right)\\+a_R^{-1}G_{2n-1}^0\left(M,M_{2n-1}'\right)+a_L^{-1}G_{-2n+1}^0\left(M,M_{-2n+1}'\right)\end{bmatrix} \tag{6}$$

which represents the contribution of the source image far from the field point.

In the following, the decomposition of G^H into two single integrals and their numerical evaluation are presented.

2. Asymptotic part of TGF and integral representations

Similar to the method pointed out by Chen (1994) and adopted in Qin & Shen (2010), assuming that the lowest number 2N in (6) is large enough to neglect the evanescent part of the open-sea Green function, the asymptotic part G^H can be rewritten as the sum including two infinite single integrals:

$$G^H = Z_0(z) a_L^{N+1} a_R^{N+1} \left\{ \begin{array}{l} \sum_{l=1}^{2} c(y_l, B)\left[I_1(y_l, B, a_L, a_R) + id(X) I_2(y_l, B, a_L, a_R)/B \right] \\ + \sum_{l=3}^{4} c(y_l, B)\left[a_R^{-1} I_1(y_l, B, a_L, a_R) + i a_L^{-1} d(X) I_2(y_l, B, a_L, a_R)/B \right] \end{array} \right\} \tag{7}$$

in which

$$Z_0(z) = \frac{-ik_0}{2k_0 h + \sinh 2k_0 h} \cosh k_0 (z+h) \cosh k_0 (z'+h) \tag{8}$$

$$c(y_l, B) = e^{i(2By_l - \pi/4)} / \left(\pi B^{1/2}\right) \tag{9}$$

$$d(X) = \left(4X^2 - 1\right)/8 \tag{10}$$

$$\begin{array}{ll} B = (k_0 + i4\alpha k_{0I}) b & ; \quad X = (k_0 + i4\alpha k_{0I})(x - x') \\ y_1 = N + 1 - (y - y')/2b & ; \quad y_2 = N + 1 + (y - y')/2b \\ y_3 = N + 1/2 - (y + y')/2b\,; & \quad y_4 = N + 1/2 + (y + y')/2b \end{array}$$

Finally, the two infinite integrals are given by:

$$I_1(y_l, B, a_L, a_R) = \int_0^\infty \frac{t^{-1/2} e^{-y_l t}}{1 - a_L a_R e^{i2B - t}} dt \tag{11a}$$

$$I_2(y_l, B, a_L, a_R) = \int_0^\infty \frac{t^{1/2} e^{-y_l t}}{1 - a_L a_R e^{i2B - t}} dt \tag{11b}$$

Using the Taylor development of e^{-t} in the denominator of two infinite integrals (11), the single integral I_1 and I_2 defined in(11) is approximated as follows:

$$\tilde{I}_1 = \pi e^A erfc\left(\sqrt{A}\right) / \sqrt{\left(1 - e^{i2B}\right) e^{i2B}} \tag{12a}$$

$$\tilde{I}_2 = \left[\sqrt{\pi / A} - \pi e^A erfc\left(\sqrt{A}\right) \right] \sqrt{\left(1 - e^{i2B}\right) / e^{i6B}} \tag{12b}$$

with $A = y_l \left(a_L^{-1} a_R^{-1} e^{-i2B} - 1 \right)$ and erfc() the complementary error function.

3. Discussions and concluding remarks

A number of studies have been performed to evaluate the side wall effects in a wave tank, such as those done by Eatock Taylor & Hung(1985), Yeung & Sphaier (1989), Kashiwagi (1991), McIver (1993), Linton (1993), Chen (1994), Xia (2002), viscous dissipation effect and side wall partial reflection effect are not considered in these previous studies.

Qin & Shen (2010) have developed Green function with viscous effect. In this paper, new formulations of the tank Green function with viscous effect and side wall partial reflection in water of finite depth are developed within the linear theory of visco-potential flow. The construction of the tank Green function by an infinite series of open-sea Green function to evaluate the partial reflection side wall effects in wave tanks is summarized in the paper. Two sources of dissipation are included in this new formulation, one is due to the fluid viscosity which is absent in the classical inviscid flow and the other is due to the partial reflection of the side walls on which special dampers might be installed. The present study on the TGF including viscous dissipation and partial reflection side wall effect is expected to give important insight on the realistic effect of side walls in wave tanks and to be able to provide closer results to the measurement of model tests.

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