Numerical Study of the Second-Order Wave Loads on a Ship with Forward Speed

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In the 25th workshop hold in Harbin, China, we have presented a domain-decomposition based method to solve the second-order diffraction/radiation problems with a current or small forward speed [1]. The boundary value problem in the inner domain is formulated in a body-fixed coordinate system, while inertial coordinate system is applied in the outer domain. The continuity of the velocity potential and the normal velocity at the control surface act as the matching conditions of the inner- and outer-domain solutions. The highlight of the method is twofold. Firstly, no higher-order derivatives appear in the body boundary conditions and thus the m_j-terms and the derivatives of the m_j-terms are avoided. Secondly, because the body boundary condition is formulated on the instantaneous position of the body, the resulting integral equation is valid for both smooth bodies and bodies with sharp corners. What we have to pay for using this method is that the number of unknowns has been increased, since singularities with unknown strengths are distributed on the control surface. See also [2], [3] for details. In the present study, we will use body-fixed coordinate system not only near the body but also in finite distance away from the body. That means the outer domain in the previous studies disappears and no control surface is needed. Perturbation method is used so that the computational domain remains unchanged. Infinite water depth will be considered.

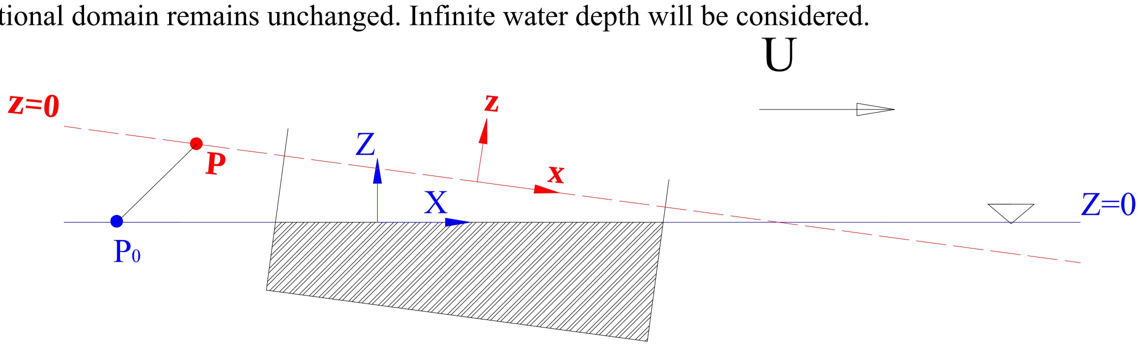


Fig.1. Definition of the problem

The fully-nonlinear formulation of the free-surface conditions in a non-inertial coordinate system can be found in, for instance [4], as

$$\eta_t = \phi_z - \phi_x\eta_x - \phi_y\eta_y - (\vec{U}' + \dot{\vec{\xi}}' + \vec{\omega}' \times \vec{r}') \cdot (-\eta_x, -\eta_y, 1) \qquad \text{on } z=\eta(x,y,t), \quad (1)$$

$$\phi_t = -\frac{1}{2}\nabla\phi\cdot\nabla\phi + (\vec{U}' + \dot{\vec{\xi}}' + \vec{\omega}' \times \vec{r}') \cdot \nabla\phi - U_g \qquad \text{on } z=\eta(x,y,t). \quad (2)$$

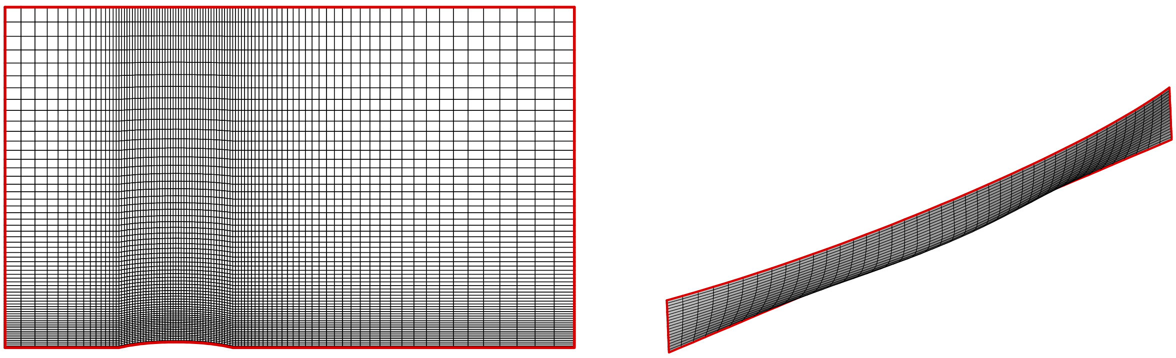
Here the subscripts x, y, z, and t indicates partial differentiation. $\vec{r}' = (x, y, \eta)$ is the position vector of a point on the free surface. $\dot{\vec{\xi}}'$ and $\vec{\omega}'$ are translatory and rotationary body motions, respectively. All the vectors are described in the body-fixed coordinate system, i.e. oxyz in Fig.1. The gradients are taken with respect to x, y and z, i.e. $\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$. U_g is the gravity potential. The free-surface conditions (1) and (2) are then approximated by introducing Stokes expansion and Taylor expanding the free-surface conditions about the oxy-plane. One should note that the oxy-plane is not necessarily the same as the calm water surface, i.e. OXY-plane in Fig.1. The oxy-plane coincides with the calm water surface when the body is at rest, and translates and rotates with the body. As shown in Fig.1, a point P_0 initially on the calm water surface will move to point P due to unsteady rigid-body motions. The Taylor expansion will not be valid if the distance $|P_0P|$ is not small compared with the characteristic dimensions of the ship (i.e. length, beam and draft), which may occur at a point far away from the body undergoing pitch/roll motions with finite amplitudes. If that happens, how could we use the body-fixed coordinate system in the whole computational domain? Our arguments are as follows: The perturbation scheme assumes that the wave amplitude and body motions are asymptotically small. In that sense, if we truncate the computational domain at a finite distance away from the body, the displacement of a point fixed on the oxy-plane (e.g. $|P_0P|$ in Fig.1) would always be small compared with the dimensions of the ship. On the other hand, if asymptotic theory is used, the Response Amplitude Operators (RAOs) of the linear results and Quadratic Transfer Functions (QTFs) of the second-order results are independent of the wave amplitudes and the ship motions. How good the asymptotic theory is needs comparison with experiments. However, experiences in ship and offshore hydrodynamics showed that the asymptotic theories are very powerful tools.

The body-boundary conditions formulated in the body-fixed coordinate system are of very simple forms without any derivatives on the right-hand sides:

$$\frac{\partial \phi^{(m)}}{\partial n} = \vec{n}' \cdot \left(\dot{\vec{\xi}}'^{(m)} + \vec{\omega}'^{(m)} \times \vec{r}' + \vec{U}'^{(m)} \right), \text{ m=1, 2, on SB.} \quad (3)$$

$\vec{U}'^{(k)}$ (k=1, 2) is the steady forward speed vector in the body-fixed coordinate system. If the Boundary Value Problem (BVP) is formulated in the inertial coordinate system, we will have higher-order derivatives on the right-hand side of (3), some of which may be not integrable for bodies with sharp corners.

A time-domain Higher-Order Boundary Element Method (HOBEM) based on cubic shape functions [5] is used as a numerical tool to solve the BVP. A 3-point upwind Finite Difference Method (FDM) is used for the calculation of the spatial derivatives in the free-surface conditions. Based on a Fourier-von Neumann stability analysis using Neumann-Kelvin linearization of the free-surface conditions, it can be shown that this scheme gives much larger stability region than that using the cubic shape functions. The other derivatives are still based on cubic shape functions, but only the 'up-stream' elements are used for stability reasons. A low-pass filter is applied near the waterline to suppress the saw-tooth behavior, which actually is stable. The wave field is decomposed into two parts, i.e. the incident wave part and the scattered wave part. The description of the incident wave field consistent to second order in a non-inertial coordinate system is made. Only the scattered wave part is solved as unknown.



(a) (b)

Fig.2. Meshes on the free surface and the wetted mean ship surface. Due to symmetric properties, only half of the free surface and ship surface are discretized. (a) Meshes on the half of the free surface. (b) Mesh on half of the Wigley hull.

Linear seakeeping analysis

Numerical examples we will report at the workshop is for Wigley Hull I with different Froude numbers in head-sea waves. The ship has length, beam and draft as 3m, 0.3m and 0.1875m, respectively. The amidships section coefficients is C_m=0.9090. See [6] for details. An example of the meshes on half of the free surface and wetted mean ship hull is shown in Fig.2. The linear hydrodynamic coefficients, excitation forces, and vertical ship motions are compared with the experimental results given in [6], showing good agreement. A strong resonant heave and pitch amplification occurs. Depicted in Fig.3 and Fig.4 are the RAOs and the corresponding phase angles for the heave and pitch motions. The phase angle is defined relative to the incident wave elevation amidships. The studied Froude number is Fr=U/$\sqrt{gL}$=0.3. The double-body flow is used as the basis flow. The double-body flow in the body-fixed coordinate system has different interpretation but the same solution as that in an inertial coordinate system.

Added resistance

The added resistance, which is simply the mean drift force at forward speed, is also studied for the Wigley Hull I. The comparison with the experimental results [6] for Fr=0.3 are shown in Fig.5. Head-sea waves are considered. It has been shown theoretically in, for instance [7, 8], that the second-order velocity potential does not contribute to the horizontal mean-drift forces. Our second-order solutions seem to agree with their conclusion, since the contributions from the second-order velocity potential are much smaller compared with the other components. The non-dimensional added resistance in the resonant heave and pitch domain is large while the non-dimensional added resistance in small wave lengths is small relative to the corresponding values for common ship forms. The fact that the measured added resistance for the smallest wave length is negative is difficult to explain.

Generalized second-order wave excitation of 2-node springing

The wave-induced sectional loads on ships are often analyzed by the blended method, which is based on the linear solution with nonlinear corrections for the Froude-Krylov and the restoring forces. Slamming-type of loads may also be added. However, the nonlinearities in the wave radiation and diffraction are not considered in this type of analysis. How important is the nonlinear wave radiation/diffraction as the excitation of nonlinear ship springing still remains as unknown.

Due to the fact that the structural natural frequencies of real ships are high, the second-order ship springing only occurs in the relatively short wave region, e.g. $\lambda/L \le 0.3$, where the ship motion is very small as it can be seen from the RAOs in Fig.3 and Fig.4. λ is the linear incident wave length. L is the ship length. Miyake et al. [9] found experimentally for a modified Wigley model that the springing of super (n-th) harmonic resonance due to the nonlinear higher hydrodynamic forces occurred, although the model is simple mathematical hull form without bulbous bow. As a starting point, we have studied the 2[nd]-order wave diffraction of the Wigley hull traveling in the regular head-sea waves with $0.25 \le \lambda/L \le 0.5$. The generalized second-order excitation of 2-node mode in the vertical plane is studied. Another equally important issue to ship springing is the damping ratio, which is not the focus of the present study. Presented in Fig.6-8 are the results for Fr=0.18, 0.20, and 0.22, respectively. $F_{7,a}^{(2)}$ is the amplitude of the total generalized second-order excitation of 2-node mode in the vertical plane. $F_{7,p2}^{(2)}$ and $F_{7,q}^{(2)}$ are the contributions from the second-order velocity potential and the quadratic velocity terms in the Bernoulli's equation, respectively. It is immediately apparent that the second-order velocity potential gives dominating effects over the quadratic terms. Comparison of the results for different Froude numbers (Fr=0.18, 0.20, 0.22) also suggests that the second-order excitation has a strong dependence on the Froude number for small wave lengths.

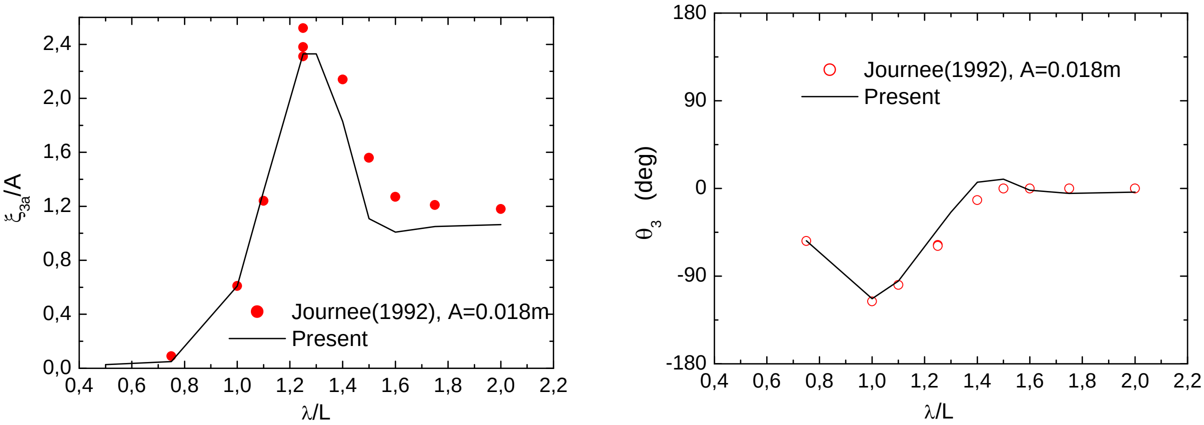


Fig.3. The amplitude and phase angle of the heave motion of Wigley hull I in head sea. Fr=0.3. The ship is restrained from surging and free to heave and pitch.

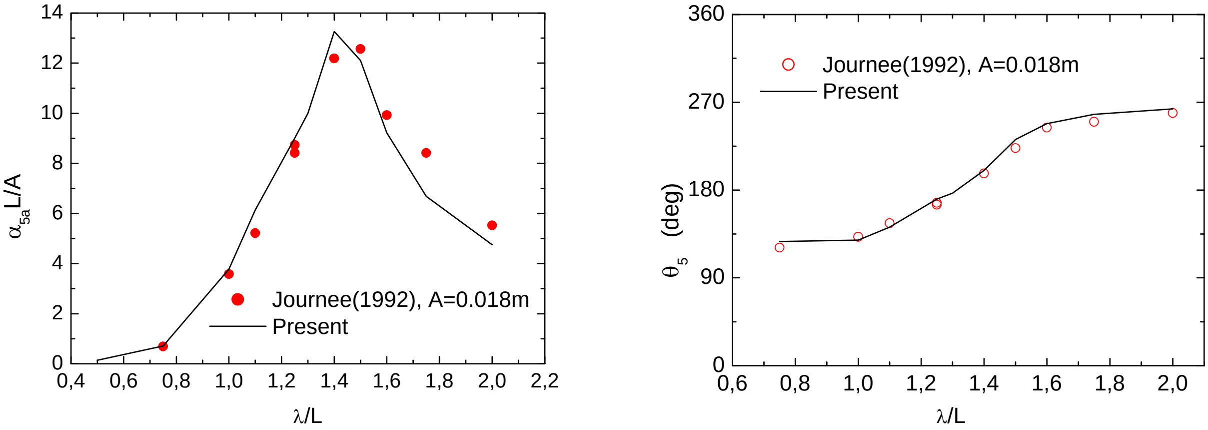


Fig.4. The amplitude and phase angle of the pitch motion of Wigley hull I in head sea. Fr=0.3. The ship is restrained from surging and free to heave and pitch.

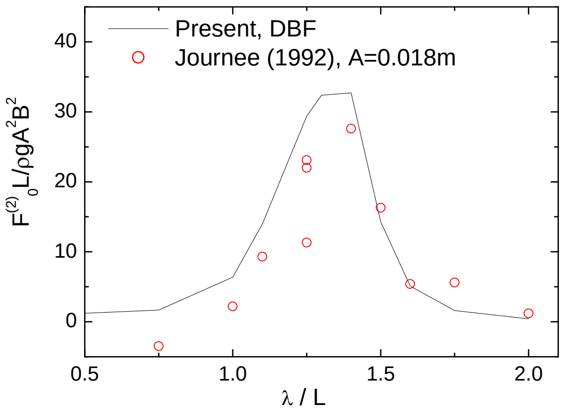


Fig.5. Added resistance on Wigley Hull I in head sea. Fr=0.3.

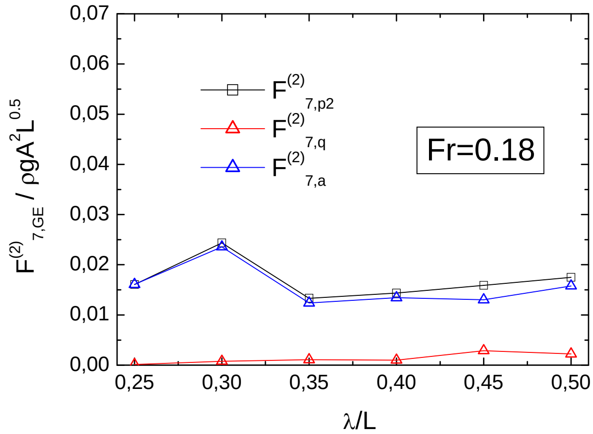


Fig.6. 2nd-order generalized excitation for 2-node mode in vertical plane with different λ/L ratio. Fr=0.18.

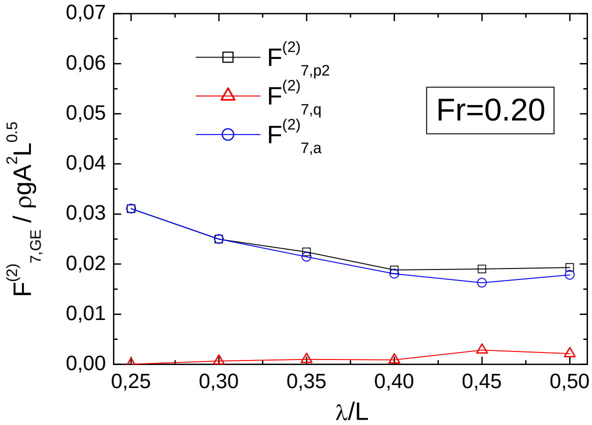


Fig.7. 2nd-order generalized excitation for 2-node mode in vertical plane with different λ/L ratio. Fr=0.20.

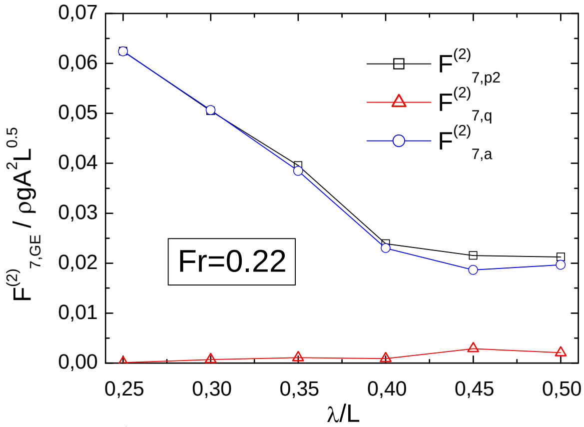


Fig.8. 2nd-order generalized excitation for 2-node mode in vertical plane with different λ/L ratio. Fr=0.22.

Acknowledgements

This work is supported financially by the Research Council of Norway through Centre for Ships and Ocean Structures.

References

[1]. Shao, Y.L. and Faltinsen, O.M. (2010). Numerical Study on the Second-Order Radiation/Diffraction of Floating Bodies with/without Forward Speed. *25th Int. Workshop on Water Waves and Floating Bodies.*

[2]. Shao, Y.L. and Faltinsen, O.M. (2010). Use of Body-Fixed Coordinate System in Analysis of Weakly-Nonlinear Wave-Body Problems. *Appl. Ocean Res.*, 32, 1, 20-33.

[3]. Shao, Y.L. and Faltinsen, O.M. (2010). Second-Order Diffraction and Radiation of A Floating Body with Small Forward Speed. In: *Proc. of the 29th international conference on ocean, offshore and arctic engineering*, OMAE2010-20073. 2010.

[4]. Faltinsen, O.M. and Timokha, A.N. (2009). *Sloshing*, Cambridge University Press.

[5]. Shao, Y.L. (2010). Numerical Potential-Flow Studies on Weakly-Nonlinear Wave-Body Interactions with/without Small Forward Speeds. Ph.D thesis. Department of Marine Technology, NTNU, Trondheim, Norway.

[6]. Journee J.M.J. (1992). Experiments and Calculations on 4 Wigley Hull Forms in Head Sea. Technical Report 0909, Delft University of Technology, Mekelweg 2, 2628 Delft.

[7]. Grue J. and Palm E. (1990). Mean Forces on Floating Bodies in Waves and Current. *5th Int. Workshop on Water Waves and Floating Bodies.*

[8]. Ferreira M.D. (1997). Second-Order Steady Forces on Floating Bodies with Forward Speed. PhD Thesis, MIT, Cambridge, MA.

[9]. Miyake, R., Matsumoto, T., Zhu, T. and Abe, N., (2008). Experimental Studies on the Hydroelastic Response due to Springing Using A Flexible Mega-Container Ship Model. *8th International Conference on Hydrodynamics*, Nantes.