Linear wave-structure interaction using overset grids

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Background

This abstract describes recent progress in the development of a finite-difference model for non-linear water waves and wave-structure interaction. Our central objective is to produce a computational tool that can accurately and efficiently simulate the interactions between waves and maritime structures. Amongst other applications, this tool will be used to analyse the performance of ocean wave energy devices featuring complex geometries.

Engsig-Karup et al. (2009) have recently developed a single-block, finite-difference potential flow model to represent fully non-linear water wave propagation and development up to the point of wave breaking. A range of validation tests have confirmed the stability, accuracy, efficiency, and robustness of this model. Of particular significance are the adoption of arbitrary-order spatial finite-difference schemes to maximise accuracy, and the use of a multigrid solver to ensure that solution effort scales linearly with problem size.

The focus of present work is to extend this model to support generalised curvilinear boundaries. This is necessary to permit the representation of structures with complex geometries, floating in water of varying depth. To achieve this, the overset approach to grid generation is employed, allowing accurate and efficient implementation of high-order algorithms on a collection of simply-generated, overlapping, structured grids. This paper addresses preliminary work to model wave-structure interaction on a simple two-dimensional, two-block grid domain using the Overture software framework. The added mass and damping of a half-submerged, heaving cylinder is determined by evaluating the force exerted on the body in response to a prescribed displacement. These numerical results are compared with an analytical solution.

Problem formulation

A Cartesian coordinate system is adopted with the xy-plane located at the free surface and the z-axis directed upwards. The depth of the undisturbed fluid is defined as $h(\mathbf{x})$, where $\mathbf{x} = (x, y)$ is the horizontal coordinate vector. The free-surface elevation is $z = \eta(\mathbf{x}, t)$. Assuming that the flow is inviscid and irrotational, the fluid velocity $(u, v, w) = (\nabla\phi, \partial_z\phi)$, can be expressed as the gradient of a scalar velocity potential $\phi(\mathbf{x}, z, t)$, where $\nabla = (\partial_x, \partial_y)$. The temporal development

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of the free surface is governed by kinematic and dynamic boundary conditions, here expressed in terms of the free surface variables $\tilde{\phi} = \phi(\mathbf{x}, \eta, t)$ and $\tilde{w} = \partial_z \phi\big|_{z=\eta}$:

$$\partial_t \eta = (1 + \nabla\eta \cdot \nabla\eta)\, \tilde{w} - \nabla\eta \cdot \nabla\tilde{\phi}, \tag{1}$$

$$\partial_t \tilde{\phi} = -g\eta - \frac{1}{2}\left(\nabla\tilde{\phi} \cdot \nabla\tilde{\phi} - (1 + \nabla\eta \cdot \nabla\eta)\, \tilde{w}^2\right), \tag{2}$$

In the linear case, where it is assumed that the wave elevation is small compared to the wavelength, the kinematic and dynamic boundary conditions simplify to:

$$\partial_t \eta = \tilde{w}, \tag{3}$$

$$\partial_t \tilde{\phi} = -g\eta. \tag{4}$$

With a knowledge of $\tilde{\phi}$ and η, and assuming an incompressible flow, these equations may be developed in time by solving Laplace's equation in the fluid domain subject to a zero-flow condition at the impermeable boundaries:

$$\phi = \tilde{\phi}, \quad z = \eta, \tag{5}$$

$$\nabla^2\phi + \partial_{zz}\phi = 0, \quad -h \le z \le \eta, \tag{6}$$

$$(\mathbf{n}, n_3) \cdot (\nabla, \partial_z)\phi = 0, \quad (\mathbf{x}, z) \in \partial\Omega, \tag{7}$$

where $(\mathbf{n}, n_3)$ is a vector pointing outwards normal to the solid boundary surface $\partial\Omega$.

Overset grids

The governing equations stated above have been solved numerically using the overset grid methodology. According to this approach, the fluid domain is decomposed into a series of overlapping, structured grid blocks (see Figure 1). Blocks with curvilinear edges or surfaces are created at the fluid boundaries to represent the geometrical profile of floating structures and the bottom. Spatial derivatives in the curvilinear, physical domain can be expressed in terms of a weighted sum of operators on a square, unit-spaced computational domain. The weightings applied to the operators on the computational grid depend on the physical geometry, but need only be calculated once when solving the linear wave problem. A comparatively small number of points along the block boundaries serve to interpolate the solution between grid blocks. Following this method, a linear system of equations is formed incorporating information from all of the component grid blocks, and solved in the conventional manner using direct or iterative methods. The Overture software framework developed at Lawrence Livermore National Laboratories provides a convenient tool for solving partial-differential equations on multiple curvilinear blocks (Henshaw 2008). This collection of high-level C++ libraries includes a powerful grid generator and advanced parallel-processing capabilities. Using this framework it is presently possible to implement spatial schemes to fourth-order accuracy.

Heaving cylinder test case

The governing equations described above have been solved in two dimensions using a method of lines approach with fourth-order accuracy in space and time. Centred, five-point, finite-difference schemes are developed on the computational grid using Taylor-series expansions. With

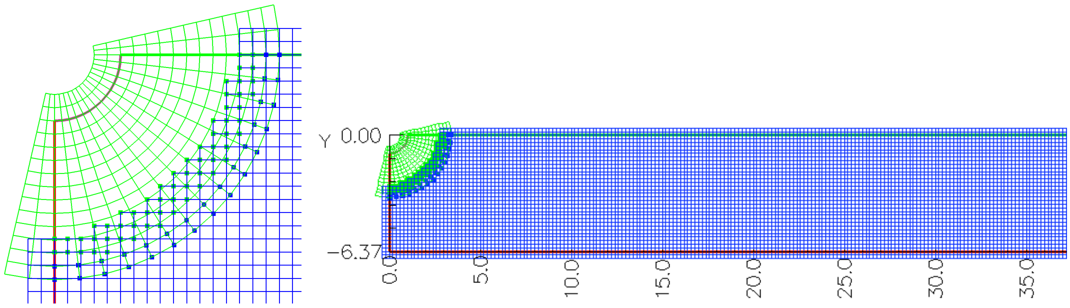

Figure 1: Details of the simple two-block overlapping grid used to simulate a heaving cylinder. The interpolation points are identified as blue and green dots.

a knowledge of the transformation function relating the physical and computational domains, the computational grid operators are weighted to satisfy the appropriate governing equations on the physical grid. Laplace's equation is satisfied at all of the internal and boundary points of the fluid with the exception of the free surface, where a Dirichlet boundary condition is applied directly. To maintain centred schemes throughout the fluid domain, ghost points are introduced at each block boundary. These points may be used to enforce Neumann boundary conditions at solid surfaces, or may provide the interpolation points required to transfer information between blocks. The resulting linear system of equations is solved directly. Having evaluated the potential field, the free-surface conditions are advanced using the classical fourth-order Runge-Kutta scheme.

Following this approach, the linear fluid motions and surface waves induced by a half-submerged heaving cylinder have been simulated using a two-block model implemented within Overture. The physical domain consists of a rectangular tank 120 m long by 6.4 m deep, with a quarter circle of radius 1 m in the top left hand corner (see Figure 1 for detail). Under the linear assumption, vertical motion of the cylinder can be represented as a non-homogeneous boundary condition at the cylinder surface. A Gaussian range of wavelengths was introduced by imposing the cylinder heave displacement profile shown in Figure 2. The pressure at the cylinder surface was evaluated and integrated to give the force acting on the cylinder in response to the displacement. After performing a discrete Fourier transform on the displacement and force signals, the real and imaginary parts of the ratio of these signals describe the added mass and damping of the cylinder respectively. As illustrated in Figure 3, the numerical results obtained compare well with the analytical results of Greenhow and Ahn (1988). Note that the discrepancy observed at low values of $\sqrt{kR}$ is expected as the analytical result assumes an infinite fluid depth. This model is presently being extended to three dimensions.

References

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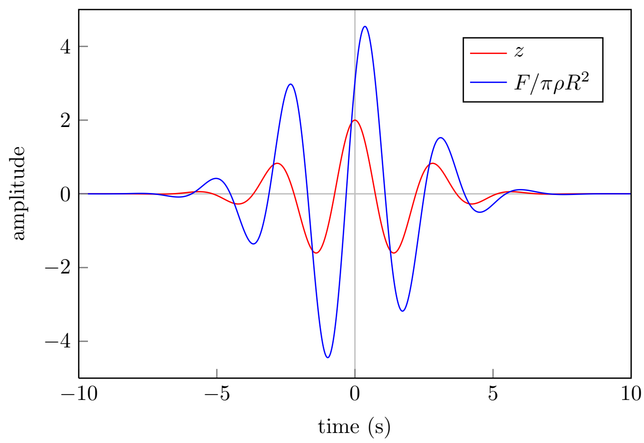

Figure 2: The displacement and force profiles associated with the heaving cylinder simulation.

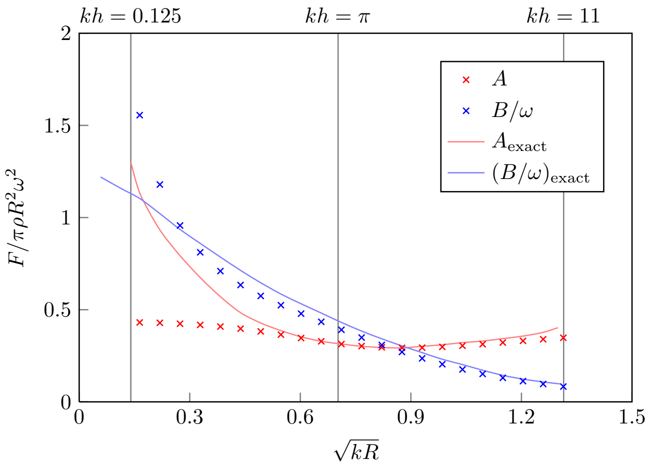

Figure 3: Comparison of analytical and numerical results for the added mass and damping of a half-submerged heaving cylinder.

Acknowledgements

The authors wish to thank the Danish Agency for Science, Technology and Innovation (grant # 09-067257) for funding, and the Danish Center for Scientific Computing for supercomputing resources.