LARGE AMPLITUDE MOTIONS AND LOADS USING A NON-LINEAR 2D APPROACH

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INTRODUCTION AND BACKGROUND

The analysis of the response of a vessel sailing in large amplitude waves is nowadays mainly conducted using linear methodologies. Linear methods are accurate for moderate sea states, but they give poor results when the wave elevation increases. This is due to the assumptions which the theories are based on. Linear theories are based on the assumption of small oscillations which limits their theoretical applicability to the prediction wave exciting forces and resulting motions in moderate seas condition. Under large amplitude waves a ship is subjected to extreme motions and loads, for example bow motions could be more than double the wave height, with the occurrence of slamming and green water on the deck. Impulsive effects and multi-harmonic behaviour and asymmetrical response characterise the wave loads distribution. In these conditions both the ship motions and wave loads are strongly non-linear and the use of a linear methodology to model the hydrodynamic behaviour of a ship in large amplitude waves could lead to inaccurate predictions. Therefore the assessment of seakeeping behaviour of a vessel sailing in high sea states should be carried out using a non-linear methodology.

Non-linear methods are very different from each other depending upon their mathematical formulations and the underpinning assumptions. The ISSC I.2 committee classified the non-linear seakeeping methodologies into four groups (Hirdaris S.E. et al., 2009). The first group consists of the "Froude-Krylov non-linear" methods. This is a simpler non-linear approach: the hydrodynamic forces are linear and all the non-linear effects are associated with the restoring and the Froude-Krylov forces. The second and the third groups include the "Body non-linear" and "Body-exact" methodologies. In these methods the radiation problem is non-linear, and solved partially in the time domain and partially in the frequency domain using a retardation function and a convolution integral. The difference between the two classes is that "Body non- linear" approach solves the radiation problem using the calm water level as a free-surface and the "Body exact" uses the oncoming wave pattern as a free-surface for the solution of the radiation problem. The last group includes the "Fully non-linear" methods; these techniques are the most complex and the most computationally intensive. They have no linear simplifications and the solution of the equations of motion is carried out directly in the time domain. The hydrodynamic problem is solved using a MEL (Mixed Euler-Lagrange) approach, they are based on the assumption of "smooth waves" therefore the wave breaking phenomena cannot be modelled. For this reason these methodologies could be instable due to the presence of wave breaking in large amplitude waves.

The aim of this project is to develop a fast and reliable seakeeping tool for the analysis of ship motions and loads subjected to large amplitude waves. The proposed method should be fast enough to be used during preliminary design and optimisation, and should be still able to model the non-linear forces accurately. The approach chosen is a two dimensional blended method. The equations of motion are numerically solved in the time domain. The radiation problem is non-linear and solved using the actual wetted hull surface at each time step and the oncoming wave pattern as the free surface, the non-linear formulation of the hydrodynamic sectional forces enables the introduction of the impulsive and lift components. The restoring and wave exciting forces are non-linear and calculated directly in the time domain. This paper describes the mathematical formulation of the proposed method for the coupled heave and pitch motions. The non-linear hydrodynamic formulations are applied to the S-175 Container Ship and the results of the computations are compared with those obtained from the formulations based on two- and three- dimensional linear approaches.

MATHEMATICAL FORMULATION

The ship is described as a rigid body with two degrees of freedom in heave and pitch motions.

The hydrodynamic forces are formulated on a 2-D strip of the hull using the potential flow assumption and using the principles of Newtonian dynamics, i.e. by assuming that the rate of change of momentum with time inside the fluid volume is equal and opposite to the sum of the external forces acting on the fluid volume. Combining the potential flow properties with the definition of momentum inside the fluid and its rate of change with time (Xia J. et al., 1998) it is possible to obtain an equation used to evaluate the sectional fluid force acting on the body.

$$\frac{d}{dt}\iint_S \rho\varphi\underline{n}ds = -\rho\iint_S\left[\left(\frac{p}{\rho}+gz\right)\underline{n}+\nabla\varphi\left(\frac{\partial\varphi}{\partial\underline{n}}-U_n\right)\right]ds \tag{1}$$

Where S is the boundary of the fluid domain, φ is the velocity potential and U_n is the component of the body forward speed normal at the surface S. Considering the potential distribution and the geometrical properties of the boundary of the fluid domain it is possible to reduce equation (1) to the integral of the fluid pressure over the prismatic hull surface S_H.

$$\iint_{S_H} p\underline{n}ds = -\rho\left(\frac{d}{dt}-U\frac{\partial}{\partial x}\right)\iint_{S_H}\varphi\underline{n}ds - \iint_{S_H}\rho gz\,\underline{n}ds \tag{2}$$

Where p is the fluid pressure along the hull section, U is the forward speed of the body and $\underline{n}$ is the normal vector at the surface S_H,

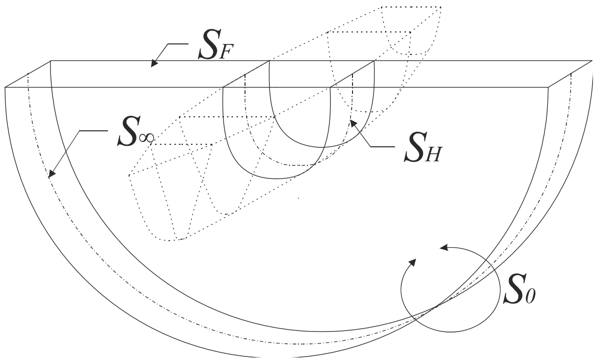

Figure 1 – Boundaries of the fluid domain

In this time domain application the velocity potential has a different formulation from the classical frequency domain approach. Here the potential is described by two terms: an impulsive terms and convolution integral in the history of motions, as proposed by Cummins (Cummins W.E., 1962).

$$\varphi_j(x,y,z;t) = \psi_j(x,y,z;t)V_j(x,t) + \int_{-\infty}^{t}\chi_j(x,y,z;t-\tau)V_j(x,\tau)\,d\tau \quad j = 3,5 \tag{3}$$

The first term of the right hand side of equation (3) is an impulsive term related to an instantaneous impulse of displacement in *j-th* motion. The second part is a convolution integral in the history of motion and describes the wave field radiated away from the body. In order to calculate the velocity potential described in equation (3) the following two boundary value problems must be solved.

$$\begin{cases} \nabla^2\psi_j = 0 & in\ the\ fluid \\ \frac{\partial}{\partial n}\psi_j = n_j & on\ S_H \\ \psi_j = 0 & on\ S_F \end{cases} \tag{4}$$

$$\begin{cases} \nabla^2\chi_j = 0 & in\ the\ fluid \\ \frac{\partial^2}{\partial\tau^2}\chi_j + g\frac{\partial}{\partial z}\chi_j = 0 & on\ S_F \\ \frac{\partial}{\partial n}\chi_j = 0 & on\ S_H \\ \chi_j(0) = 0 & \\ \frac{\partial}{\partial\tau}\chi_j(0) = -g\frac{\partial}{\partial z}\psi_j & on\ S_F \end{cases} \tag{5}$$

Both systems are solved at each time step using the actual wetted hull surface and the oncoming wave elevation as free surface plane. The solution for the impulsive boundary value problem is given by analogy solving the problem of a body oscillating at infinite frequency. The memory effect term is not solved directly in the time domain, but is found using the inverse Fourier transform of the sectional damping coefficient in the frequency domain.

Equations (4) and (5) are numerically solved using the boundary elements method. Since the hydrodynamic forces are non-linear, the boundary value problem should be solved for the same section for different combinations of immersion and heel angle. In order to avoid numerical problems due to the occurrence of irregular frequencies the direct method approach introduced by Sclavounos (Sclavounos P.D. et al., 1985) has been used.

$$-\frac{1}{2}\varphi(\underline{x}) + \frac{1}{2\pi}\int_{S_H}\varphi(\underline{\xi})\frac{\partial G(\underline{\xi},\underline{x})}{\partial n_{\underline{\xi}}}d\xi = \int_{S_H} G(\underline{\xi},\underline{x})\frac{\partial\varphi(\underline{\xi})}{\partial n_{\underline{\xi}}}d\xi \tag{6}$$

Where $G(\underline{\xi},\underline{x})$ is the two dimensional Green's function based on Newman (Newman J.N., 1985).

The hydrodynamic forces are non-linear, hence all the term of equation (2) are time dependent. This consideration leads to the final formulation used for the evaluation of the sectional hydrodynamic forces, as described in the following equation:

$$f_{Hj}(x;t) = \sum_i [-a_{ij}^{\infty}\frac{D}{Dt}V_i + U\frac{\partial}{\partial x}a_{ij}^{\infty}V_i - \frac{\partial}{\partial z}a_{ij}^{\infty}V_i^2 - \frac{D}{Dt}\int_{-\infty}^{t}K_{ij}(x;t-\tau)V_i(x,\tau)\,d\tau] \tag{7}$$

Equation (7) gives the possibility to model lift and impulsive effects, which are represented by the second and third terms respectively on the right hand side of equation (7), which are neglected in a linear approach.

The restoring and exciting forces are time dependent and are calculated directly in the time domain as a function of the actual wetted hull geometry and the wave elevation plane as free surface. The Froude-Krylov and diffraction forces are calculated using the strip theory approach (Salvesen et al., 1970).

The system of equations of motion includes non-linear terms as described in equation (8). These coupled equations are solved numerically in the time domain using a fourth order Runge-Kutta method.

$$\begin{cases}(M_{33}+A_{33}^{\infty})\ddot{\eta}_3 + (M_{53}+A_{53}^{\infty})\ddot{\eta}_5 = F_3^E - F_3^{Dam} - F_3^{Imp} - F_3^{Lift} - F_3^R \\ (M_{55}+A_{55}^{\infty})\ddot{\eta}_5 + (M_{35}+A_{35}^{\infty})\ddot{\eta}_3 = F_5^E - F_5^{Dam} - F_5^{Imp} - F_5^{Lift} - F_5^R\end{cases} \tag{8}$$

Where F^E_j are the exciting forces, F^{Dam}_j are the damping components of the hydrodynamic forces, F^{Imp}_j are the impulsive terms, F^{Lift}_j are the lift components and F^R_j are the restoring forces.

COMPARISONS AND CONCLUSIONS

The proposed method is applied to predict the motions and loads of the S175 container ship travelling in regular head seas. The results of these non-linear predictions are compared with those obtained from a linear frequency domain two-dimensional strip theory, and a three-dimensional boundary elements method. The comparisons with the linear frequency domain methods are carried out in small amplitude waves. This was necessary to compare the results obtained from the non-linear predictions with those obtained from the linear methods for the validation of the non-linear technique developed. In this validation process it was considered that the response of a non-linear system in small amplitude waves is not affected by any non-linear behaviour and can be predicted using a linear method.

In figures 2-3 the comparisons for heave and pitch motions at zero forward speed are presented. The prediction of the vertical wave bending moment at amidship $x=87.5m$ from AP is shown in Figure 4. The longitudinal mass distribution used in the calculations is given in Figure 5.

The comparisons based on the non-linear time domain technique developed in this study show a good agreement with the techniques developed by others for both the heave and pitch motions well as amidships vertical wave bending moment in regular head waves, at zero forward speed.

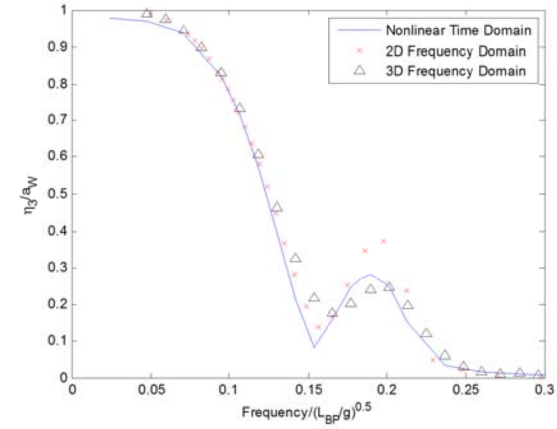

Figure 2: *Comparison of heave response between nonlinear time domain (a_W = 0.05m) 2D and 3D linear frequency domain methods*

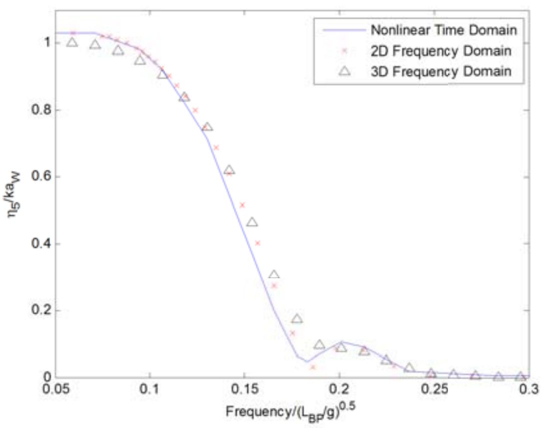

Figure 3: *Comparison of pitch response between nonlinear time domain (a_W = 0.05m) 2D and 3D linear frequency domain methods*

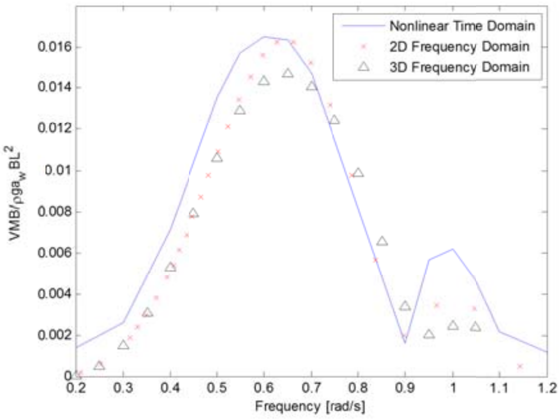

Figure 4: *Vertical bending moment at x=87.5m from AP*

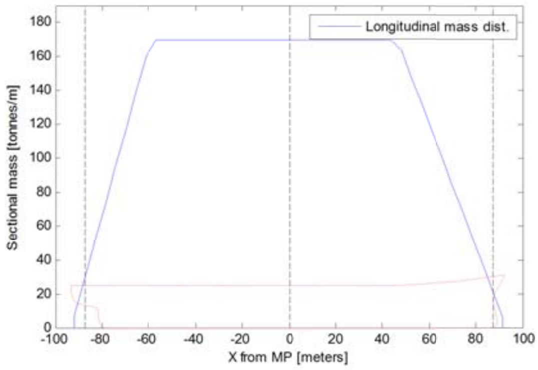

Figure 5: *Longitudinal mass distribution for the S175 Containership*

Further comparison will be carried out to analyse the effect of non-linearities on motions and loads prediction, with and without forward speed for regular waves for the head sea condition.

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