Slow-drift excitation in varying bathymetry

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There has been an increasing interest lately for marine operations in restricted waterdepths; examples are nearshore pipe-laying and LNG terminals. There are many associated hydrodynamic issues, one of them being the prediction of the wave induced mooring loads. In deep water the usual procedure is to compute the drift forces in regular waves, then to recur to some Newman approximation to predict the slowly-varying second-order loads in irregular waves. As the waterdepth decreases it is known that the contribution of the second-order incident potential at the difference frequencies can no longer be neglected. However it has often been observed that the flat bottom expression of this component leads to over-conservative second-order loads (e.g. see Weiler *et al.* 2009): as the second-order long waves shoal on the varying sea-floor they do not develop to the extent predicted by the flat bottom theory, unless the bottom slope be very very small. Another important feature that has strong implications on the second-order loading is that their phase relationship with the short wave envelope also deviates from the flat bottom model.

In this paper we focus on the effect of the shoaling of the long wave on the second-order low-frequency loading. We assume the bathymetry to be two-dimensional and the incoming wave system to be normal to the parallel depth contours. So we do not consider the effect of the angular spreading which is also known to strongly affect the low-frequency loading (e.g. see Molin & Fauveau 1984). However one of the model that we propose accounts for wave components at any angles with respect to the depth contours; application of this model to angular spread wave systems will be shown elsewhere.

Low-frequency second-order loads in unidirectional multichromatic waves take the general form

$$F_-^{(2)}(t) = \Re\left\{\sum_i \sum_j A_i\, A_j\, f_-^{(2)}(\omega_i,\omega_j)\, \mathrm{e}^{-\mathrm{i}\,(\omega_i-\omega_j)\,t+\mathrm{i}\,(\theta_i-\theta_j)}\right\} \tag{1}$$

with the first-order incoming wave elevation given as

$$\eta_I^{(1)}(x,t) = \sum_i A_i\ \cos(k_i\,x - \omega_i\,t + \theta_i) \tag{2}$$

The complex Quadratic Transfer Function (QTF) $f_-^{(2)}(\omega_i,\omega_j)$ can be separated into a real part and an imaginary part

$$f_-^{(2)}(\omega_i,\omega_j) = P(\omega_i,\omega_j) + \mathrm{i}\ Q(\omega_i,\omega_j) \tag{3}$$

Newman approximations provide an estimate of the real part P. In flat bottom the second-order incident potential mostly contributes to the imaginary part Q since, in a bichromatic wave system, the associated acceleration is 90 degrees out of phase with the short wave envelope.

If the bottom slope is small the first-order wave system will shoal according to ray theory and it can be assumed that the regular wave drift forces are still reasonably well obtained through the flat bottom assumption. However a mild bottom slope to first-order waves can be very steep to the second-order accompanying long waves which, consequently, do not reach the extent predicted by the flat bottom model. This means that their contribution to the low-frequency loading must be corrected with a factor $R(\omega_i,\omega_j)\ \exp\{\mathrm{i}\ \alpha(\omega_i,\omega_j)\}$ so that the QTF becomes

$$\begin{aligned} f_-^{(2)}(\omega_i,\omega_j) &= P(\omega_i,\omega_j) + \mathrm{i}\ Q(\omega_i,\omega_j) \times R(\omega_i,\omega_j)\ \mathrm{e}^{\mathrm{i}\ \alpha(\omega_i,\omega_j)} \\ &= P - Q\ R\ \sin\alpha + \mathrm{i}\ Q\ R\ \cos\alpha \end{aligned} \tag{4}$$

When $R < 1$ and $0 \leq \alpha \leq \pi/2$ the modulus of the QTF is decreased as compared to its flat bottom reference value. It can even get lower than its Newman approximation P alone (this was apparently the case in the model tests reported in Liu *et al.* 2010).

In Liu *et al.* (2010) two models have been proposed to compute the correction factor $R(\omega_i, \omega_j) \exp\{\mathrm{i}\ \alpha(\omega_i, \omega_j)\}$ for an arbitrary two-dimensional bathymetry. In these models the long wave associated with a bichromatic wave system is propagated from the deep water end and no consideration is given to its eventual partial reflection at the shore end or to other long wave components being generated in the surf zone (for a review on shoaling of infra-gravity waves, see e.g. Battjes *et al.* 2004). One model is an extension of Schäffer's model for a straight beach (Schäffer 1993). The other one consists in idealizing the variable bathymetry zone as a succession of steps and using eigen-function expansions to solve the second-order problem in the successive rectangular sub-domains. Both models have been found to be in excellent agreement with each other, and with experimental results given in Van Dongeren *et al.* (2004) (see also Van Noorloos 2003 where more results from the experiments are given). In Liu *et al.* (2010) it is demonstrated that application of the correction factor $R(\omega_i, \omega_j)$ $\exp\{\mathrm{i}\ \alpha(\omega_i, \omega_j)\}$, as obtained from the step model, to the flat bottom QTFs, leads to improved agreement between the measured and calculated slow-drift motion of a rectangular barge moored over a sloping bottom.

In this short paper we address the problem of determining a priori whether the shoaling of the accompanying long wave to an irregular wave system will lead to significant corrections with regards to the flat bottom model. For the sake of simplicity we assume that the bathymetry consists in a constant depth zone followed by a rectilinear ramp. Even with this simplification there remain many parameters coming into play like the initial depth, the bottom slope, the mean wave period and the resonant period of the mooring system.

We consider 4 slope cases, of 0.5 %, 1 %, 2 % and 4 %, typical wave periods of 10 s and 15 s, and typical resonant periods of 50 s and 100 s. We propagate bichromatic wave systems of periods (T_1, T_2) over the varying bathymetry.

In figure 1 we vary the initial depth, which takes the values 100 m, 80 m, 60 m and 40 m, and we show the R and α values obtained in the [5 m 40 m] depth range, for $T_1 = 10$ s and $T_{\mathrm{beat}} = T_1\, T_2/(T_2 - T_1) = 50$ s. It can be seen that 60 m, 80 m and 100 m initial depths lead to more or less identical R and α values in the depth range of interest. (Interestingly the R ratio oscillates and is locally larger than 1 in the 40 m initial depth case; this is a somewhat unexpected feature). So the initial depth is not a critical parameter.

In figure 2 we take the initial depth equal to 100 m and we vary the wave and beat periods. When the bottom slope, wave period and beat period increase, the R coefficient takes smaller and smaller values, while the phase shift α increases. This means that the low-frequency second-order loads will be strongly reduced as compared to their flat bottom reference values.

References

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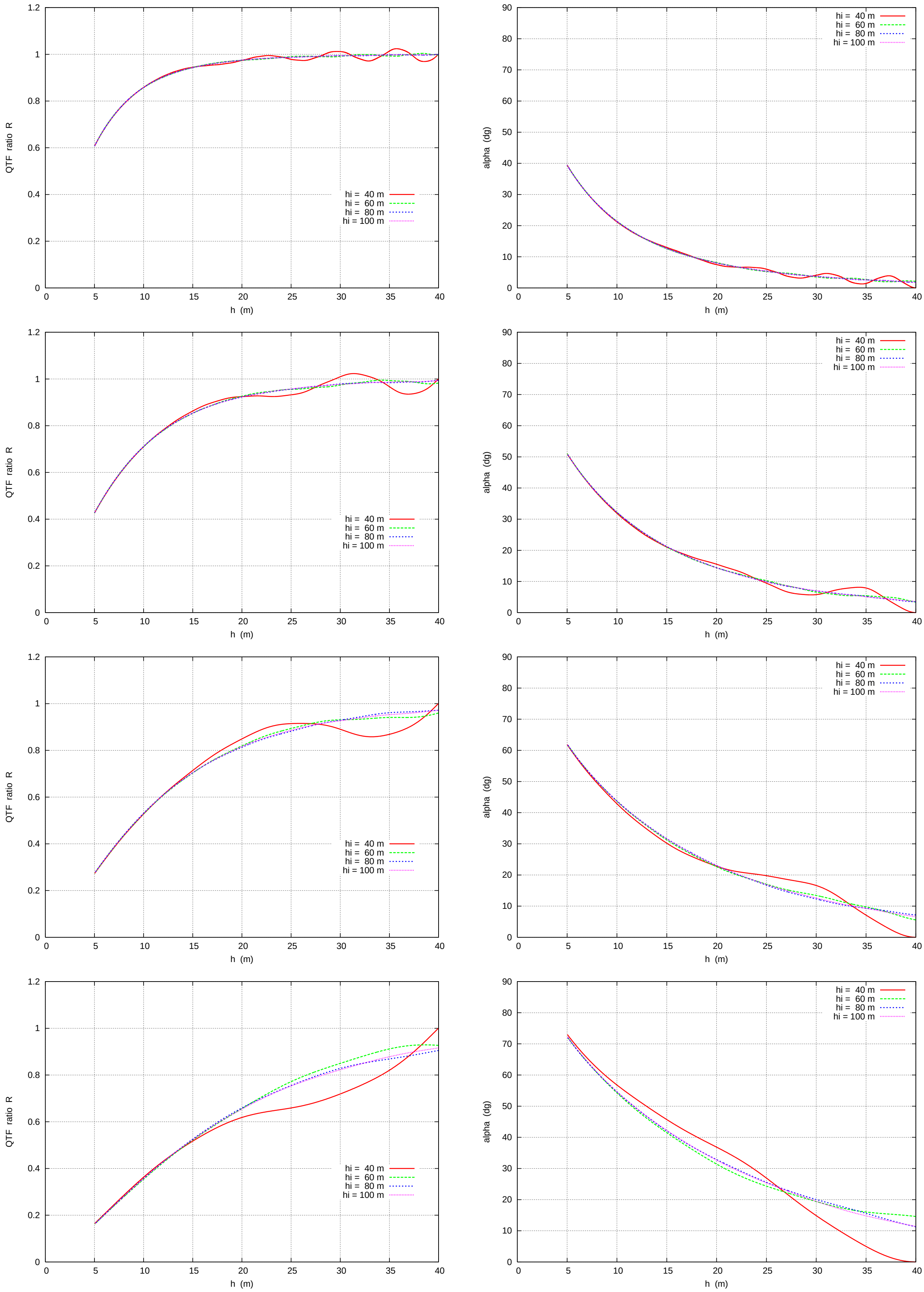

Figure 1: QTF ratios R (left column) and phase shifts α (right column) for $T_1 = 10$ s, $T_{\text{beat}} = 50$ s, vs. local depth h for different initial depths. From top to bottom slopes of 0.5 %, 1 %, 2 % and 4 %.

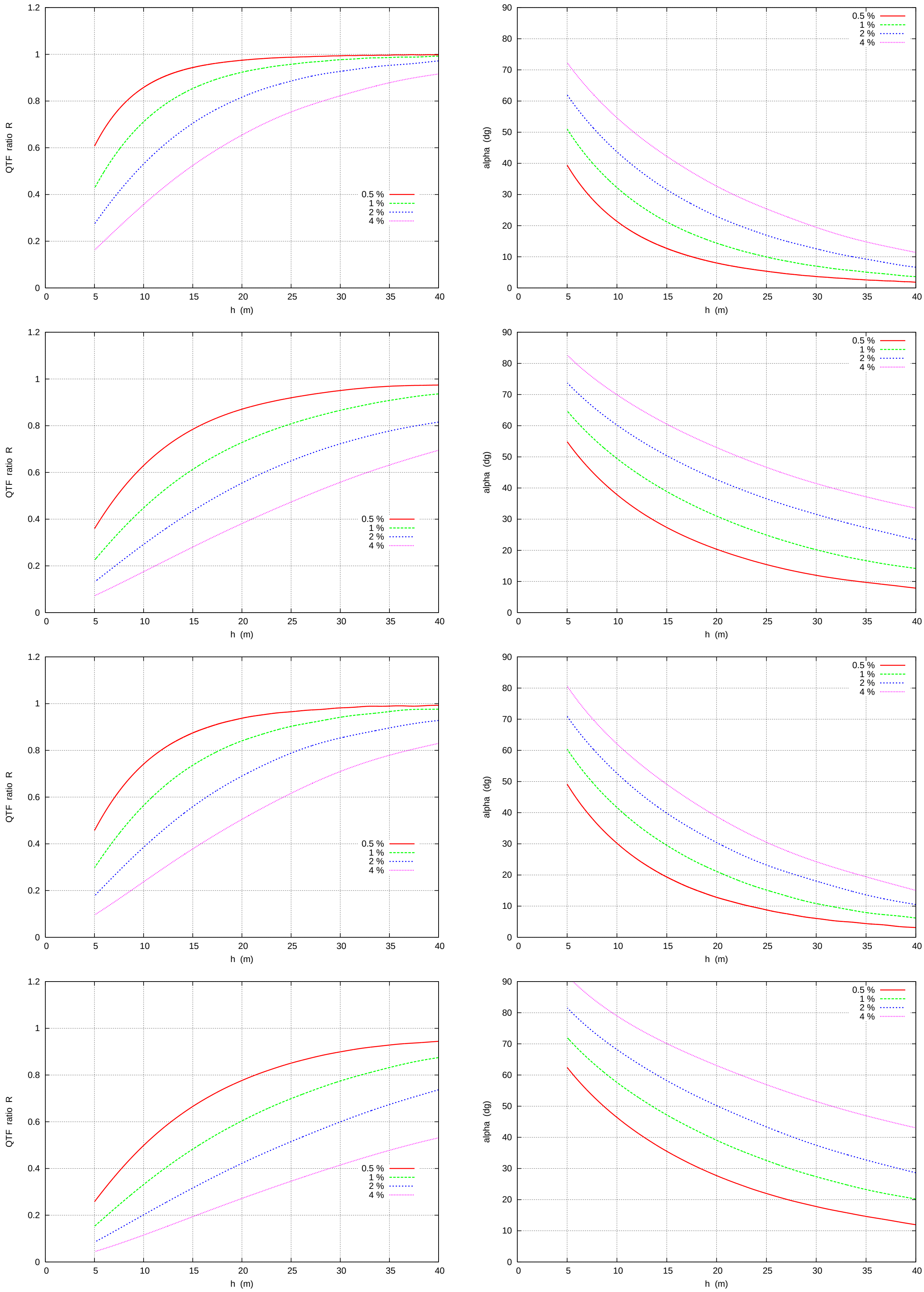

Figure 2: QTF ratios R (left column) and phase shifts α (right column), vs. local depth h, for different slopes. $T_1 = 10$ s, $T_{\mathrm{beat}} = 50$ s (top); $T_1 = 15$ s, $T_{\mathrm{beat}} = 50$ s; $T_1 = 10$ s, $T_{\mathrm{beat}} = 100$ s; $T_1 = 15$ s, $T_{\mathrm{beat}} = 100$ s (bottom).