Global hydroelastic model for LNG ships

Malenica Š.[1], Bigot F.[1], Chen X.B.[1] & Bralic S.[2],

(1) BUREAU VERITAS - Research Department, Paris, France

(2) BVB - Zagreb, Croatia

Introduction

It is nowadays well recognized that the global hydroelastic response of ships can become an important part of the total ship structural response influencing both the extreme structural response as well as the fatigue life of some structural details. This is particularly true for large ships since their natural frequencies are much lower. In the case of the ships without large liquid cargo tanks, such as large container ships, the overall methodology for calculating the global hydroelastic response is fairly well established and an example of these types of models is presented in [2, 3]. This method is based on the modal approach and is valid both for the simplified beam structural model as well as for the 3DFEM structural model. In this paper we use similar principles and we add the influence of the liquid cargo. This influence is accounted for using the approach presented in [1] where the seakeeping of the rigid ship with internal liquids was considered.

Mathematical model

The basic configuration is shown in Figure 1. As already mentioned, a solution for the seakeeping of the rigid body with internal liquids was presented in [1], and here below we extend the same approach for the elastic body using the so called modal approach. The main principles of the modal approach for

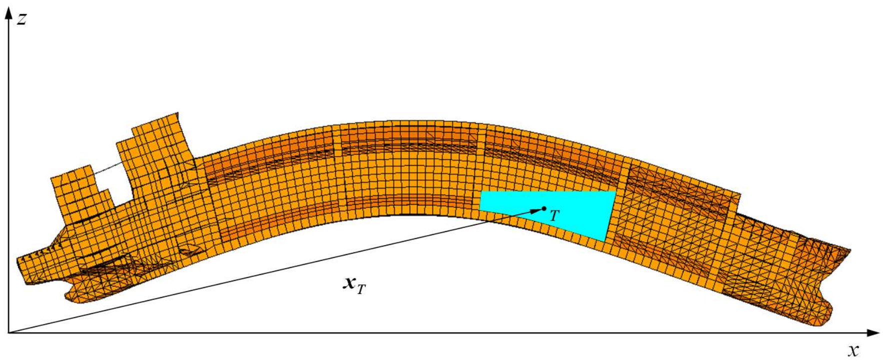

Figure 1: Basic configuration.

elastic body are presented in [2, 3] and we follow them closely here. The original six degrees of freedom dynamic system of the floating rigid body is extended with a certain number of elastic structural modes. The frequency domain approach is adopted, and we formally write for the displacement of one point on the body:

$$\boldsymbol{H}(\boldsymbol{x},\omega)=\sum_{i=1}^{N}\xi_i(\omega)\boldsymbol{h}^i(\boldsymbol{x}) \tag{1}$$

where N is the total number of modes (rigid + elastic), $\boldsymbol{x}=(x,y,z)$ describes the position of one point in the structure and $\boldsymbol{h}^i(\boldsymbol{x})$ is i^{th} modal displacement vector:

$$\boldsymbol{h}^i(\boldsymbol{x})=h^i_x(\boldsymbol{x})\boldsymbol{i}+h^i_y(\boldsymbol{x})\boldsymbol{j}+h^i_z(\boldsymbol{x})\boldsymbol{k} \tag{2}$$

where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the unit vectors defining the global coordinate system.
It should be kept in mind that the modal definition formally includes also the local rotation angles (ϕ, θ, ψ) and we can write:

$$\boldsymbol{h}_r^i(\boldsymbol{x}) = h_\phi^i(\boldsymbol{x})\boldsymbol{i} + h_\theta^i(\boldsymbol{x})\boldsymbol{j} + h_\psi^i(\boldsymbol{x})\boldsymbol{k} \tag{3}$$

Even if these rotation angles are usually not explicitly necessary in the overall analysis, their use might sometimes be helpful, as we will see later.

Seakeeping

Compared to the classical rigid body seakeeping analysis the overall procedure remains the same except that the additional Boundary Value Problems (BVP) "elastic" radiation modes are defined by the following body boundary condition:

$$\frac{\partial \varphi_{Rj}}{\partial n} = \boldsymbol{h}^j \boldsymbol{n} \tag{4}$$

Once the different BVP's solved the resulting pressure field is integrated over the wet surface of the body and the different hydrodynamic coefficients (added mass, damping and excitation) are calculated and the final generalized dynamic motion equation is written:

$$\left\{-\omega^2([\,\mathbf{m}\,] + [\,\mathbf{A}\,]) - i\omega[\,\mathbf{B}\,] + ([\,\mathbf{k}\,] + [\,\mathbf{C}\,])\right\}\{\boldsymbol{\xi}\} = \{\boldsymbol{F}^{DI}\} \tag{5}$$

where:

$[\,\mathbf{m}\,]$	-	genuine mass
$[\,\mathbf{k}\,]$	-	structural stiffness
$[\,\mathbf{A}\,]$	-	added mass
$[\,\mathbf{B}\,]$	-	damping
$[\,\mathbf{C}\,]$	-	hydrostatic stiffness
$\{\xi\}$	-	modal amplitudes
$\{\boldsymbol{F}^{DI}\}$	-	distributed pressure excitation (diffraction & incident)

The dimension of the above linear system is $N = 6 + N_{flex}$ with N_{flex} denoting the number of flexible modes.

Sloshing

The procedure for sloshing is very similar. Indeed the additional "elastic" sloshing BVP's are defined by projecting the mode shapes onto the internal wet structure of the tank:

$$\frac{\partial \varphi_{Rj}^{T}}{\partial n} = \boldsymbol{h}^j \boldsymbol{n} \tag{6}$$

Special care should be given to the free surface condition inside the tank. Here we follow the method proposed in [1] and we write:

$$-\nu\varphi + \frac{\partial \varphi}{\partial z} = -i\omega\zeta_v^A \tag{7}$$

Due to the arbitrary tank deformation which is assumed, the vertical displacement of the waterplane ζ_v^A can not be expressed simply as in [1] because the volume of the tank can change as shown in Figure 2! That is why it is necessary to generalize the definition of the displacement of the waterplane.
We can formally write the following expression for the conservation of the liquid volume:

$$\Delta V - S_W \zeta_v^A = 0 \tag{8}$$

where ΔV is the change of the tank position and volume due to the motions/deformations, and S_W is the waterplane area.
The waterplane area can be easily calculated, and for the change of the tank position and volume we can write:

$$\Delta V = \iint_{S_T} \boldsymbol{H}\boldsymbol{n} dS = \sum_{i=1}^{N} \xi_i \iint_{S_T} \boldsymbol{h}^i \boldsymbol{n} dS \tag{9}$$

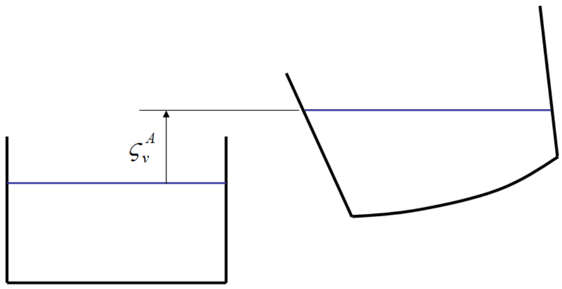

Figure 2: General tank motion/deformation.

In order to simplify the notations let us write the above expressions in a more compact form:

$$\zeta_v^A = \sum_{i=1}^{N} \xi_i \zeta_v^{Ai} \qquad , \qquad \zeta_v^{Ai} = \frac{\iint_{S_T} \boldsymbol{h}^i \boldsymbol{n} dS}{S_W} \tag{10}$$

With this in mind the final BVP for the sloshing potential becomes:

$$\left.\begin{array}{lr} \Delta \varphi_{Rj}^{T} = 0 & \text{in the fluid} \\ -\nu \varphi_{Rj}^{T} + \dfrac{\partial \varphi_{Rj}^{T}}{\partial z} = \zeta_v^{Aj} & z = 0 \\ \dfrac{\partial \varphi_{Rj}^{T}}{\partial n} = \boldsymbol{h}^j \boldsymbol{n} & \text{on } S_T \end{array}\right\} \tag{11}$$

The solution of the above BVP gives the sloshing potential from which the pressure can easily be calculated and integrated over the wet surface of the tank and give the added mass corresponding to the different modes of motions/deformations. This added mass matrix can be simply added to the dynamic motion equation (5) and there is no need for any special coordinate transformations, as it was the case in [1].

Hydrostatic restoring

The calculation of the hydrostatic restoring for the seakeeping part does not change and we can directly use the expressions given in [2, 3]. However, it should be noted that, when calculating the part of the restoring which is related to the structural mass i.e. C_{ij}^m, the volume integral should exclude the mass of the liquid in the tanks!

Concerning the hydrostatic restoring related to the liquid cargo, we need to combine the approaches presented in [1] and [2]. The general expression for hydrostatic force at instantaneous position (˜ sign is used to indicate that the values are taken at the instantaneous position) is written in the form:

$$\tilde{\boldsymbol{F}}_{ij}^{Hs} = -\varrho g \iint_{\tilde{S}_B} (z - \zeta_v^{Aj} - Z_0^T) \tilde{\boldsymbol{h}}^i \tilde{\boldsymbol{n}} d\tilde{S} = -\varrho g \iint_{S_B + \delta S_B} (Z - \zeta_v^{Aj} - Z_0^T + \delta Z)(\boldsymbol{h}^i + \delta \boldsymbol{h}^i)[\boldsymbol{n} dS + \delta(\boldsymbol{n} dS)] \tag{12}$$

After few manipulations, similar to those presented in [2], the expression for the hydrostatic restoring becomes:

$$C_{ij}^{Hs} = \varrho g \iint_{S_B} \left\{ (Z - Z_0^T)[\nabla_X \boldsymbol{h}^j \boldsymbol{h}^i \cdot \boldsymbol{n} + (\underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j - \underline{\underline{\nabla_X \boldsymbol{h}^j}} \cdot \boldsymbol{h}^i) \cdot \boldsymbol{n}] + (h_Z^j - \zeta_v^{Aj}) \boldsymbol{h}^i \boldsymbol{n} \right\} dS \tag{13}$$

where Z_0^T denotes the vertical position of the free surface in the tank.

Simplified model

We discuss here below a simplified model which can be used to describe the sloshing part of the problem. The main advantage of the simplified model is that we can use the existing codes and there is no need

for further developments! However, for the moment it is not completely clear how accurate is this model and this should be carefully investigated.
Anyhow, the idea is to take into account only the rigid body motion of the tanks and disregard the local tank deformations! In that respect we write for the motions of the representative point of the tank and for the corresponding simplified modes:

$$\boldsymbol{H}(\boldsymbol{x}_{_T},\omega)=\sum_{j=1}^{N}\xi_j(\omega)\boldsymbol{h}^j(\boldsymbol{x}_{_T}) \quad , \quad \boldsymbol{h}^j(\boldsymbol{x})=\sum_{i=1}^{6}h_i^j(\boldsymbol{x}_{_T})\boldsymbol{h}^{Ri}(\boldsymbol{x}) \tag{14}$$

where it should be noted that the above expression formally includes both the translations (2) and the rotations (3) i.e. $h_i^j=(h_x^j,h_y^j,h_z^j)$ for $i=1,3$ and $h_i^j=(h_\phi^j,h_\theta^j,h_\psi^j)$ for $i=4,6$. The local rigid body modes $\boldsymbol{h}^{Ri}(\boldsymbol{x})$ define the motion of the tank with respect to $\boldsymbol{x}_{_T}$.
Concerning the hydrodynamic part of the problem we can write for the sloshing potential the following expressions:

$$\varphi^{^T}_{_{Rj}}=\sum_{i=1}^{6}h_i^j(\boldsymbol{x}_{_T})\varphi^{^R}_{_{Ri}} \tag{15}$$

where $\varphi^{^R}_{_{Ri}}$ represent the rigid body sloshing potential defined by (46) in [1].
With this in mind, the expression for the added masses becomes:

$$A^{^T}_{ij}=\varrho\iint_{S_T}\varphi^{^T}_{_{Rj}}\boldsymbol{h}^i\boldsymbol{n}dS=\varrho\sum_{k=1}^{6}\sum_{l=1}^{6}h_k^j(\boldsymbol{x}_{_T})h_l^j(\boldsymbol{x}_{_T})\iint_{S_T}\varphi^{^R}_{_{Rl}}\boldsymbol{h}^{Rk}\boldsymbol{n}=\varrho\sum_{k=1}^{6}\sum_{l=1}^{6}h_k^j(\boldsymbol{x}_{_T})h_l^j(\boldsymbol{x}_{_T})A^{^R}_{kl} \tag{16}$$

where $A^{^R}_{kl}$ is the rigid body added mass of the tank defined by (49) in [1].
Finally, concerning the hydrostatic restoring matrix, the expression (13) remains valid provided that the above defined mode shapes (14) are used.

A few comments

Even if the above procedure looks rather straightforward the implementation should be done very carefully. Here below we summarize a few important comments.
Since we use the modal approach there is no "real" coordinate system associated with the definition of the different modes of motion so that there is no need for the coordinate transformations from the local (tank) to the global coordinate system. The mass matrix in the dynamic motion equation does not include the mass of the liquid in the tanks! It is also important to note that, contrary to what was presented for the rigid body case in [1], the gravity part of the seakeeping hydrostatic restoring, does not include the liquid cargo mass neither, and that is why it is not necessary to deduce it from the hydrostatic part of the liquid cargo. Concerning the simplified sloshing method and the choice of the representative point in the tank, in principle this point can be arbitrarily chosen and the final result should not depend very much on this choice because otherwise the overall simplified assumptions become questionable. However the choice of the tank center of gravity on which the average of the overall tank motion/deformation is applied, looks reasonable. Indeed, in that way the possible local deformations of the tank will be filtered out.

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