Deformation of Free Surface Due to a Water Droplet Impact

S.H. Kwon[(1)], C.W. Park[(1)], S.H. Lee[(1)], J.Y. Shin[(1)], Y.M. Choi[(1)], J.Y. Chung[(1)], H. Isshiki[(2)]

,

(1) Department of NAOE, Pusan National University, Busan, Republic of Korea
(shkwon, pcw8127, lsh1098, jyshin, chungjy)@pusan.ac.kr

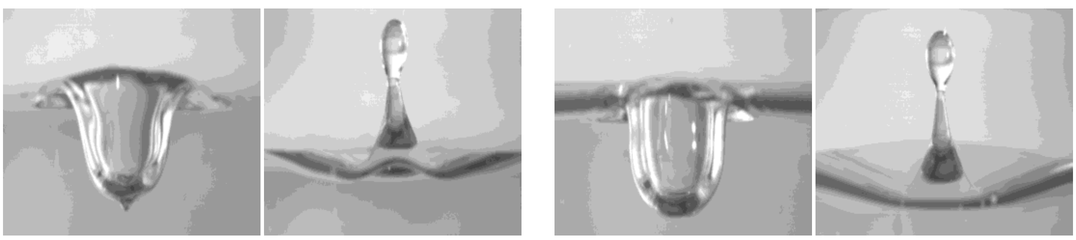
(2) School of NAOE, University of Ulsan, Ulsan, Republic of Korea, isshiki@dab.hi-ho.ne.jp

Introduction

The authors observed bubbles generated on the free surface and in the water while they were performing sloshing experiment. Those generated bubbles influence a lot on the magnitude of the impact pressure. To investigate the bubble generation mechanism the authors started with very simple experiment. A water droplet was released at various heights above the calm water surface. Experiments were done for various masses to see mass effect on the free surface deformation. To investigate the effect of surface tension experiments with and without surfactant were done. During this investigation it was desired to compute the deformation of free surface due to a water droplet impact. This problem can be considered to be initial value problem, i.e., the classical Cauchy-Poisson problem. The deformation of free surface was recorded by high speed camera. This measured free surface deformation was used as initial condition for this initial value problem. The order zero Hankel and Laplace integral transform was applied to the governing equation, kinematic and dynamic boundary conditions. Combining these results the expression for the free surface elevation can be obtained. The computed deformation of the free surface was compared with that of the experiment.

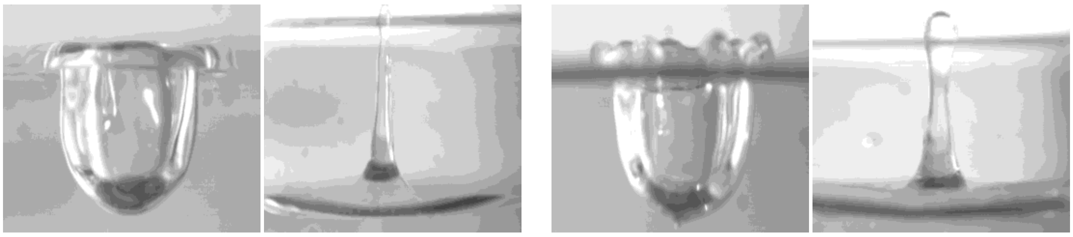
Experiment

The experiment was carried out in a small wave tank whose dimensions are 60mm×60mm×60mm (L×B×H). The deformation of free surface was recorded by high speed camera. The maximum frame rate of the high speed camera is 64,000 frames/s. The frame rate for this study was 2,000 frames/s & 1,000 frames/s. Two different masses of water drops were tested to see the effect of the variation of the mass on the free surface deformation. To investigate the effect of the surface tension experiments with and without surfactant were done. To reduce the surface tension, NP-10 solution was added to the water. The surface tension was reduced from 72.7dyne/cm to 36.5dyne/cm after the solution was added. Fig. 1 shows the free surface profiles with and without surfactant when the mass of the water drop was 16mg. The height of the drop origin was 150mm above the calm water surface.



Without Surfactant   With Surfactant

Fig. 1 Comparison of the Effect of Surfactant (16mg)



Without Surfactant With Surfactant

Fig. 2 Comparison of the Effect of Surfactant (65mg)

Figs. 2 present the free surface profile when the mass of the water drop was 65mg. The same drop height was maintained for both experiments. It can be shown that the difference due to mass effect gets smaller when the mass gets heavier. It means that when the size of disturbance gets larger the effect of surface tension gets smaller.

Theoretical approach (Debnath, 1994)

The velocity potential is a function of r, z, and t. In other words, we are talking about axisymmetric case. The governing equation can be written in a form

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=0 \tag{1}$$

On the free surface two boundary conditions are needed to be imposed.

The linearized kinematic free surface boundary condition is given

$$\frac{\partial\eta}{\partial t}=\frac{\partial\phi}{\partial z} \quad on \quad z=0 \tag{2}$$

The linearized dynamic free surface boundary condition is

$$\frac{\partial\phi}{\partial t}+g\eta=0 \quad \text{on z=0} \tag{3}$$

Along with these boundary conditions, we need to specify initial conditions for surface elevation and potential. To solve this problem integral transformation method is employed.

Application of order zero Hankel and Laplace transform to velocity potential is defined by

$$\bar{\phi}(k,z,s)=\int_0^\infty e^{-st}dt\int_0^\infty rJ_0(kr)\phi(r,z,t)dr \tag{4}$$

Its inverse can be deified by

$$\phi(r,z,t)=\frac{1}{2\pi i}\int_\Gamma e^{st}ds\int_0^\infty kJ_0(kr)\bar{\phi}(k,z,s)dk \tag{5}$$

Application of Eq. (4) to Eq. (1) yields (Dautray, 1985)

$$-k^2\bar{\phi}(k,z,s)+\frac{d^2\bar{\phi}(k,z,s)}{dz^2}=0 \tag{6}$$

When we consider z<0, we can have the solution under deep water assumption as

$$\bar{\phi}(k,z,s)=A(k,s)e^{kz} \tag{7}$$

Transformation of Eq. (2) gives us

$$s\bar{\eta}(k,s)-\bar{\eta}(k,0)=kA(k,s) \quad on\ z=0 \tag{8}$$

Eq. (3) becomes

$$sA(k,s)-\bar{\phi}(k,0,0)+g\bar{\eta}(k,s)=0 \quad on \quad z=0 \tag{9}$$

We can write the expression for the elevation by solving Eq. (8) and Eq. (9) with an assumption of banishing of initial potential $\bar{\phi}(k,0,0)$.

$$\bar{\eta}(k,s) = \frac{1}{s^2 + gk} s\bar{\eta}(k,0) \quad on \quad z = 0 \tag{10}$$

Inversion of Hankel Laplace transformation will give us the free surface elevation

$$\eta(r,t) = \int_0^\infty kJ_0(kr)\bar{\eta}(k,0)\cos\sqrt{gk}tdk \tag{11}$$

Let's consider the initial displacement

$$\eta(r,0) = de^{-\left(\frac{r}{a}\right)^2}\left\{1 - \left(\frac{r}{a}\right)^2\right\} \tag{12}$$

The Hankel transform of Eq. (12) results in (Debnath, 1994 ; Miles, 1968)

$$\bar{\eta}(k,0) = \frac{1}{8}da^2(ka)^2 e^{-\frac{1}{4}(ka)^2} \tag{13}$$

Substitution of Eq. (13) into (11) gives solution for the free surface elevation in a form

$$\eta(r,t) = \frac{1}{8}da^4\int_0^\infty k^3 J_0(kr)e^{-\frac{1}{4}(ka)^2}\cos\sqrt{gk}tdk \tag{14}$$

Results and discussion

The related parameters of Eq. (12) are 'a' and 'd'. These parameters were computed from the digitized free surface deformation captured by high speed camera. The two curves are shown in Fig. 3. The symbols represent measured free surface deformation. The solid line shows the curves calculated using Eq. (12). The units of the axes are cm.

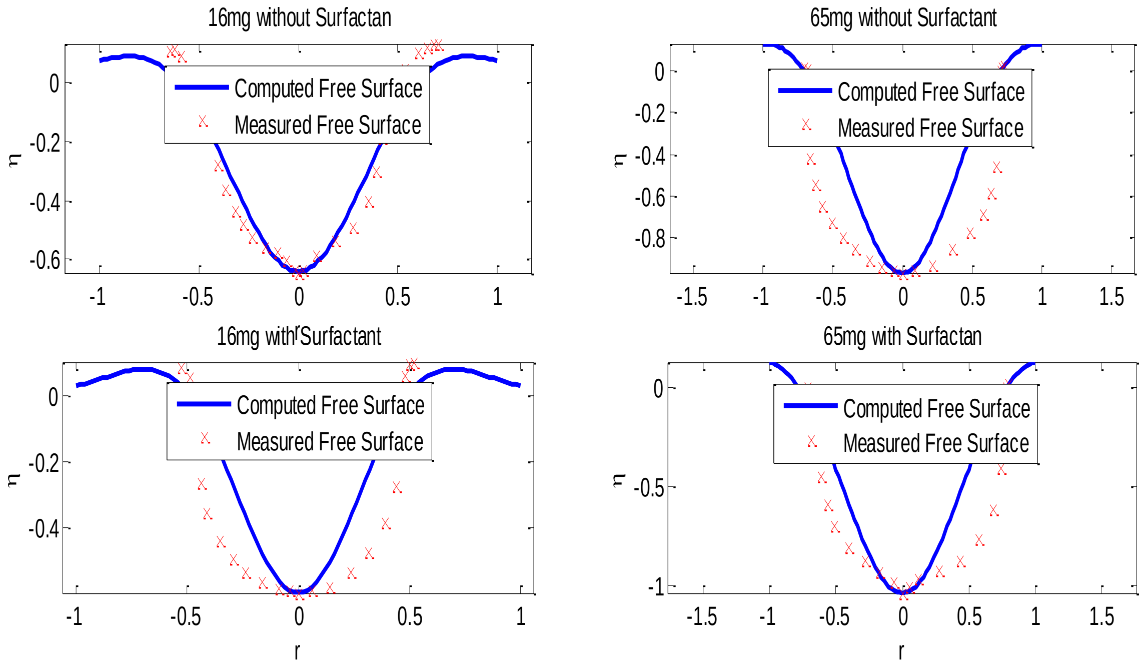


Fig. 3 Initial Condition for the Free Surface Deformation

Having known these 'a' and 'd' values we can compute the free surface deformation making use of Eq. (14). The computed elevation moves much slower than that of measured one. The maximum surface elevations for both computed and measured are shown in Fig. 4. It seems that the slope of the computed results is smaller than that of the measurement. For the 16 mg case the deviation with surfactant is smaller. It seems that this is due to the fact that the computation did not include the surface tension effect in the dynamic free surface boundary condition. The computed profile of the free surface deviates a lot at the end of the surface elevation. The reason of the deviation might be due to some factors. First of all, the calculation was based on the potential theory which ignores the effect of viscosity. Secondly the free surface boundary conditions are linearized. Thirdly the surface tension was not included in the dynamic free surface boundary condition. In addition to the above mentioned reasons, the initial free surface deformation calculated from Eq. (12) does not match well with the measured profile which fact is featured in Fig. 3.

References

Robert Dautray, Jacques-Louis Linos, 1985, "Mathematical analysis and numerical methods for science and technology, vol. 2, functional analysis and variational methods, Springer

Lokenath Debnath, 1994, "Nonlinear water waves", Academic Press, INC,

John W. Miles, 1968, "The Cauchy-Poisson problem for a viscous liquid", J. Fluid Mech. Vol. 34, part 2, pp359-370

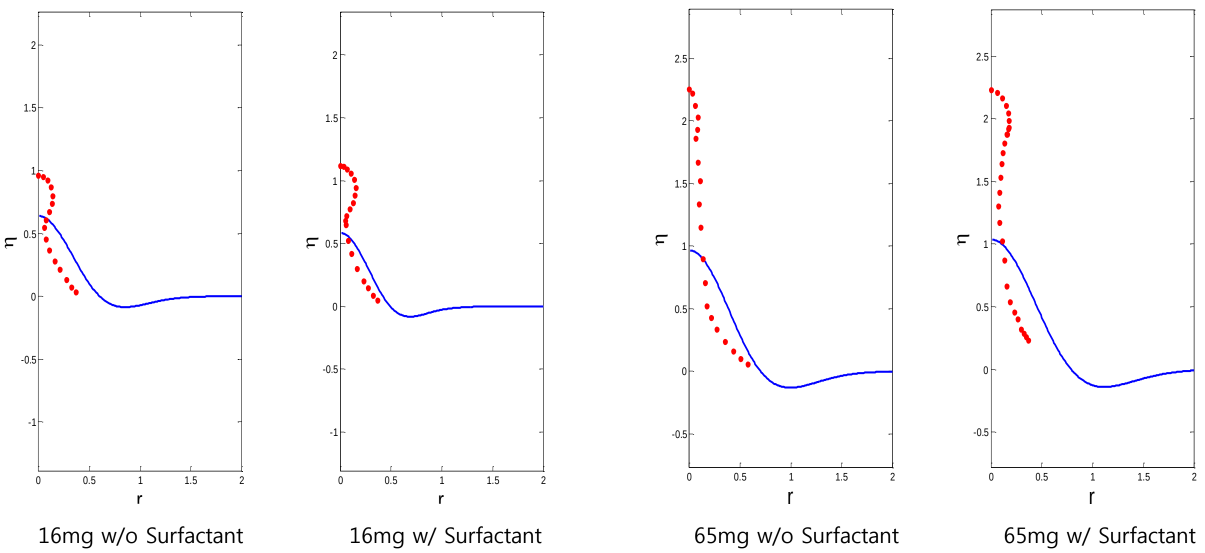


Fig. 4 Comparison of the Maximum Surface Elevations.