FLUID IMPACT ONTO A CORRUGATED PANEL WITH TRAPPED GAS CAVITY

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Initial stage of incompressible liquid impact onto a corrugated elastic panel with account for a compressible gas trapping between the corrugations is studied. The liquid free surface is flat and parallel to the panel before the impact. The corrugations are modelled as rigid short structures on the surface of the main panel. Part of the panel between two corrugations is elastic. The liquid closes the gas cavity between two corrugations and compresses the gas before the fluid comes in contact with the elastic part of the panel. The elastic deflections of the plate and bending stresses due to impact are determined accounting for the presence of the gas between the corrugations. The hydroelastic problem is solved within the Wagner approach. The effect of gas compressibility on the elastic behaviour of the corrugated elastic plate is investigated.

1. Introduction

The problem of fluid impact onto an elastic panel is of importance in many engineering applications. For example, sloshing in LNG (liquefied natural gas) tank can be so violent that the LNG may hit the walls and ceiling of the tank with strong force [Bredmose et al., 2002]. In calculations of the tank wall response to such fluid impacts, flexibility of the containment system is important. The problem should be treated as a coupled problem of hydroelasticity, where the hydrodynamic loads and elastic deflections of the containment system have to be determined simultaneously. This problem was studied in [Korobkin et al., 2008] for compressible jet impact onto elastic panel without corrugations. Approximate solutions of the corresponding three-dimensional problem were investigated in [Khabakhpasheva, 2008]. The inner surfaces of the containment systems Mark-III and NO-96 are corrugated to increase damping of sloshing motion and to reduce thermal stresses in the outer membrane.

The corrugations modify the hydrodynamic loads during fluid impact and may increase/decrease the bending stresses in the containment system. The effect of NO-96 corrugations on the bending stresses in the main panel was studied in [Khabakhpasheva, Korobkin, 2009]. The NO-96 corrugations were modelled as rigid short plates (tongues) which are perpendicular to the main elastic panel. The gas trapped before the impact between two tongues was assumed to be mixed instantly with the fluid. The gas-fluid mixture was modelled as a fictitious compressible media with reduced sound speed.

The present analysis of fluid impact onto a corrugated elastic panel is similar to that in [Korobkin, Khabakhpasheva, 2006], where elastic plate impact without corrugations was studied. They used the Wagner approach for the hydrodynamic part of the problem and normal mode method for the elastic plate vibrations. Impact of a rigid body with attached cavity was studied in [Korobkin, 1996].

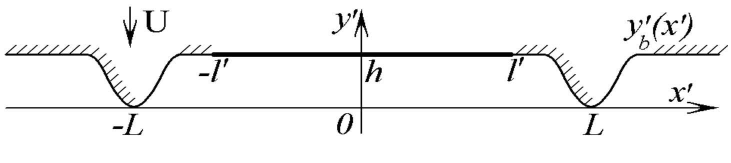

Fig. 1.

We consider the early stage of liquid impact onto a flexible corrugated panel. The panel is flat with two rigid symmetric corrugations (Fig. 1). Part of the plate $-l' < x' < l'$ is flexible and the rest of the panel including corrugations is rigid. At the initial time instant $t' = 0$ the corrugations touch the free surface at $x' = \pm L$ enclosing the gas cavity. Than the panel starts to penetrate the liquid with the constant velocity U. It is assumed that the gas pressure inside the cavity is a function of time only. The pressure in the gas cavity depends only on the cavity volume which is determined by the length of the the wetted part of the panel, elastic deflection of the panel and by the shape of the cavity free surface.

Elastic deflection of the panel is described by the Euler beam equation. The beam is clamped to the main structure at $x' = \pm l$. The coupling between the fluid flow, gas pressure in the cavity and the deflection of the beam is achieved through the dynamic and kinematic conditions on the wetted surface of the panel and on the cavity free surface. The Wagner approach is used in the present analysis to determine the fluid flow, pressure distribution and the free-surface shape.

2. Formulation and solution

Entry of an elastic panel with corrugations onto the initial calm fluid is considered. The panel is symmetric with respect to the vertical axis Oy' (Fig. 1). The line $y' = 0$ corresponds to the initial undisturbed position of the free surface. Dimensional variable are denoted by a prime. At the initial time instant ($t' = 0$), the panel touches the liquid surface at two points $x' = \pm L$. The cavity filled with compressible gas is bounded by the free surface, $|x'| < L$, $y' = 0$, from below and by the panel surface from above. The hight of corrugations is h. We assume that $h/L \ll 1$. Then the panel starts to penetrate the liquid vertically with the initial impact velocity U.

We assume that the pressure $P'(t')$ inside the cavity is a function of time only, $P'(0) = 0$. The equation of state of the gas is taken in the Tate form $P' = P^*[(\rho'/\rho_0)^\nu - 1]$, where P' is the deviation of the pressure from its initial atmospheric value, ρ' is the gas density, ρ_0 is the initial gas density, ν and P^* are constants which depend on the gas properties, $P^* = \rho_0 c_0^2/\nu$, c_0 is the sound speed in the gas at rest. The mass conservation law $\rho' V' = \rho_0 V_0$ is hold, where $V'(t')$ is the cavity volume, $V_0 = V'(0)$. The position of the entering body is given by the equation $y' = f'(x') - s'(t')$, where $s'(t')$ is the depth of penetration, $s'(0) = 0$, $(ds'/dt')(0) = U$.

The problem is considered in non-dimensional variables, where L is the length scale, h is the displacement scale, U is the velocity scale, $\rho_l U^2 L/h$ is the hydrodynamic pressure scale, ρ_l is the liquid density, P^* is the scale of pressure in the cavity, the ratio h/U is taken as the time scale.

In the dimensionless variables, which are designated by the above-mentioned terms without primes, the boundary conditions are imposed on the undisturbed initial level of the liquid and linearized together with the equations of liquid motion.The linearization leads to the well-known Wagner approach [Wagner, 1932] where the velocity potential satisfies the Laplace equation in the lower half-plane $y < 0$ and mixed boundary conditions on the line $y = 0$. The division of the liquid boundary (see Fig. 2) into the cavity surface, $|x| < c_1(t)$, the contact region $c_1(t) < |x| < c_2(t)$ and the outer free surface, $|x| > c_2(t)$ is unknown and must be determined together with the solution using the condition that displacements of liquid particles are bounded during the initial stage.

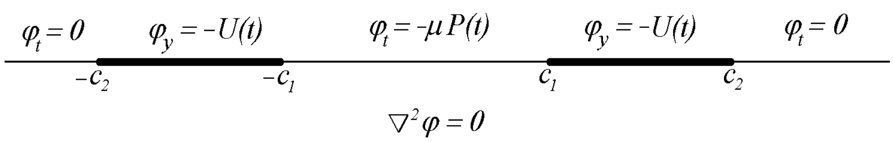

Fig. 2.

The initial stage is subdivided into two phases. During the first phase (Fig. 3,a) the elastic part of the panel is still dry, $l < c_1(t) < 1$, and flexibility of the panel modifies only the volume of the gas cavity and the pressure in the cavity but not directly the fluid flow. In this phase, the hydrodynamic part of the problem is the same as for a corresponding rigid panel. However, the pressure in the gas cavity has to be computed with account for the panel deflection. During the second phase (Fig. 3,b), $0 < c_1(t) < l$, both the pressure in the cavity and the flow must be computed with account for the panel deflection. In this study, only the first phase of the impact is considered.

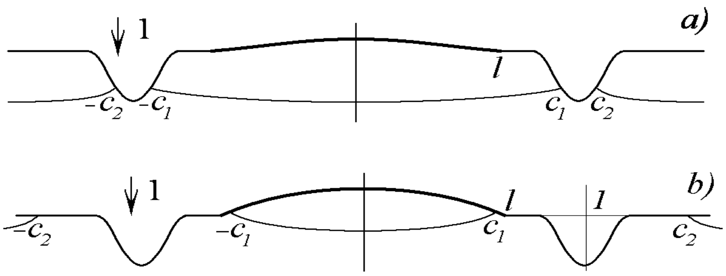

Fig. 3.

In the non-dimensional variables the boundary-value problem for the velocity potential $\varphi(x, y, t)$ and the elastic deflection of the panel $w(x, t)$ has the form

$$\nabla^2\varphi = 0 \qquad (y < 0), \tag{1}$$

$$\varphi_y = -1 + w_t(x, t) \qquad (y = 0, \quad c_1 < |x| < c_2), \tag{2}$$

$$\varphi_t = -\mu P(t) \qquad (y = 0, \quad |x| < c_1), \tag{3}$$

$$\varphi = 0 \qquad (y = 0, \quad |x| > c_2), \tag{4}$$

$$\alpha w_{tt} + \beta w_{xxxx} = Q(x, t), \tag{5}$$

$$w(\pm l, t) = 0, \qquad w_x(\pm l, t) = 0, \tag{6}$$

$$w(x, 0) = w_t(x, 0) = 0, \qquad \varphi(x, y, 0) = 0, \qquad P(0) = 0. \tag{7}$$

Here

$$\mu = \frac{1}{\nu}\frac{\rho_0}{\rho_l}\frac{h}{L}\left(\frac{c_0}{U}\right)^2, \qquad \alpha = \frac{m_b}{\rho_l L}, \qquad \beta = \frac{EJh^2}{\rho_l L^5 U^2}, \qquad \kappa = \frac{hL}{V_0},$$

$$P(t) = V^{-\nu}(t) - 1, \qquad V(t) = \tilde{V}(t) + 2\kappa\int_0^l w(x, t)\, dx.$$

The function $\tilde{V}(t)$ is obtained by integration of the following differential equation

$$\frac{d\tilde{V}}{dt} = -2\kappa\left[c_1(t) + \int_0^l \varphi_y(x, 0, t)dx\right], \qquad \tilde{V}(0) = 1.$$

The function $Q(x, t)$ in (5) is given by $Q(x, t) = \mu P(t)$ during the fist phase, and $Q(x, t) = \mu P(t)$, where $|x| < c_1(t)$, $Q(x, t) = -\varphi_t$, where $c_1 < |x| < l$, during the second phase. In the body boundary condition (2), $w(x, t) \equiv 0$ during the first phase.

The boundary-value problem (5)–(6) is solved with the help of the normal mode method. This method leads to infinite system of ordinary differential equations with respect to principal coordinates of the beam deflection $w(x, t)$, $-l < x < l$. Within the framework of the normal mode method the beam deflection $w(x, t)$ is sought in the form

$$w(x, t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(x).$$

Here $\psi_n(x)$ are non-trivial solutions of the homogeneous boundary-value problem

$$\frac{d^4\psi_n}{dx^4} = \lambda_n^4\psi_n \;\; (-l < x < l), \qquad \psi_n = \psi_n' = 0 \;\; (x = \pm l),$$

and λ_n are the corresponding eigenvalues. The eigenfunctions $\psi_n(x)$ are orthonormal. In the case of clamped beam and symmetric configuration (see Fig.1) the eigenfunctions $\psi_n(x)$ are given by

$$\psi_n(x) = B_n\left[\cos(\lambda_n x) - \frac{\cos(\lambda_n l)}{\cosh(\lambda_n l)}\cosh(\lambda_n x)\right], \qquad \tan(\lambda_n l) = -\tanh(\lambda_n l) \qquad (n \geq 1).$$

It is convenient to introduce new unknown functions

$$A_n(t) = \frac{a_n(t)}{\mu C_n\sqrt{l}}, \qquad C_n = -\frac{4B_n}{\lambda_n\sqrt{l}}\cos(\lambda_n l)\tanh(\lambda_n l).$$

During the first phase the functions $A_n(t)$ satisfy the following differential equations

$$\alpha A_n'' + \beta\lambda_n^4 A_n = P(t) \quad (t > 0) \qquad A_n(0) = A_n'(0) = 0. \tag{8}$$

The cavity volume is calculated now as

$$V(t) = \tilde{V}(t) + \kappa\mu l\sum_{n=1}^{\infty} C_n^2 A_n(t).$$

In order to derive equations for the functions $c_1(t)$ and $c_2(t)$, we consider the vertical displacement of the liquid free surface

$$Y(x, 0, t) = \frac{1}{\pi W(x)}\left(\int_{-c_2}^{-c_1}\frac{y_b(\tau, t)W(\tau)}{\tau - x}d\tau - \int_{c_1}^{c_2}\frac{y_b(\tau, t)W(\tau)}{\tau - x}d\tau + F(t)\right),$$

where $y_b(x,t) = f(x) - t$, and $F(t)$ is undetermined function of time. The characteristic function $W(\tau)$ in the symmetrical contact region is given as

$$W(\tau) = \sqrt{(\tau^2 - c_1^2)(c_2^2 - \tau^2)}.$$

The vertical displacement $Y(x,0,t)$ is bounded at the contact points $x = \pm c_1$ and $x = \pm c_2$ if and only if

$$-2\int_{c_1}^{c_2} y_b(x,t)\sqrt{\frac{(c_2^2 - x^2)(x + c_1)}{(x - c_1)}}\,dx + F(t) = 0,$$
$$2\int_{c_1}^{c_2} y_b(x,t)\sqrt{\frac{(x^2 - c_1^2)(x + c_2)}{(c_2 - x)}}\,dx + F(t) = 0.$$

Differentiation of these equations with respect to time leads to the following system for the derivatives $\dot{c}_2$ and $\dot{c}_1$

$$a_{11}\dot{c}_1 + a_{12}\dot{c}_2 = b_1, \qquad (9)$$
$$a_{21}\dot{c}_1 + a_{22}\dot{c}_2 = b_2 + \dot{F}(t),$$

initial conditions for which are $c_1(0) = c_2(0) = 1$. Here a_{11}, a_{12}, a_{21}, a_{22}, b_1 and b_2 are given functions of c_1, c_2 and t.

During the first phase of the impact the hydrodynamic part of the problem (1)–(7) is the same as for the rigid panel. The solution of the impact problem for the rigid body provides [Korobkin, 1996]

$$\frac{d\tilde{V}}{dt} = \frac{\kappa}{K(m)}\left[\,2\,W(t)\,K(1-m) - \pi c_2(t)\right], \qquad \tilde{V}(0) = 1, \qquad (10)$$

$$\frac{dF}{dt} = c_2^2(t)\frac{E(m)}{K(m)} - c_2(t)\frac{W(t)}{K(m)} - \frac{c_1^2(t) + c_2^2(t)}{2}, \qquad F(0) = 0. \qquad (11)$$

where $m(t) = 1 - c_1^2(t)/c_2^2(t)$ and $W(t)$ is obtained by integration of the differential equation

$$\frac{dW}{dt} = \mu P(t), \qquad W(0) = 0. \qquad (12)$$

Here $K(m)$ and $E(m)$ are the complete elliptic integrals of the first and second kind, respectively.

The infinite system of ordinary differential equations (8)–(12) is integrated numerically in time by the Runge-Kutta method. Distribution of strains $\varepsilon(x,t)$ in the elastic part of the panel is calculated by the formula $\varepsilon(x,t) = (h_b h/2L^2)w_{xx}(x,t)$ where h_b is the beam thickness.

Results of calculations will be presented at the Workshop.

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