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Damping of oscillatory body-motion at large forward speed

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We consider a slender body moving with forward speed in otherwise calm water. The body is undergoing an oscillatory motion in some of its six degrees of motion. Calculation of the wave radiation due to the body's oscillatory motion, particularly the damping forces, enables an investigation of the efficiency and the energy loss of the motion. This depends on the details of the hull shape (and trivially on the motion amplitude). We have in mind human powered vessels like a kayak. The paddling motion results in significant yaw and roll motions which generate waves that are superposed on the steady wave pattern lagging behind the vessel.

The calm-water wave and drag resistance of kayaks have been investigated by L. Lazauskas, J. Winters and E. O. Tuck - Hydrodynamic Drag of Small Sea Kayaks - in 1997, and is available on the Internet. They calculated the wave resistance using the Michell's integral, obtained the skin friction drag from the 1957 ITTC line, and compared the calculations to experiments, finding generally a good agreement. Effects due to trim and sinkage were not included in their predictions. In addition to the energy loss due to the steady waves left behind the kayak, and the skin friction, there is energy loss due to the oscillatory motion resulting from the paddler's alternating oare motion, particularly in yaw and roll. This loss may eventually be a small fraction in the big picture. However, in competitions, a reduction in the energy loss, even on the level of one percent, results in a gain in the forward speed and is an advantage. Our purpose is to model the energy loss due to the oscillatory motion in the body's motion modes, varying the vessel shape.

The mathematical problem relates to theories for oscillatory ship motion at forward speed, and have been developed by Salvesen, Tuck and Faltinsen (1970), Faltinsen and Zhao (1991), see also Faltinsen (2005). Slender ship theories may also be found in Newman (1977, Ch. 7). We consider a slender body of length l, draft D and beam B, moving with forward speed U on the surface of otherwise calm water, under the effect of gravity, with g the acceleration of gravity. The body is undergoing oscillatory motion with frequency ω. In the actual application l is about 5 m, D about 0.1-0.2 m, B about 0.5 m, U about 5 m/s and $\omega = 2\pi/T$ with T about 1 second. Assuming application of potential theory, the velocity potential of the motion, considered in the frame of reference $O - xyz$ moving with the forward speed of the kayak, with x along the forward speed direction and z vertical upward, is decomposed by

$$\Phi = -Ux + U\chi + \phi_R \tag{1}$$

where χ represents the steady motion and ϕ_R the radiation potential. We study the oscillatory effect in yaw. The motion in the other modes modes of motion may be studied similarly.

The steady potential χ

Consider the steady part of the potential, χ. This is a Laplacian velocity potential satisfying the body boundary condition $\partial\chi/\partial n = n_1$ where $n_1 = \mathbf{n}\cdot\mathbf{i}$, $\mathbf{n}$ the normal vector along the hull and $\mathbf{i}$ unit vector along x-direction.

We introduce a slenderness parameter by $\epsilon = B/L$. We also introduce a slow variable along the x-direction by $X = \epsilon x$. This means that $\partial/\partial x = \epsilon\partial/\partial X$, while $\partial/\partial X \sim \partial/\partial y \sim \partial/\partial z$. This means that $n_1 = O(\epsilon)$ and that χ is $O(\epsilon)$.

The slenderness assumption here differs from that in classical theories, e.g. in Faltinsen and Zhao (1991) where they employ a slow variable by $X_1 = \epsilon^{\frac{1}{2}}x$, such that $n_1 = O(\epsilon^{\frac{1}{2}})$ and that $\chi = O(\epsilon^{\frac{1}{2}})$. The resulting boundary conditions at the free surface are then expressed in a different way, including some more terms than those that appear in the present analysis.

In our case the free surface condition for χ reads

$$\frac{U^2}{g}\frac{\partial^2\chi}{\partial x^2} + \frac{\partial\chi}{\partial z} + O(\epsilon^2) = 0 \tag{2}$$

at the surface, or, alternatively, close to the body

$$\frac{\epsilon^2 U^2}{g}\frac{\partial^2\chi}{\partial X^2} + \frac{\partial\chi}{\partial z} + O(\epsilon^2) = 0 \tag{3}$$

at the surface, which means that the steady flow at the body may be represented by the double body motion. The far-field waves may eventually be evaluated by a far-field analysis.

The oscillatory motion

The oscillatory potential assumes the form

$$\phi_R = Re\ (\mathrm{i}\omega e^{\mathrm{i}\omega t}\xi_6\psi) \tag{4}$$

We consider the free surface condition for this potential, which is linearized in ψ but the coupling effect to χ is accounted for, giving

$$-\frac{\omega^2}{g}\psi + \frac{\partial\psi}{\partial z} + \frac{2\mathrm{i}U\omega}{g}\Big(-\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\chi}{\partial x} + \frac{\partial\psi}{\partial y}\frac{\partial\chi}{\partial y} + \tfrac{1}{2}\psi\frac{\partial^2\chi}{\partial z^2}\Big) + \frac{U^2}{g}\frac{\partial^2\psi}{\partial x^2} = 0 \tag{5}$$

at $z = 0$. Introducing the slow variable X in the x-direction, we obtain

$$-\frac{\omega^2}{g}\psi + \frac{\partial\psi}{\partial z} + \frac{2\mathrm{i}\epsilon U\omega}{g}\Big(-\frac{\partial\psi}{\partial X} + \epsilon^2\frac{\partial\psi}{\partial X}\frac{\partial\bar{\chi}}{\partial X} + \frac{\partial\psi}{\partial y}\frac{\partial\bar{\chi}}{\partial y} + \tfrac{1}{2}\psi\frac{\partial^2\bar{\chi}}{\partial z^2}\Big) + \frac{\epsilon^2 U^2}{g}\frac{\partial^2\psi}{\partial X^2} = 0 \tag{6}$$

at $z = 0$, where $\chi = \epsilon\bar{\chi}$ and $\bar{\chi} = O(1)$. Neglecting terms of the order $O(\epsilon^2)$ we obtain

$$-\frac{\omega^2}{g}\psi + \frac{\partial\psi}{\partial z} + \frac{2\mathrm{i}\epsilon U\omega}{g}\Big(-\frac{\partial\psi}{\partial X} + \frac{\partial\psi}{\partial y}\frac{\partial\bar{\chi}}{\partial y} + \tfrac{1}{2}\psi\frac{\partial^2\bar{\chi}}{\partial z^2}\Big) = 0 \tag{7}$$

at $z=0$. If the contributions from $\bar{\chi}$ are small, such as in the flat ship approximation, or, χ is not small, but the motion is observed at some distance from the body, the free surface condition becomes

$$-\frac{\omega^2}{g}\psi + \frac{\partial\psi}{\partial z} - \frac{2\mathrm{i}\epsilon U\omega}{g}\frac{\partial\psi}{\partial X} = 0 \tag{8}$$

at $z=0$.

The body boundary condition reads $\psi_n = n_6 + Um_6/(\mathrm{i}\omega)$, where $n_6 \simeq xn_2$ and $m_6 = -n_2 + X\partial\bar{\chi}_y/\partial n$, and is applied at the average position of the body.

The free surface condition (7) and the mathematical formulation including the body boundary condition, are similar to those in the slow forward speed problem. This has been investigated in several publications in the late 1980s and the 1990s. By that time the calculation of the so-called wave drift damping was of the main interest. Here, the task is easier, as the goal is to evaluate the linear damping force.

We note that, in the small forward speed problem, the parameter $U\omega/g$ is small, because the forward speed is small. Here, where the forward speed is great, the product $U\omega/g$ is not small. However, because the slenderness parameter ϵ is small, the product

$$\tau = \frac{\epsilon U\omega}{g} \tag{9}$$

is small.

The boundary value problem for ψ may be solved by means of integral equations. A Green function G satisfies the linear free surface boundary condition, i.e.

$$-KG + G_z - 2\mathrm{i}\tau G_X = 0 \tag{10}$$

where $K = \omega^2/g$. The Green function is obtained on the form

$$G = \frac{1}{r} - \frac{1}{r_1} + G_{wave} \tag{11}$$

where $r = |(x,y,z) - (x',y',z')|$, r_1 its image with respect to $z=0$ and G_{wave} is obtained from the free surface condition (10) by inverse Fourier transform.

The boundary value problems for the steady flow field $\bar{\chi}$ and the oscillatory motion represented by ψ can be treated using the formulation derived by Nossen, Grue and Palm (1991) and Grue and Biberg (1993). The following boundary integral equation is obtained for ψ, which differs somewhat from the equations in Nossen et al. and Grue and Biberg on a few points, because here $\partial/\partial x << \partial/\partial y$, $\partial/\partial x << \partial/\partial z$. We obtain

$$\int_B \Big[\psi G_n - \Big(G - \frac{\tau}{\mathrm{i}K}\nabla G\cdot\nabla(\bar{\chi} - X)\Big)n_6\Big]dS - 2\mathrm{i}\tau\int_F (\psi G_y\bar{\chi}_y - \tfrac{1}{2}G\bar{\chi}_{zz}) = -2\pi\psi \tag{12}$$

for (x,y,z) on the body boundary B, where in the equation F denotes integration over the mean free surface. In the equation $\bar{\chi}$ is precalculated. The integral over F converges rapidly.

A local variant of the Green function may be obtained assuming that $G = G_0 + \tau G_1$, where G_0 denotes the zero speed Green function, satisfying $-KG_0 + G_{0,z} = 0$ at the mean free surface, and G_1 is a correction because of the forward speed, and is obtained by

$$G_1 = 2\mathrm{i}\frac{\partial^2 G_0}{\partial K \partial X} \tag{13}$$

$G_0 + \tau G_1$ satisfies the free surface condition (10). Similarly, the potential is obtained by an expansion $\psi = \psi_0 + \tau\psi_1$ where ψ_0 satisfies the linearized free surface condition with $U = 0$.

The oscillatory force

The oscillatory force on the body is obtained by

$$F_i = \mathrm{Re}(\mathrm{i}\omega\xi_6 e^{\mathrm{i}\omega t} f_{i6}) \tag{14}$$

where

$$f_{i6} = \mathrm{i}\omega a_{i6} + b_{i6} = \rho \int_B (\mathrm{i}\omega\psi + U\epsilon\nabla(\bar{\chi} - X)\cdot\nabla\psi) n_i dS \tag{15}$$

where ρ denotes density. Evaluation of the radiation potential and the force coefficients a_{i6} and b_{i6}, particularly b_{66}, may be obtained by solving the integral equation (12) and then integrating the expression in (15). However, it is well known from e.g. Nossen et al. (1991) that there is no forward speed effect on the force coefficients along the diagonal, in the small forward speed problem (or in the slender body approximation), and therefore, particularly the damping coefficient b_{66}, may be obtained by the zero speed value. Calculation of the forces and the energy loss in the yaw mode of motion with comparison to the wave and drag resistance loss is in progress.

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