Hydrodynamic aspects of a floating fish farm with circular collar

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The desire to move fish farms to more wave exposed sites increases the attention to marine technology. Mooring-line loads, cage-volume reduction, sufficient water exchange in a cage and contact between chains or ropes with the net in combined waves and current are, for instance, of concern. Our objective is to develop hydrodynamic methods for the commonly used floating fish farms with circular plastic collar made of high-density polyethylene (HDPE) pipes (see Figure 1). Two concentric pipe circles that are linked together are often used. Each pipe circle is a torus in calm water. The floater may also consist of one or three pipe circles. Our focus is first on the wave effects on the floater by neglecting current. Wave lengths of interest are long relative to the cross-section. Wave radiation and scattering are secondary. A slender-body theory based on a rigid free-surface condition will be derived. Three-dimensional flow is essential for the vertical loads. Wave-induced elastic behavior of the floater matters as demonstrated in Figure 1.

Figure 1. To the left: Details of circular HDPE cages (SINTEF Fisheries and Aquaculture). To the right: Circular floating collar of HDPE without attached net cage in bad weather (David Kristiansen).

We will study either one torus or two closely spaced concentric tori. The torus is semi-submerged in water of infinite depth and infinite horizontal extent. We define a Cartesian coordinate system (x, y, z) with the mean free surface at $z = 0$. The $z-$axis is the torus axis and upwards. The incident waves propagate along the x-axis. A far-field view is presented in Figure 2 where the radius R is introduced. When one torus is investigated, R means the radius of the circular centre-line curve of the torus. When two tori are studied, R refers to the radial distance from the $z-$axis to midways between the two tori. The vertical vibration velocity of the circular floater can be expressed as the Fourier series $\dot{a}_0 + \sum_{n=1}^{\infty} \dot{a}_n(t) \cos n\beta$ where β is related to the $x-$ and $y-$coordinates on the torus (tori) by $x = R\cos\beta,\ y = R\sin\beta$ and $\dot{a}_0$ and $\dot{a}_1 \cos\beta$ are due to heave and pitch. Potential flow theory of incompressible water is assumed. We consider the limiting case that the forcing frequency $\omega \to 0$ which means that a rigid free-surface condition can be used. The crossdimensional radius c of a torus is assumed small relative to R so that slender-body theory is appropriate. Matched asymptotic expansions with a far-field and near-field description are used. We do not see the details of the cross-dimensions of the floating torus (tori) in the far-field and the dominant flow appears as a line distribution of sources with density $Q\cos n\beta$ along the centre line of the torus (tori). The source points are at $\xi = R\cos\alpha,\ \eta = R\sin\alpha,\ \zeta = 0$ and the field point is at $\left(x = a\cos\beta, y = a\sin\beta, z\right)$. The far-field velocity potential can be expressed as

$$\varphi^F = \frac{QR}{4\pi}\cos n\beta\int_0^{2\pi}\frac{\cos n\alpha_1 d\alpha_1}{\sqrt{r^2+2aR\left[1-\cos\alpha_1\right]}},\quad \alpha_1 = \alpha-\beta,\quad r = \sqrt{\left(a-R\right)^2+z^2}$$

A first term φ_I^F of the inner expansion is:

$$\varphi_I^F = \frac{Q\cos n\beta}{2\pi}\left[-K_n + \ln\left(\frac{8R}{r}\right)\right], K_n = \frac{1}{2\sqrt{2}}\int_0^{2\pi}\frac{1-\cos(nx)}{\sqrt{1-\cos(x)}}dx \Rightarrow$$

$$K_0 = 0, K_1 = 2, K_2 = \frac{8}{3}, K_3 = \frac{46}{15}, K_4 = \frac{352}{105}, K_5 = \frac{1126}{315}, K_6 = \frac{13016}{3465}, K_7 = \frac{176138}{45045},..$$

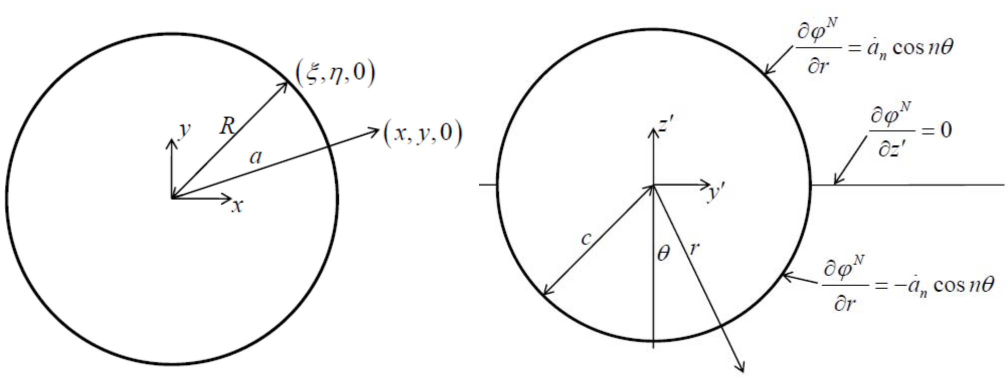

Figure 2. To the left: Far-field view of a circular floater with either one torus or two tori. To the right: Cross-section of a torus with coordinate system and boundary conditions for the near-field solution of the velocity potential associated with forced vertical oscillations of Fourier component n.

We start with discussing the near-field solution for one torus. A semi-circular submerged cross-section of radius c is forced vertically with velocity $\dot{a}_n\cos n\theta$. We can image the semi-circle about the mean free surface and consider the problem where the image semi-circle is moving vertically with opposite sign to the submerged semi-circle. The consequence is that the rigid free-surface condition is satisfied. We define a local polar coordinate system $\left(r,\theta\right)$ and Cartesian coordinate system $\left(y',z'\right)$ with origin in the centre of the circle (see Figure 2). Their relationship is $y' = r\sin\theta,\ z' = r\cos\theta$ with $\theta = 0$ corresponding to the negative z' – axis. It follows by matching that the complete near field solution of the velocity potential can be expressed as

$$\varphi^N = \dot{a}_n\cos n\theta\left\{\frac{2c}{\pi}\left[\ln\left(\frac{8R}{r}\right)-K_n\right]-\sum_{m=1}^{\infty}c^{2m+1}\frac{3\cos\left(m\pi\right)+\cos\left(3m\pi\right)}{\pi 2m\left(4m^2-1\right)}\frac{\cos 2m\theta}{r^{2m}}\right\}$$

The matching determines a constant in the near-field solution that makes the solution unique. If a purely two-dimensional solution with the classical frequency-domain free-surface condition with gravity is used for deep water, the constant goes to infinity when the frequency $\omega \to 0$ and thereby causes infinite added mass for vertical modes of a 2D free-surface piercing body. We have used a BEM to formulate the near-field solution for two tori. Having obtained the velocity potential we can derive the added mass. We introduce a generalized added mass coefficient $A_{(n+3)(n+3)}$ by multiplying the two-dimensional vertical linear hydrodynamic force associated with each mode with $\cos n\theta$ and integrate along the centerline of the torus (tori). It follows that

$$A_{(n+3)(n+3)} = a_{33}^{(n)} K 2\pi R,\ K = 2 \text{ for } n = 0,\ K = 1 \text{ for } n = 1, 2, \ldots$$

$$a_{33}^{(n)} / \left(\rho c^2\right) = \frac{2}{\pi}\left[\ln\left(\frac{8R}{c}\right) - K_n\right] + \underbrace{\sum_{m=1}^{\infty} \frac{\left(3\cos m\pi + \cos 3m\pi\right)\cos m\pi}{\pi 2m\left(4m^2 - 1\right)^2}}_{0.07238725793},\ \text{for one torus}$$

$$a_{33}^{(n)} = f + \rho\frac{16c^2}{\pi}\left(\ln\left(\frac{8R}{c}\right) - K_n\right),\ \frac{f}{\rho c^2} = 5.74604 - 5.76835\left(\frac{2p}{c}\right) + 1.55575\left(\frac{2p}{c}\right)^2$$

$$-0.21295\left(\frac{2p}{c}\right)^3 + 0.01128\left(\frac{2p}{c}\right)^4 \text{ for two tori with } 2.0 < 2p/c < 6.0$$

Here $2p$ is the distance between the axes of the two semi-submerged circular cylinders that each has a radius c. The values of f have been numerically obtained by Shao (personal communication, 2010). The hydrodynamic interaction between the cylinders is verified for large $2p/c$ by assuming a source behavior of the individual cylinders.

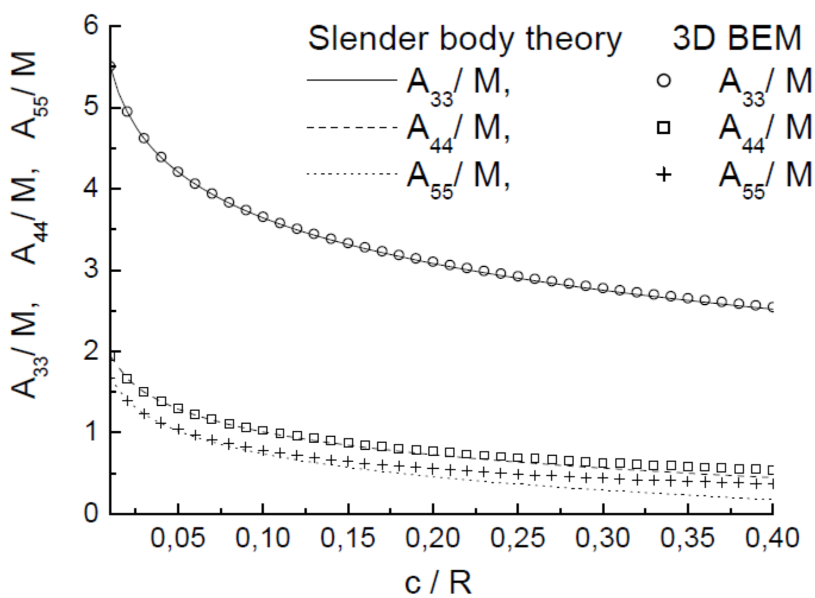

Figure3. Asymptotic zero-frequency added mass A_{33}, A_{44} and A_{55} due to heave, pitch and lowest elastic mode of a semi-submerged torus in deep water. M = displaced mass. c= radius of torus cross-section. R= torus radius.

Figure 3 presents A_{33}/M , A_{44}/M and A_{55}/M for one torus as a function of c/R. $M=\rho\pi^2 Rc^2$ is the displaced mass. The present slender-body theory agrees well with a 3D HOBEM for $c/R-$ values of interest for fish farms. Convergence studies showed a relative numerical error of 10^{-4} (Shao, personal communication, 2011). The slender-body predictions of A_{33} agree well even for the non-small largest tested value $c/R=0.4$.

The vertical excitation loads assume that the incident wave potential is expressed as $\varphi_0=(g\zeta_a/\omega)\exp(kz+\mathrm{i}kx-\mathrm{i}\omega t)$, $k=\omega^2/g$, $\mathrm{i}=\sqrt{-1}$. The velocity potential along the centerline of the torus (tori) can be expanded in a Fourier series involving Bessel functions. When solving the diffraction problem, we can utilize the solutions obtained for forced vertical oscillations. The vertical Froude-Kriloff force per unit length on the cross-section of the floater can be approximated as

$$f_3^{FK}=\rho g\zeta_a \mathrm{i}\left[J_0(kR)+\sum_{m=1}^{\infty}2\mathrm{i}^m J_m(kR)\cos m\beta\right]b_w\exp(-\mathrm{i}\omega t),$$
$$b_w=2c\,(\text{one torus}),\; b_w=4c\,(\text{two tori})$$

Here $J_m(kR)$ are Bessel functions of the first kind. The vertical diffraction loads per unit length is

$$f_3^{D}=-\mathrm{i}\,\omega^2\zeta_a\left[J_0(kR)a_{33}^{(0)}+\sum_{m=1}^{\infty}2\mathrm{i}^m J_m(kR)a_{33}^{(m)}\cos m\beta\right]\exp(-\mathrm{i}\omega t)$$

The fact that strip theory is appropriate for lateral loads follows by matched asymptotic expansions and that the far-field behavior is a distribution of lateral dipoles. The beam equation can approximate the structural influence of the floater. The torus (tori) radius R must be accounted for in the beam equation of the lateral floater deflections. The rigid body surge motion has to be separately considered. Loads associated with the mooring system, the netting and weights restraining the netting must be incorporated and provide coupling between lateral and vertical motions of the fish farm. Viscous loads play an important role in the analysis of the netting. Tangential force and pressure loss coefficients across the netting have been derived by accounting for the effect of Reynolds number of a twine, solidity ratio and the local orientation of the netting relative to the inflow. Continuity of the flow through the net, strip theory and drag coefficients for a single circular cylinder has been used. Error sources are due to flow around the net, detailed cross-sectional shape of a twine and the mesh orientation. The solidity ratio must be less than ~ 0.5. There is reasonable agreement with experimental results for planar netting panels with different orientations. If there is a current, the wake flow inside the net cage plays an important role in our comparisons with experiments for a cylindrical net mounted to a fixed frame. The wake is estimated by superposing the wake from individual twines. The deformation of the net is important for a floating fish farm. An essential factor is membrane forces. The elastic elongation of the twines changes the solidity ratio. An error source is the influence of the fish on the flow.

Parametric resonance in surge associated with large change of wetted area of a floater without net in 2D flow was detected by experiments, CFD and a simplified model accounting for nonlinear Froude-Kriloff and restoring forces. It depends on natural periods and damping if this can happen for a complete floating fish farm.