A wave energy converter with an internal water tank

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(26th IWWWFB – Athens, Greece – 17-20 April 2011)

This Workshop is dedicated to Odd Faltinsen, who has made so many valuable contributions to our field and has been a regular participant at the Workshops. It is appropriate therefore that we should address the topic of sloshing in tanks, one of Odd's many special interests and the subject of his latest book (Faltinsen & Timokha, 2009).

Introduction

Many wave energy converters (WECs) rely on resonance for efficient power conversion and the theory is well understood for devices operating in a single degree of freedom. The introduction of further resonances can improve power conversion by extending the frequency band over which the mean power is maximised. A recent review of various types of WECs which exploit this idea is given by Evans & Porter (2010) where the idea of including an internal water tank in the WEC and extracting power from the sloshing motion is introduced. The use of internal tanks as tuned liquid dampers to reduce resonant oscillations of tall buildings has been known for some time. The aim there is to match the lowest sloshing frequency with the resonant frequency of the building, thereby reducing its motion. In the present study we also exploit the coupling between the sloshing frequencies and the natural resonance of the WEC while at the same time extracting energy from the system.

Formulation of the problem

We consider a particular case of a class of WECs introduced by Evans & Porter (2010), shown in Figure 1. It consists of two concentric horizontal circular cylinders of lengths L with closed ends and radii a and b, where $0 < b < a$. The annular region $b < r < a$ forms an internal tank which is partially filled to a depth such that there are two separate free surfaces. The buoyant WEC is totally submerged and held by vertical moorings

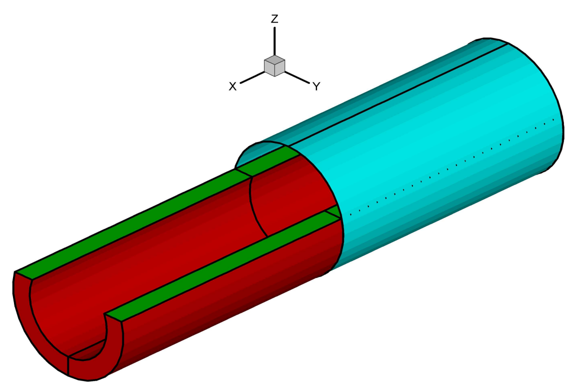

Figure 1: Perspective view of the cylinder and coaxial annular tank. The length is 8m and the outer radius is 1m. The tank inner radius is 0.7m and its free surface is 0.25m above the axis. The left half of the cylinder is removed to show the tank.

extending from each end down to the sea-bed, so that it can make angular motions about its sea-bed attachment. In a Cartesian co-ordinate system with z measured vertically upwards from the mean external free surface, the axis of both cylinders occupy $0 < x < L$, $y = 0$, $z = -f$, where $f > a$, and the level of the internal water surfaces in equilibrium is at $z = -f + c$ so that $c = 0$ corresponds to the annular region being half full.

For simplicity we consider beam waves of frequency $\omega/2\pi$, so that the WEC makes small sway oscillations of the same frequency due to the horizontal component of the tension in the moorings. Thus we can regard the motion of the enclosed water as being two-dimensional in the (y, z)-plane and to be odd in y. The equation of motion of the converter is

$$X_w + X_i - i\omega^{-1}CU = -i\omega MU. \qquad (1)$$

Here the complex time-dependent factor $e^{-i\omega t}$ is assumed, U is the sway velocity, X_w, X_i the sway forces on the converter due to the exterior hydrodynamic pressure and the internal sloshing respectively, and $C \equiv M_w(1-s)g/l$ is the restoring force due to buoyancy, where $M_w \equiv \pi\rho a^2 L$ is the mass of the displaced water, M is the mass of the converter excluding the internal water, $s = M_i/M_w$ is its specific gravity, where M_i is the mass of the WEC including the internal fluid, and l is the length of the mooring lines measured from the centre of the cylinder. The mean power from the incident waves is $W = \Re X_w\overline{U}/2 = -\Re X_i\overline{U}/2$ from (1).

In the absence of any damping of the enclosed water the effect of the tank is simply to exert a sway force on the WEC in the form of $X_i = i\omega A_u U$ where A_u is the added mass of the enclosed water, and the power is zero. In order to extract power from the system we need to introduce damping. We assume the free surface of the enclosed water to occupy the intervals $S^+(S^-)$ in $y > 0$ $(y < 0)$ respectively and that the air trapped above the free surface S^+ is forced by the antisymmetric motion to pass into the region above the free surface S^- by way of a turbine contained in a thin rigid vertical baffle connecting the two sides of the tank above the free surfaces. Thus the excess pressure on, and the volume flux across the internal free surface, will be $\pm P$ and $\pm Q$ on $S^\pm$ respectively.

It follows that we need only consider $y > 0$ provided that the harmonic velocity potential Φ of the internal water satisfies $\Phi(0,z) = 0$. We also require

$$\Phi_r = U\sin\theta \tag{2}$$

on S_B, the internal surfaces of the tank bounding the enclosed water in $y > 0$, where $r\cos\theta = z + f$, $r\sin\theta = y$. On the internal free surface, (see for example, Falnes (2002) §7.1, equn. (7.15)), it can be shown that

$$K\Phi - \Phi_z = -i\omega P/\rho g, \qquad z \in S^+ \tag{3}$$

It is convenient to write

$$\Phi = U\phi^{(u)} + P\phi^{(p)}, \tag{4}$$

where $\phi^{(u)}$ satisfies (2) with $U = 1$, and (3) with $P = 0$, whilst $\phi^{(p)}$ satisfies (2) with $U = 0$, and (3) with $P = 1$. It follows that $\phi^{(u)}$ is real and $\phi^{(p)}$ is pure imaginary. The force exerted by the internal water is

$$X_i = 2i\omega\rho L\int_{S_B}\Phi(y,z)n_y ds \equiv Uf_u + Pf_p, \tag{5}$$

where

$$f_{u,p} = 2i\omega\rho L\int_{S_B}\phi^{(u,p)}(y,z)n_y ds, \tag{6}$$

and the factor of 2 arises since $\phi^{(u)}$ is odd in y and S_B accounts for only one half of the total symmetrical tank wetted surface.

Thus f_p is real and f_u is pure imaginary so that we write $f_u = i\omega A_u$, with A_u real. The volume flux across the free surface S^+ is

$$Q = \int_{S^+} L\Phi_z(y,-f+c)dy = (Uq_u + Pq_p), \tag{7}$$

where we define

$$q_{u,p} = \int_{S^+} L\phi_z^{(u,p)}(y,-f+c)dy. \tag{8}$$

Hence q_u is real and q_p is pure imaginary so that we write $q_p = i\omega A_p$, with A_p real. The turbine characteristics are modelled by a constant λ linking the volume flux Q through the turbine to the pressure drop $2P$ across it via a linear damping law, Thus we assume

$$Q = 2\lambda P, \tag{9}$$

the factor of two arising since the difference in pressure across the turbine equates to $2P$ here. Returning to (1), we write $X_w = (i\omega A - B)U + X_s$, where A, B, X_s are the added mass, damping in sway and sway exciting force respectively for the cylinder. We obtain

$$UZ = X_s + X_i \tag{10}$$

where

$$Z \equiv B - i\omega(M + A - \omega^{-2}C). \tag{11}$$

It follows from (10) and (5) that

$$Z_1 U = Pf_p + X_s, \quad \text{where } Z_1 = Z - i\omega A_u, \tag{12}$$

whilst from (7) and (9),

$$Uq_u = (2\lambda - i\omega A_p)P. \tag{13}$$

It follows from (12) that we may write (13) as

$$2Z_1(\lambda + Z_2)P/q_u = X_s, \tag{14}$$

where

$$Z_2 = q_u^2/Z_1 - \tfrac{1}{2}i\omega A_p, \tag{15}$$

and we have used the result $f_p = -2q_u$, which can be proved by a simple application of Green's second identity.

We assume the mean power generated at the turbine is

$$W = \tfrac{1}{2}\Re\{Q\overline{P}\} = (\lambda + \overline{\lambda})|P|^2, \qquad (16)$$

where, we have assumed that λ may be complex so that, from (14),

$$W = \frac{(\lambda + \overline{\lambda})q_u^2|X_s|^2}{8|Z_1|^2|\lambda + Z_2|^2}. \qquad (17)$$

We wish to maximise this as a function of the complex turbine characteristic λ. This is most easily done by noting the identity

$$\frac{(\lambda + \overline{\lambda})}{|\lambda + Z_2|^2} = \frac{1}{\Re Z_2}\left(1 - \frac{|\lambda - \overline{Z}_2|^2}{|\lambda + Z_2|^2}\right). \qquad (18)$$

It follows from (17), (18) after using the identity $|Z_1|^2\Re Z_2 = q_u^2 B$ that the maximum power is given by the well-known result

$$W = |X_s|^2/8B \qquad (19)$$

and is achieved when $\lambda = \overline{Z_2}$.

Results

Computations are presented for the WEC shown in Figure 1, with its axis at depths between 1.05m and 1.25m below the free surface, and moored 3m above the sea-bed. The specific gravity s=0.5, and the resonant frequency for sway motion of the cylinder with a 'frozen' tank coincides with the wavenumber $Ka \simeq 0.1$.

Figure 2 shows the optimum capture width, or the ratio of the maximum power divided by the mean incident power per unit crest length of the incident wave. For large Ka the curve approaches $L/2 = 4$, consistent with the maximum efficiency of a symmetric two-dimensional WEC. For small Ka the curve asymptotes to $2/Ka$, the capture width of a three-dimensional axisymmetric WEC operating in sway.

In order to determine the effect the internal sloshing has on the overall power absorption of the WEC it is necessary to compute A_u, the added mass of the internal water, and the volume fluxes $q_p \equiv i\omega A_p$, and q_u given by (8). It has proved possible to adapt WAMIT to such problems, and results are shown in Figure 3 for the tank in Figure 1. Each of these coefficients is singular at the natural frequencies associated with the tank and behaves like $\sim (\omega_n^2 - \omega^2)^{-1}$ as $\omega \sim \omega_n$. For the range of Ka covered only the lowest natural frequency for this tank, with $Ka = K_1 a \sim 0.61$, shows up.

With the inclusion of the motion of the internal water the maximum power is achieved by choosing $\lambda = \overline{Z}_2$ at each frequency. Figure 4 shows the computed value of λ to achieve this condition. In the vicinity of the resonant frequency where A_u, f_p, q_u and A_p are singular, λ is bounded. Thus in the term $q_u^2/|Z_1|^2$, where the singularity in the denominator arises from the term A_u in the definition of Z_1, the singularities can be seen to cancel. Also, from (15) we have $Z_2 \sim (f_p q_u/f_u - q_p)/2$, $\omega \sim \omega_n$. But this is bounded using the general result

$$f_p q_u - f_u q_p = O(\omega_n^2 - \omega^2)^{-1} \quad \omega \sim \omega_n, \qquad (20)$$

which can be proved by writing (5), (7) in matrix form and considering its determinant as $\omega \sim \omega_n$.

This is a special case of a more general result for the added-mass coefficients A_{ij} which include pressure modes as well as rigid-body modes, since $A_u = A_{22}$ and the coefficients f_p and A_p can be related to A_{ij} using the free-surface condition (3). At the resonant frequency the equations of motion based on A_{ij} have a nontrivial homogeneous solution, and thus the determinant of the left side is zero. It follows that, for any pair of antisymmetric modes i and j,

$$A_{ii}A_{jj} - A_{ij}A_{ji} = \mathrm{O}((\omega_n^2 - \omega^2)^{-1}, \qquad (21)$$

which is one order less singular than the products of the separate coefficients. This relation has been established by Faltinsen (Faltinsen & Timokha, 2009, §5.4.1.3) for coupled sway-roll motions of a tank, using the properties of the eigensolutions.

Notice from Figure 4 that $\Im Z_2$ vanishes just once in the range, at $Ka = K_1 a$ so that for real $\lambda = \Re Z_2 = 5.85$ the maximum capture width can be achieved.

In practice it would be simpler to fix λ, so we define the efficiency of the WEC to be $E = W/(8B/|X_s|^2)$ where W is given by (17). Thus E measures how close the WEC performs to its maximum possible capture width. Figure 5 shows the variation of E with Ka for different fixed real values of λ. Apart from the case $\lambda = 1.0$, all the curves peak close to the lowest natural frequency $K_1 a$, with the maximum capture width being attained for $\lambda = 5.85$ as expected. The narrow bandwidth of the curves is disappointing although it can be seen from Figure 1 that the maximum capture width exceeds 4 for $Ka < 1.5$ so that for example the *actual* capture width when $\lambda = 1.0$ exceeds 2 for $0.6 < Ka < 1$.

Conclusion

We have explored one example of the idea of installing an internal water tank in a WEC in order to extract power from an incident wave field by coupling the resonant motions of the tank and the WEC. A complete analysis has been developed based on linear water wave theory and some computations made of the various hydrodynamic coefficients needed to compute the power absorbed. There are many parameters involved and further work is needed to optimise the system and improve predicted performance. One improvement would be to alter the shape of the internal tank so as to lower the natural frequencies to coincide with the dominant frequency of the incident waves. This could be done for example by increasing the size of the internal free surfaces whilst preserving the internal water mass.

References

Evans, D.V. & Porter, R. 'Wave energy extraction by coupled resonant absorbers,' 2010, (Submitted for publication)

Falnes, Johannes, 'Ocean Waves and Oscillating Systems,' Cambridge Univ. Press, 2002

Faltinsen, O.M. & Timokha, A.N., 'Sloshing,' Cambridge Univ. Press, 2009

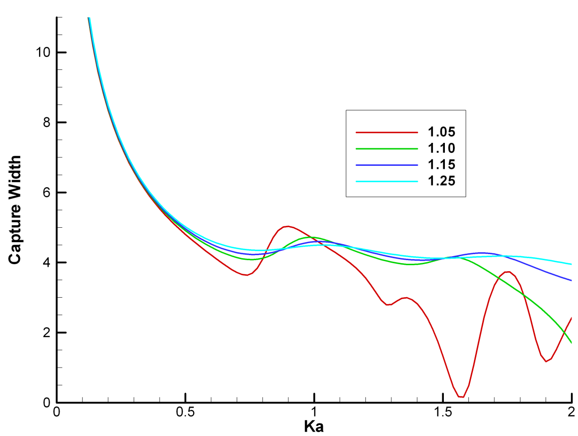

Figure 2: Maximum capture width in metres. The cylinder axis is submerged at the depths indicated in the legend. Ka is the product of ω^2/g and the cylinder radius.

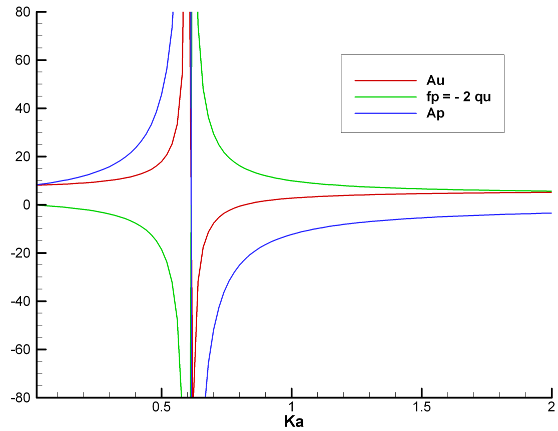

Figure 3: Parameters A_u, $f_p = -2q_u$ and A_p for the tank shown in Figure 1. A_u is normalized by ρV where V is the tank volume. f_p and A_p are normalized by ρS_f where S_f is the free-surface area.

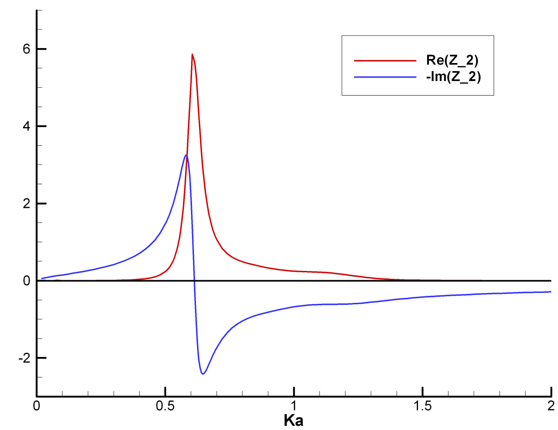

Figure 4: Real and imaginary parts of the optimum value of the turbine parameter λ for the submergence 1.10m.

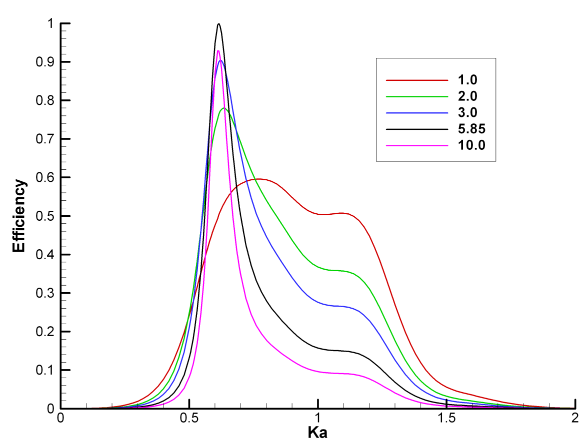

Figure 5: Efficiency based on the five fixed real values of λ shown in the legend.