Energy and damping analysis of the wet-modes of an elastic floating structure

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ABSTRACT
In this paper, the bending modes of an elastically scaled segmented hull model are identified using the proper orthogonal (or Karkun-Loeve) decomposition. Despite the conceptual simplicity of the method, its application to floating structures requires some cares and motivated the analysis of two sets of data, accelerations and strains, with two different tailoring of the original method, that allows to provide information about both the damping and the energy distribution among the identified modes.

INTRODUCTION
Structural dynamics has been covering in last decades new topics, encouraging applications that in the past missed proper theoretical support. The identification of ship vibration modes, objective of the present work, is one of the problems in ship engineering benefitting of recent theoretical achievements in structural dynamics. The advantages of identifying modal parameters by performing modal tests using the ambient excitation and measuring only the responses of the structure, i.e., ambient or operational conditions, made the output-only modal testing very popular [1] in recent years. In fact, the test procedure consists only in measuring the response of the system, resulting then an easier way for characterizing the dynamic behavior of the structure with respect to the traditional experimental modal analysis. Furthermore, with this approach, it is possible to identify the dynamic properties of the system in real operative conditions where the loading conditions are, in general, unknown or substantially different from those simulated in modal tests. In this paper a time-domain procedure to identify the vibration modes of a floating structures, based on the analysis of both displacements and accelerations, is presented. The implemented time-domain technique is the proper orthogonal decomposition (POD) that provides the functional basis that accounts for more captured energy than any other orthogonal one. The POD has been applied in its straightforward formulation and in a slightly different version as well, named band-pass POD, that exploits preliminary filtering around resonant peaks of the analyzed signals to enhance the convergence of the proper orthogonal modes (POMs) to the linear normal modes (LNM) in case of poor information about the mass distribution. The presented procedure has been employed to analyze the experimental data provided by accelerometers and strain-gages applied to the flexible backbone of an elastically scaled segmented-hull model sailing in both irregular sea and regular waves in the towing-tank. The comparison between the modes shapes identified with the two different procedures (original POD on the displacements and band-pass POD on the accelerations) allows to show the effectiveness of this method and the possibilities and limitations related to the use of each procedure. Some results related to the present application, like energy ordering of the wet-modes and its dependence on the encountered sea pattern, as well as the modal damping variation with ship forward speed, are discussed in the paper, disclosing the POD capability to provide new insights in the analysis of hydroelastic phenomena.
the decomposition $\mathbf{w}(t) = \sum_{k=1}^{L} w_k(t) \mathbf{p}_k$, that gives the best representation of the solution $\mathbf{w}$ in the sense already specified for the continuous problem, where the vectors $\mathbf{p}_k$ are the proper orthogonal modes. At this point, it is useful to introduce the following transformation in Eq. 3, $\mathbf{w} = \mathbf{M}^{-1/2} \hat{\mathbf{w}}$, thus obtaining $\hat{\mathbf{w}} + \mathbf{M}^{-1/2} \mathbf{K} \hat{\mathbf{w}} = \mathbf{f}$, that can be recast finally as

$$\hat{\mathbf{w}} + \mathbf{K} \hat{\mathbf{w}} = \mathbf{f},$$

(4)

where the matrix $\mathbf{K}$ is still symmetric. Equation 4 defines an undamped mechanical system with uniform mass distribution ($\mathbf{M} = \mathbf{I}$ in this particular case) for which the linear normal modes are directly provided by the proper orthogonal decomposition. The $\mathbf{M}$ components of the vector $\hat{\mathbf{w}}(t) = [\hat{w}_1(t), \ldots, \hat{w}_m(t), \ldots, \hat{w}_N(t)]^T$ represent the transformed displacements in the same points where, for instance, the measurements were performed. If $N$ ‘observations’ for each of the $\mathbf{M}$ components of the vector $\hat{\mathbf{w}}$ are available, let us define a new vector variable, $\hat{\mathbf{w}}^{(m)} = [\hat{w}_m(t_1), \ldots, \hat{w}_m(t_N)]^T$, that is the sampled time history relative to the generic component $\hat{w}_m(t)$ of the state space vector $\hat{\mathbf{w}}(t)$, assuming that the mean value was previously subtracted. Thus, the $N \times M$ response ensemble matrix is constructed as

$$\tilde{\mathbf{W}} = [\hat{\mathbf{w}}^{(1)}, \hat{\mathbf{w}}^{(2)}, \ldots, \hat{\mathbf{w}}^{(M)}],$$

(5)

that allows to obtain the sample covariance matrix as

$$\mathbf{R}_{\tilde{\mathbf{W}}} = (1/N)\tilde{\mathbf{W}}^T \cdot \tilde{\mathbf{W}},$$

(6)

where the symbol · denotes the inner product. Considering the system response in its continuous form (Eq. 2), it emerges that the averaged auto-correlation function $\mathcal{R}(x, y)$ has been replaced by the $M \times M$ sample covariance matrix $\mathbf{R}_{\tilde{\mathbf{W}}}$. The proper orthogonal modes are calculated as the eigenvectors of the covariance matrix $\mathbf{R}_{\tilde{\mathbf{W}}}$, i.e.,

$$\mathbf{R}_{\tilde{\mathbf{W}}} \mathbf{p}_k = \sigma_k \mathbf{p}_k,$$

(7)

where $\sigma$ is the corresponding proper orthogonal value. The proper orthogonal values give an indication of the level of excitation of the correspondent proper orthogonal mode. In fact, if $\mathcal{E}$ is the energy associated to the random field, it can be expressed as

$$\mathcal{E} \propto \sum_{i=1}^{M} \sigma_i,$$

(8)

and the relative energy captured by the $k$-th proper orthogonal mode is $\varepsilon_k = \sigma_k / \mathcal{E}$. It is worth to remark that energy is defined as the norm of the signal and not as mechanical energy. Defining the $M \times M$ eigenvector matrix $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_M]$, it follows that the values of the proper orthogonal coordinate (POC) $a_k$ at the different time instants $t_i$ can be expressed in terms of the columns of the $N \times M$ matrix $\mathbf{A} = [\mathbf{a}_1^{(1)}, \mathbf{a}_2^{(2)}, \ldots, \mathbf{a}_M^{(M)}]$ given by

$$\mathbf{A} = \mathbf{W}^T \cdot \mathbf{P},$$

(9)

with $\mathbf{a}_k^{(h)} = [a_k(t_1), \ldots, a_k(t_N)]^T$. Thus, one finally obtains, by suitable interpolation over the proper coordinate vector $\mathbf{a}_k^{(h)}$

$$\hat{\mathbf{w}}(t) = \sum_{k=1}^{L} a_k(t) \mathbf{p}_k,$$

(10)

that is the discretised form of Eq. 1, where, in general, $L \leq M$, i.e., the number of assumed modes is less or equal than the number of measurement points. This procedure to compute the proper orthogonal decomposition is more efficient with respect to the snapshot method (see Lumley [3]) from a computational point of view when the number of time instants $N$ is larger than the number of measurements points $M$.

**MODEL DESCRIPTION**

In the present case, the segmented model technique with an elastic backbone (rectangular, hollow and made of an aluminium alloy was built with 20 elements of constant stiffness and shear area) was adopted in order to scale the bending stiffness of the fast ferry Fincantieri MDV3000 (for more details, refer to [6]). Each segment is connected to the elastic beam with short legs and the gaps between adjacent segments are made water-tight by using rubber strips (Fig. 1). The short legs were built by using steel, whereas the hull segments are of fiber-glass. The materials employed in the model construction were chosen for several technological reasons; among them, the limitation of the total weight was one of the main concerns. Thus, the model-scale was set equal to $\lambda = 1/30$, whereas the number of segments was set to six.

![Fig. 1 Sketch of the segmented model.](image)

**EXPERIMENTAL INVESTIGATION**

**Analysis of strain-gauge signals**

The identification procedure that makes use of the strain-gage signals exploits the time-histories of the vertical bending moment relative to each measurement point. Accordingly to the Euler-Bernouilli beam model, the elastic displacement is given by

$$w(x,t) = c_1 + c_2 x + \int_{-\ell_1}^{x} \left[ \int_{-\ell_1}^{x_1} \frac{M_s(x_1,t)}{E I_{yy}(x_1)} dx_1 \right] dx_2,$$

(11)

with $E I_{yy}$ the bending rigidity, where $E$ is the Young modulus, $I_{yy}(x)$ is the sectional moment of inertia with respect to the $y$ axis, $\ell_1 > 0$ is the absolute distance of the center of gravity from the beam end. The presence of the constants of integration, due to the free-end boundary conditions of the floating beam, indicates that the elastic mode shapes may appear arbitrarily translated and rotated, depending on the choice of the constants $c_i$ (note that they can be even time-dependent). To avoid this trouble, the total displacement $w(x, t)$ instead of $w(x, t)$ is then processed with the POD. In fact, due to the orthogonality relationships between the proper orthogonal modes, the elastic modes ($j = 3, \ldots$) has to be orthogonal to the rigid-body modes ($i = 1, 2$) and this condition implies the correct translation and rotation of the mode shapes.

It is important to recall that information about the mass distribution is needed to get the POMs converging to the LNMs. This requirement would be unnecessary if the input signals were preliminary filtered so as to leave just a single mode contribution. However, the number of measurement points ($M = 12$) is large enough to build a sufficiently accurate mass matrix and, avoiding filtering, the energy distribution among modes can be retrieved. The added mass of the segment has to be taken into account too and was calculated numerically by integrating along the segments the distribution of the sectional added mass provided by the Lewis’ infinite-frequency approximation. In Figs. 2, 3 and 4 the identified proper orthogonal modes are plotted for the case $Fr = 0.44$ and an irregular sea with a Jonswap spectrum, being $H_s = 2 m$ and $T_s = 7.5 s$ the correspondent significant wave height and period, respectively, at full-scale. It is worth to note that the chosen parameters
for the sea do not cause any large amplitude motion for the scaled ship. Each identified mode is compared with that one calculated via modal analysis carried out on a finite element model relative to the complete backbone model. From the observation of these figures it emerges that the differences between the computed and experimentally identified modes are significative only for the 4-nodes mode.

The POMs do not present any apparent variation in shape if the encountered sea changes its main spectrum parameter (significant wave height and period) or if the ship sails through regular waves. In fact, a POM variation can occur only if the system hydrodynamic coefficient (principally the added mass and damping) be affected by the different (but physically admissible) seaways encountered by the ship. However, since the wave elevation is a zero-mean process, it follows that despite the nonlinear relationship relating the hydrodynamic coefficients to the wave elevation, the average of their perturbation is close to zero and POD is not sensitive to this small coefficients fluctuations, thus not providing any clear POM shape alteration. On the other hand, the level of excitation of each identified mode may significantly change from one mode to the other, as it will be shown in the following. Considering as a starting point the sea state already considered for the identification of the POM shapes (\(H_2/3 = 2\) m and \(T_1 = 7.5\) s), in Fig. 5 the proper orthogonal values are represented for all the modes (i.e., rigid-body and elastic modes), using a logarithmic scale on the y-axis, highlighting the different energy content associated to each identified mode. It is evident that the first bending mode (2-nodes mode) is highly predominant on the others bending modes, as confirmed also by the analysis of the correspondent POC time-history.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>(\lambda w/ L_{pp})</th>
<th>(\bar{\sigma}_1)</th>
<th>(\bar{\sigma}_2)</th>
<th>(\bar{\sigma}_3)</th>
<th>(\bar{\sigma}_4)</th>
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<td>0.60</td>
<td>97.15728</td>
<td>2.80306</td>
<td>0.03866</td>
<td>0.00085</td>
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<tr>
<td>1.35</td>
<td>98.76840</td>
<td>1.21798</td>
<td>0.01307</td>
<td>0.00051</td>
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<tr>
<td>1.75</td>
<td>99.75023</td>
<td>0.23554</td>
<td>0.01549</td>
<td>0.00063</td>
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<tr>
<td>2.30</td>
<td>99.807933</td>
<td>0.17508</td>
<td>0.01626</td>
<td>0.00192</td>
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</tr>
</tbody>
</table>

Table 1 POV percentage calculated on the elastic modes for different encountered seas \((Fr = 0.44)\).

It has been shown that regular waves, despite of their apparent sinusoidal wave form, do not determine a monochromatic load excitation at all and the resulting load spectrum amplitudes, in the frequency range of the vertical bending modes, vary barely so as to excite rather uniformly all the vibration modes. Therefore, it is not surprising that the vertical bending modes can be identified in regular waves as well, via the POMs converging to the corresponding LNM. However, the POMs appear more sensitive to the excitation features, as it appears from the results shown in Tab. 1, where the percentage of energy for each elastic mode \(\bar{\sigma}_i\) is calculated with respect to the overall ‘elastic’ energy alone (more precisely, to the summation of the energy associated to the computed (bending modes), i.e., \(\bar{\sigma}_i = \sigma _i / \sum _{j=1} ^{M} \sigma_j\). The trends provided by Tab. 1 show the tendency of the first elastic POM to increase with the wave length and, on the opposite, a rather negative slope for the second POM (further considerations for the subsequent modes may be affected by low excitation levels). These trends can be explained recalling the general form of the response of a linear structural system to external excitation, since hydroelastic coupling for the bending modes is low. Thus, the role of the load projection upon the bending modes is much more relevant in amplifying the response than the possible resonance effect due to the load spectrum. It seems reasonable to observe that, as expected, if \(\lambda w/ L_{pp}\) is close to 1, the waveform is close to the shape of the second bending mode (higher value of the 2nd POM), whereas if \(\lambda w/ L_{pp} \rightarrow 2\), the waveform resembles more to the 1st mode. Returning back to the irregular sea case, there is not any clearly predominant

Fig. 2 Identified 2-nodes POM at \(Fr = 0.44\).

Fig. 3 Identified 3-nodes POM at \(Fr = 0.44\).

Fig. 4 Identified 4-nodes POM at \(Fr = 0.44\).

Fig. 5 POVs spectrum (irreg. sea).
The modal damping was evaluated by analyzing the modal coordinates associated to the identified modes. In particular, the logarithmic decrement method was applied to the auto-correlation functions of the modal coordinates, obtaining a percentage modal damping as shown in Tab. 3. This technique allows then to obtain an estimation of the damping variation with the forward speed, as shown in Fig. 6, that is not easy to be obtained theoretically.

REFERENCES


