Dissipation effect in potential flows of fairly perfect fluid

X.B. Chen[1,2], W.Y. Duan[3] & H.X. Liu[3]

[1]Research Department, BV, 92570 Neuilly-Sur-Seine (France)
[2]Professorship, HEU, 150001 Harbin (China)
[3]Shipbuilding Engineering College, HEU, 150001 Harbin (China)

Further to the recent work in Chen & Dias (2010) on the introduction of viscous terms in the boundary condition at the free surface which is critically important to describe wave motions with decay factors in time and space, we present our new development to include the dissipation effect in wave diffraction and radiation flow around a floating body which is essential to provide a realistic prediction of resonant motions. The notion of *fairly perfect* fluid by Guével (1982) introducing a dissipation force proportional to the magnitude of fluid velocity but in the opposite direction, is adopted and extended to define a pressure loss across some *dissipative* surfaces in the vicinity of body hull where large dissipation occurs in a real fluid. The multi-domain boundary element method developed in Chen (2010) is then extended to include the dissipation and applied to bottomless cylinders with zero- or finite-thickness wall. Comparisons of numerical results for a bottomless cylinder with semi-analytical results and model test measurements given in Miloh (1983) and Mavrakos (1985) validate the present method and show that it is indeed efficient and reliable to provide realistic predictions.

1. Potential flow of fairly perfect fluid

We consider one body floating on the free surface in the presence of incident propagative waves. The reference system of Cartesian coordinates is defined by letting (x, y) plan coincide with the mean free surface and z-axis be positive upwards. The fluid is assumed to be incompressible and inviscid while the fluid motion irrotational. Under these assumptions of a perfect fluid, the flow velocity $\mathbf{v} = (v_1, v_2, v_3)$ can be expressed as the gradient of a scalar potential $\Phi(M, t)$ in the space $M = (x, y, z)$ at the time t. The mass conservation law is then respected for the velocity potential $\Phi(M, t)$ to satisfy the Laplace equation. The fluid is under the action of gravity. Besides this gravitational field, an internal force defined as

$$\mathbf{f} = -\mu\mathbf{v} \tag{1}$$

is assumed to apply to the fluid particle as well. The parameters μ being assumed to be positive and small, the force $\mathbf{f}$ is proportional to the magnitude of fluid velocity but in the opposite direction. Although $\mathbf{f}$ plays the same role of damping fluid motion and dissipating energy as that of fluid viscosity, it does not introduce any vorticity so that the existence of velocity potential is safeguarded. The inviscid and irrotational fluid with the dissipative force $\mathbf{f}$ is called here as *fairly perfect* fluid in Guével (1982) and Chen (2004). The momentum equation in the fairly perfect fluid is written as :

$$(\partial/\partial t + \mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla(P/\rho + gz) - \mu\mathbf{v} \tag{2}$$

in which P stands for the pressure and g the gravitational acceleration. Introducing $\mathbf{v} = \nabla\Phi$ into above momentum equation, the modified Bernoulli equation is obtained and expressed as :

$$P/\rho + gz + \Phi_t + \nabla\Phi \cdot \nabla\Phi/2 + \mu\Phi = c(t) \tag{3}$$

with $c(t)$ an arbitrary function of t usually omitted by redefining Φ without affecting the velocity field. On the free surface $z = \mathcal{E}$, the dynamic condition requires that the pressure given from (3) is equal to the atmospheric pressure ($P = 0$), i.e.

$$\mathcal{E} = -(\Phi_t + \nabla\Phi \cdot \nabla\Phi/2 + \mu\Phi)/g \tag{4}$$

and the kinematic condition is expressed as :

$$\Phi_z - \Phi_x\mathcal{E}_x - \Phi_y\mathcal{E}_y - \mathcal{E}_t = 0 \tag{5}$$

to guarantee that a particle in motion at the free surface stays always on the same surface. In the presence of a body, the boundary condition on body's hull is expressed classically as $\Phi_n = \mathbf{v}^H \cdot \mathbf{n}$ where $\mathbf{v}^H$ stands for the local velocity vector of body's hull. If the sea bed $B(z = -h)$ is present, an additional condition $\Phi_z = 0$ is written on B.

To further introduce the dissipation effect due to the interaction of bodies with fluid flows, we first define one or several dissipative surfaces D in the vicinity of body hull, like sharp corners, bilge keels, entrance of moonpool or gap between side-by-side ships, where the large dissipation occurs in a real fluid. Across the dissipative surface D, there is a difference of dynamic pressure :

$$[P] = f(\Phi_n) \tag{6}$$

as a function dependent on the normal velocity Φ_n which can be linear or quadratic while Φ_n is continuous across D. The form (6) is in agreement with (1) and represents the energy loss due to the internal dissipative force (1) in the region around the dissipative surface.

2. Boundary element method with dissipations

By assuming a small steepness of incident waves, we make use of linearization of above equations. Furthermore, the fluid motion is assumed to be harmonic in time with the circular frequency ω in such a way that we can write the velocity potential

$$\Phi(M,t) = \Re_e\{\phi(M)e^{-i\omega t}\} \tag{7}$$

in which $\Re_e\{\cdot\}$ stands for taking the real part. The linear problem of wave radiation and diffraction is then defined by :

$$\nabla^2\phi = 0 \qquad M \subset V \tag{8a}$$

$$\phi_z - K_0(1+i\epsilon')\phi = 0 \qquad M \subset F \tag{8b}$$

$$\phi_n = v_n \qquad M \subset H \tag{8c}$$

$$\phi_z = 0 \qquad M \subset B \tag{8d}$$

$$[\phi] = i\epsilon\phi_n/K_0 \quad \text{and} \quad [\phi_n] = 0 \qquad M \subset D \tag{8e}$$

in which $K_0 = \omega^2/g$ and $\epsilon' = \mu/\omega$, V stands for the fluid domain limited by the mean free-surface F, the body surface H, and the sea bed B. The term v_n on the right hand side of (8c) is the amplitude function of $(\mathbf{v}^H \cdot \mathbf{n})$ given following the radiation and diffraction problems. The condition (8e) across D is derived from (6) by assuming a linear dependence of dynamic pressure change with respect to the fluid velocity. On the left side of (8e), $[\phi]$ and $[\phi_n]$ stand for the difference of ϕ and ϕ_n along the positive direction of the normal vector, respectively. The coefficient ϵ is a positive constant to characterize the dissipation effect.

To solve the first-order boundary value problem defined by (8), we consider the Green function which satisfies the following equations :

$$\nabla^2 G(M,Q) = 4\pi\delta_{MQ} \qquad M \subset V \tag{9a}$$

$$G_z - K_0(1+i\epsilon')G = 0 \qquad M \subset F \tag{9b}$$

$$G = 0 \qquad M \subset B \tag{9c}$$

in which (M,Q) are respectively the field point $M(x,y,z)$ and singular point $Q(x',y',z')$, and the Dirac function $\delta_{MQ} = \delta(x-x')\delta(y-y')\delta(z-z')$.

Applying the Green's second formula to the couple of harmonic functions (ϕ, G) in the domain V limited by the hull H, the free surface F, the sea bad B, a cylindrical surface C^∞ at infinity and two sides of D, we have the integral representation of $\phi(M)$ for $M \subset V$:

$$4\pi\phi(M) = \iint_H ds\,(v_n G - \phi\, G_n) + (i\epsilon/K_0)\iint_D ds\,\psi\, G_n \tag{10}$$

in which the normal vector $\mathbf{n}$ on H is oriented positively toward into fluid and that on D is chosen from one side to another. The left hand side in (10) is the result of the domain integral and the terms on the right side come from the transformation of the domain integral to the surface integral on the boundaries according to the formula of Ostrogradsky among which the boundary integrals on F, B and C^∞ are nil. The integral on both sides of D is reduced to one on the side with positive normal vector and ψ stands for ϕ_n on D. From (10), the integral equations for the unknowns ϕ on H and (ϕ, ψ) on D can be derived and written as :

$$2\pi\phi(M) + \iint_H ds\,\phi\, G_n - (i\epsilon/K_0)\iint_D ds\,\psi\, G_n = \iint_H ds\, v_n G \quad \text{for} \quad M \subset H \tag{11a}$$

$$4\pi\phi(M) + \iint_H ds\,\phi\, G_n - (i\epsilon/K_0)\iint_D ds\,\psi\, G_n = \iint_H ds\, v_n G \quad \text{for} \quad M \subset D \tag{11b}$$

$$4\pi\psi(M) + \iint_H ds\,\phi\,\partial_n G_n - (i\epsilon/K_0)\iint_D ds\,\psi\,\partial_n G_n = \iint_H ds\, v_n \partial_n G \quad \text{for} \quad M \subset D \tag{11c}$$

In (11b), $\phi(M)$ for $M \subset D$ stands for the average value since, following (8e), we can write :

$$\phi^{\pm}(M) = \phi(M) \mp i\epsilon\psi(M)/(2K_0) \tag{12}$$

on the positive and negative sides of D, respectively. Furthermore, the derivatives of the Green function in (11) are understood as :

$$G_n = \partial G(M,Q)/\partial n(Q) \; ; \quad \partial_n G = \partial G(M,Q)/\partial n(M) \; ; \quad \partial_n G_n = \partial[\partial G(M,Q)/\partial n(Q)]/\partial n(M) \tag{13}$$

The integration in (11c) of the second derivatives of the Green function associated with its Rankine part is hypersingular and to be evaluated as a Hadamard finite-part integral.

Frequently, the hull H presents the surfaces parallel and very close to each other. The integral on H in the integral equation (11a) leads to a degenerated system as analyzed in Martin & Risso (1993) so that the results are, if not singular, very poor in convergence. The work in Chen (2010) dealt with this issue successfully by developing a multi-domain boundary element method. A control surface is designed to isolate the parallel surfaces by dividing the fluid domain into two subdomains in each of which only one concerned surface is present. The integral representation in each subdomain is augmented by the part of control surface but not singular any more, so that the convergent results have been achieved. The multi-domain boundary element method is now extended to include the integral equations (11b) and (11c) to take into account the dissipation.

3. Semi-analytical solution for a bottomless cylinder

In order to validate the boundary element method with dissipations, we consider the case of a bottomless cylinder same as one studied in Miloh (1983) with zero-thickness wall which was extended in Mavrakos (1985) for bottomless cylinders with finite wall thickness. The cylinder is measured by its radius a and draught d in water of depth h. The fluid domain is divided into two by the control surface extended from the hull end $(z=-d)$ to sea bottom $(z=-h)$. The incoming wave potential ϕ_I, diffraction potential ϕ_{DI} in the inner domain $(r<a)$ and that ϕ_{DE} in the outer domain $(r>a)$ can be written in the cylindrical coordinate system :

$$(\phi_I, \phi_{DI}, \phi_{DE}) = -\frac{Ag}{\omega}\sum_{\ell=0}^{\infty}(\phi_I^\ell, \phi_{DI}^\ell, \phi_{DE}^\ell)\cos\ell\theta \tag{14}$$

where the incoming wave potential

$$\phi_I^\ell = e_0^\ell Z_0(z)\mathbf{J}_\ell(k_0 r) \quad \text{with} \quad e_0^\ell Z_0(0) = \begin{cases} 1 & \text{for} \quad \ell = 0 \\ 2i^\ell & \text{for} \quad \ell \geq 1 \end{cases} \tag{15}$$

Following Garrett (1970), the diffraction potentials are written as :

$$\phi_{DI}^\ell = [a_0^\ell - e_0^\ell \mathbf{J}_\ell'(k_0 a)]Z_0(z)\mathbf{J}_\ell(k_0 r)/\mathbf{J}_\ell'(k_0 a) + \sum_{m=1}^{\infty} a_m^\ell Z_m(z)\mathbf{I}_\ell(k_m r)/\mathbf{I}_\ell'(k_m a) \tag{16a}$$

$$\phi_{DE}^\ell = [a_0^\ell - e_0^\ell \mathbf{J}_\ell'(k_0 a)]Z_0(z)\mathbf{H}_\ell(k_0 r)/\mathbf{H}_\ell'(k_0 a) + \sum_{m=1}^{\infty} a_m^\ell Z_m(z)\mathbf{K}_\ell(k_m r)/\mathbf{K}_\ell'(k_m a) \tag{16b}$$

with

$$Z_m(z) = \begin{cases} \cosh k_0 h \cos k_0(z+h)/(2k_0 h + \sinh 2k_0 h) & \text{for} \quad m = 0 \\ \cos k_m(z+h)/(2k_m h + \sin 2k_m h) & \text{for} \quad m \geq 1 \end{cases} \tag{17}$$

with $k_0 \tanh k_0 h = K_0$ and $k_m \tan k_m h = -K_0$ for $m \geq 1$.

In (15), (16a) and (16b), $(\mathbf{J}_\ell, \mathbf{H}_\ell)$ are the ℓth-order (Bessel, Hankel) functions of the first kind, $(\mathbf{I}_\ell, \mathbf{K}_\ell)$ are the ℓth-order modified Bessel functions of the (first, second) kinds, respectively, defined in Abramowitz & Stegun (1965). To determine the unknown coefficients (a_0^ℓ, a_m^ℓ) for $m \geq 1$, the dissipative condition on the control surface and boundary condition on the cylinder :

$$\phi_{DE}^\ell - \phi_{DI}^\ell = \frac{i\epsilon}{K_0}\frac{\partial}{\partial r}\phi_{DI}^\ell \quad \text{for} \quad -h \leq z < -d \quad \text{and} \quad \frac{\partial}{\partial r}(\phi_{DI}^\ell + \phi_I^\ell) = 0 \quad \text{for} \quad -d \leq z < 0 \tag{18}$$

are to be satisfied in the sense of Galerkin integral associated with the eigenfunctions in z :

$$\int_{-H}^{-D} dz\,(\phi_{DE}^\ell - \phi_{DI}^\ell)\cosh k_0(z+h) + h\int_{-D}^{0} dz\,\frac{\partial}{\partial r}(\phi_{DI}^\ell + \phi_I^\ell)\cosh k_0(z+h) = \frac{i\epsilon}{K_0}\int_{-H}^{-D} dz\,\frac{\partial}{\partial r}\phi_{DI}^\ell \cosh k_0(z+h) \tag{19a}$$

$$\int_{-H}^{-D} dz\,(\phi_{DE}^\ell - \phi_{DI}^\ell)\cos k_n(z+h) + h\int_{-D}^{0} dz\,\frac{\partial}{\partial r}(\phi_{DI}^\ell + \phi_I^\ell)\cos k_n(z+h) = \frac{i\epsilon}{K_0}\int_{-H}^{-D} dz\,\frac{\partial}{\partial r}\phi_{DI}^\ell \cos k_n(z+h) \tag{19b}$$

for $n \geq 1$. The equations (19) give a linear system to determine (a_0^ℓ, a_m^ℓ) for $1 \leq m \leq M$ with M the truncated number of the infinite series in (16).

4. Discussions and conclusions

We have introduced the dissipation across the control surface for the sake of having least modification to the classical semi-analytical solution. Indeed, if $\epsilon = 0$ in (19), we obtain exactly the same linear system for the unknowns as in Miloh (1983). For comparison, we have evaluated the wave elevation η_0 at the cylinder center :

$$\eta_0/(-iA) = a_0^0 Z_0(0)/\mathbf{J}_0'(k_0 a) + \sum_{m=1}^{M} a_m^0 Z_m(0)/\mathbf{I}_0'(k_m a) \tag{20}$$

and the surge forces F_x :

$$\frac{F_x}{i\pi\rho g A a^2} = \frac{2i(e_0^1 \mathbf{J}_1' - a_0^1) Z_0(0)}{\pi k_0 a \mathbf{H}_1' \mathbf{J}_1'} \frac{\sinh k_0 h - \sinh k_0(h-d)}{k_0 a \cosh k_0 h} + \sum_{m=1}^{M} \frac{a_m^1 Z_m(0)}{k_m a \mathbf{K}_1' \mathbf{I}_1'} \frac{\sin k_m h - \sin k_m(h-d)}{k_m a \cos k_m h} \tag{21}$$

The semi-analytical results of the amplitude of wave elevation (20) for a bottomless cylinder ($a = 15$cm, $d = 8$cm and $h = 60$cm) are depicted on the left of Figure 1 by the dashed and solid lines for $\epsilon = 0$ and $\epsilon = 0.08$. The results from the boundary element method with dissipation for $\epsilon = 0.08$ using the mesh (in the middle of the figure) composed of 400 flat panels on the both sides of cylinder hull and 1600 flat panels on the dissipative (control) surface, are illustrated by the crossed symbols while the measurements of model tests in Miloh (1983) by the squares. The results for the amplitude of surge forces (21) are depicted on the right of Figure 1. The abscissa in the figure represent the value of $\sqrt{k_0 a}$ as in Miloh (1983). The values of (η_0, F_x) are obtained by taking a truncated number $M = 100$ in the infinite series of (16) slightly different from those of Miloh (1983) in which $M = 20$ was used. It's shown that the results with dissipation are largely reduced for

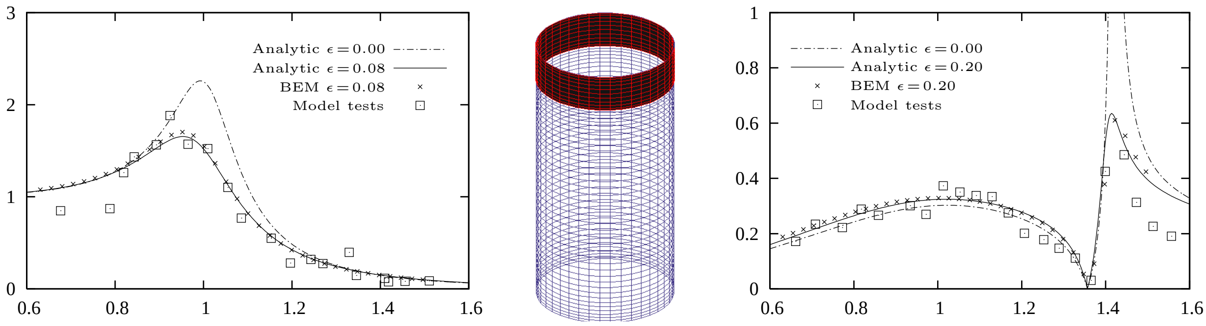

Figure 1: Wave elevation at cylinder center (left), BEM mesh (middle) and surge forces (right)

wavenumbers close to that of resonance and much more closer to the measurements of model tests than those without dissipation, as expected. The results using the boundary element method agree well with those of semi-analytical solution. The excellent level of comparisons validates the multi-domain boundary element method with dissipation which is an efficient and reliable way to provide realistic results, particularly, in the cases of resonances in which exaggerations can be made by classical methods without considering the dissipation.

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