Investigation of geometric nonlinear potential flow effects on free surface flows

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1 Summary

The aim of this work is to report the first steps of ongoing research towards the investigation of geometric non linear potential flow effects on free surface flows and, eventually, on the seakeeping problem in time domain using the mixed Eulerian Lagrangian (MEL) description of the fluid flow. In particular, once the method is working properly, special interest is going to be devoted to the effects of non linear effects on the radiation potential. The foundation of the MEL method was established originally to simulate steep waves in two dimensions by Longuet-Higgins and Cokelet [1]. The main idea behind the MEL scheme is to approximate the nonlinear solution by solving a linear problem at each time step, the so called mixed boundary value problem.

MEL schemes, because of their flexibility, have been applied to a broad range of hydrodynamic problems, eg. Subramani et al [2] and Liu et al [3]. Unfortunately, although relatively simple in theory, MEL implementations bring their own problems, especially in the presence of the floating body, due to the mixed nature of the boundary value problem, to instabilities associated with the free surface and to wave breaking phenomena. Some of these problems are discussed by Bai and Eatock Taylor [4] .

In order to try to overcome some of the problems associated with the MEL description of the fluid flow and enhance its applicability, the present methodology describes the geometric domain by means of signed distance functions. In this context, grid (re)generation is performed using the algorithm develloped by Persson [5]. This method allows for flexible and simple grid (re)generation schemes to be implemented, which are not only capable of accounting for the instantaneous water line intersection between the free surface and the floating body but also provide ways of handling different kinds of domain geometry once intersections and unions of different domains are performed by basic operations between the corresponding distance functions. On the other hand, the additional problem of evolving signed distance functions in time, on a tridimensional background grid, is introduced once the meshing scheme relies on this domain representation. In this work two alternatives are currently being investigated to tackle this problem: in the first approach, for a given velocity field, the single level set method is used to advect the distance function in time; in the second approach, the interface is explicitly moved in a Lagrangian fashion and represented by means of a family of radial basis function (RBF) from which a new signed distance function is then derived for the free surface.

In what follows of the present work a brief description of how the present methodology can be incorporated in the context of the mixed Eulerian Lagrangian description of the fluid flow is presented. In addition, numerical tests using both approaches are also presented for simple waves highlighting the issues currently being addressed.

2 Methodology

In the current context the geometric domain is represented by a fixed cartesian grid equiped with the three-dimensional Euclidean norm $||.||$ in which the fluid domain Ω is embeded. Thus, for a given point $\vec{x}_g$ on the background grid, the signed distance function $d(\vec{x}_g, \vec{x}, t)$ is defined as:

$$\begin{aligned} d(\vec{x}_g, \vec{x}, t) &= ||\vec{x}_g - \vec{x}|| \;\; for \;\; \vec{x} \notin \Omega \\ d(\vec{x}_g, \vec{x}, t) &= -||\vec{x}_g - \vec{x}|| \;\; for \;\; \vec{x} \in \Omega \\ d(\vec{x}_g, \vec{x}, t) &= 0 \;\; for \;\; \vec{x} \in \partial\Omega. \end{aligned} \tag{1}$$

From the above defintion it also follows that $|\nabla d(\vec{x}_g, \vec{x}, t)| = 1$.

In the fluid domain Ω , the formulation of the problem is made in a similar fashion to the usual MEL description of the fluid flow (see for instance Liu et al [3]). In this context, Laplace's equation governs the fluid flow, and the velocity potential $\phi(x, y, z, t)$ satisfies:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0. \tag{2}$$

In addtion on the impervious boundaries moving with velocity $\vec{U}$(x,y,z,t) where the normal unit vector is $\vec{n}$(x,y,z,t) the von Neumann boundary conditions are written as:

$$\nabla\phi \cdot \vec{n} = \vec{U} \cdot \vec{n}. \tag{3}$$

At points where Dirichlet boundary conditions are applied, meaning that the velocity potential is known at the prescribed point, the boundary value problem is solved for the normal velocity.

It follows from Green's second identity and the divergence theorem that Laplace's equation together with the von Neumann and Dirichlet boundary conditions can be represented as Fredholm integral equations of first and second kind which upon a domain discretization, can be reduced to a linear system of equations(see Liu [6]). In the present framework, when the RBF approach is used, a direct constant rankine panel method is used to solve the mixed boundary value problem.

After the mixed boundary value problem is solved the potential value and the free surface position are updated according to the dynamic and kinematic boundary conditions, respectively.The kinematic boundary condition under the MEL framework is often written in Lagrangian coordinates, hence for a vector $\vec{x}$=(x(t),y(t),z(t)) one have:

$$\frac{D(\vec{x}(t))}{Dt} = \nabla\phi. \tag{4}$$

The dynamic boundary condition follows directly from Bernoulli's equation applied on the free surface for the case of unsteady irrotional potential flow (see Newman [7]).

$$\frac{D(\phi)}{Dt} = \frac{1}{2}\nabla\phi \cdot \nabla\phi - g\zeta. \tag{5}$$

However, within the context of the single level set approach the kinematic boundary condition is replaced by a convection of the distance function. This way, equation 4 is changed, and the movement of nodes are implicitly performed by convecting the distance function with the normal velocity $V_n = \nabla\phi \cdot \vec{n}$ according to the following hyperbolic partial differential equation:

$$\frac{\partial d}{\partial t} + V_n \cdot |\nabla d| = 0. \tag{6}$$

In order to solve equation 6, which can also be regarded as particular case of Hamilton-Jacobi equation, a first order upwind scheme is used and the spatial derivative of the gradient is approximated by a first order space convex scheme proposed by Sethian [8].

It is also worth noting that in order to evolve the distance function according to equation 6 the normal velocity, Vn, needs to be defined on every point on the background grid whereas the solution of the mixed boundary value problem yields the velocity field only in Ω and $\partial\Omega$. Hence, the normal velocity V_n is extended to the background grid by a scheme proposed by Peng et al [9], such that:

$$\frac{\partial d}{\partial \tau} + S(d)\vec{n} \cdot \nabla V_n = 0. \tag{7}$$

Equation 7 is then iterated in the non physical time parameter τ until convergence by a first order upwind scheme. The function $S(d)$ is the signature function and a central difference scheme is used to approximate the normal vector components.

An unplesant feature of level set methods is that by advecting the signed distance function by 6 it doesn't necessarily remain a signed distance function [9]. In order to fix this, the distance function is reinitialized every time step according to:

$$\begin{aligned} \frac{\partial d}{\partial \tau} + S(d_0)(|\nabla d| - 1) = 0 \\ d(x,0) = d_0(x). \end{aligned} \tag{8}$$

For reinitialization a first order upwind - Godunov scheme is used, as described by Peng et al [9].

Alternatively, when the nodes are moved explicitly, in a Lagrangian fashion, the approach proposed by Biausser et al [10] is followed so that a second-order explicity time stepping scheme is used to update the free surface position according to equation 4. This explicit time stepping scheme is also used to update the potential value on the free surface nodes in both formulations according to equation 5. In this context once the new position of the nodes of the free surface is given the wave elevation at a given point $\zeta(x, y, t)$ is approximated, at each time step, in terms of a family of radial basis function (RBF) as:

$$\zeta(x, y, t) = \sum_{i=1}^{n} c_i f(r_i) + r_0 \tag{9}$$

In equation 9 r_i is the two dimensional euclidian distance function, namely

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}, \tag{10}$$

$f(r_i)$ is the RBF centered at the point (x_i, y_i), r_0 is a constant and c_i are constant coefficients which are calculated by imposing $\zeta(x_j, y_j, t) = \zeta_j$, which, for j=1 to n, leads to a linear system that is solved by LU decomposition. The wave elevation is then interpolated on a given triangular mesh and the signed distance function is then explicitly calculated, for each grid point $\vec{x}_g$ as :

$$d(\vec{x}_g, \vec{x}, t) = min\left\{S(\vec{n} \cdot (\vec{x}_g - \vec{x}))||\vec{x}_g - \vec{x}||\right\} \forall \vec{x} \in \partial\Omega. \tag{11}$$

3 Preliminary Results

In all preliminary results discussed below the background grid is a cartesian grid with dimensions of (2.8 x 2.8 x 2.8) units and grid spacing $h_{grid} = 0.025$ units. The edge element size function is of the form $f_h = 1 + (1.90 - z)$. Mesh quality is measured by twice the ratio of the radii of inscribed to circumscribed circles of the triangles. The edge size of an element is related to f_h and the reference edge size h_0.

At the present moment both approaches for tackling the distance evolution in time have been implemented and coupled with the mesh generating algorithm to generate simple waves. Figure 1 shows the mesh generated at t=0.01s, using the distance function estimation of equation 11 , after the mixed boundary value is solved by imposing von Neumann impervious boundary conditions on the five sides of the domain and by imposing an initial potential on the free surface of the form $\phi(\vec{x}, t) = \frac{gA}{\omega} e^{(z_0 - z)} sin(ky - \omega t)$. In this case figure 1 indicate that equation 11 is not estimating a smooth distance function although the domain reconstruction is satisfatory in most regions, except in the vicinity of the boundary in the direction of wave propagation, as the element quality is resonably good as can be seen in figure 1. The problems with this approach are currentely being investigated.

In figure 2, an evolution of an analitycal prescribed normal velocity field, V_n, of the following form $V_n = (\frac{\pi}{5} \frac{sinh(z)}{sinh(z_0)} \cdot sin(4\pi(x + t))$ is used as numerical test case for the single level set method, i.e. using equations 6, 7 and 8. As time evolves, for larger values than 0.5 seconds it can be seen that the shape of the interface starts to distort even with reinitialization being performe at each time step. This sort of behavior has also been described by Peng et al [9] where some issues relating to movement or distorting interfaces due to reinitialization are discussed. The reasons why these effects are being obeserved in a such an early stage of the simulation with a relatively simple normal velocity field are also under investigation.

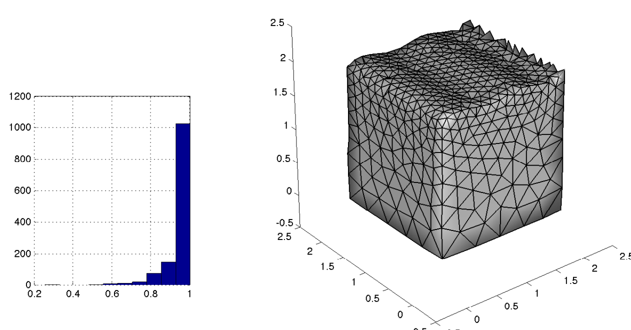

Figure 1: Mesh Generated for t=0.01s (1276 triangles), RBF approach , $h_0 = 0.10$ units, the mesh quality is also provided

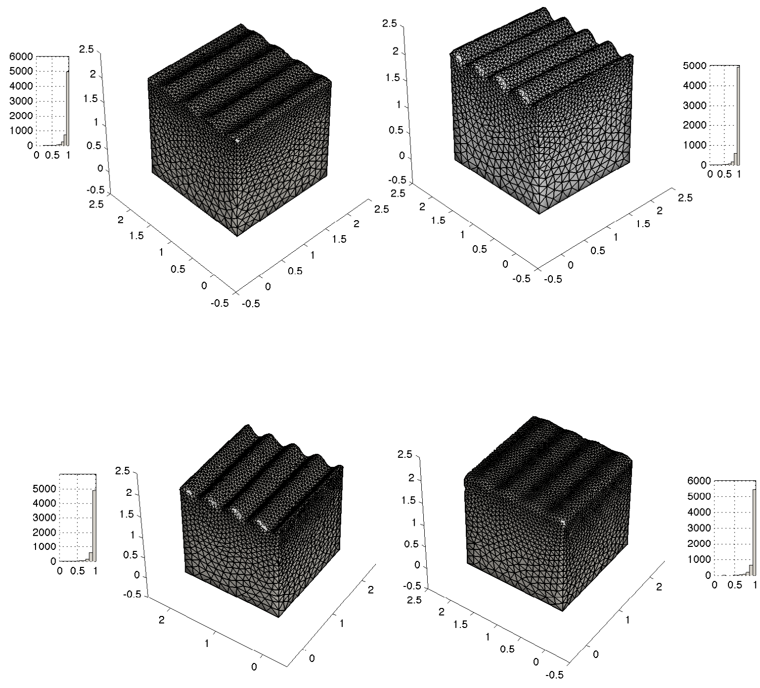

Figure 2: Level Set results of the numerical experiment for t=0.05,0.1, 0.3 and 0.5s from top to bottom; $h_0 = 0.05$ units and average mesh size is approximately 6000, $z_0 = 2$.

References

[1] M.S Longuet-Higgins and E.D Cokelet. The deformation of steep waves on water. *Proc. R. Soc. Lond.*, A(350):1–28, 1976.

[2] A. Subramani, R.F. Beck, and S. Scorpio. Fully nonlinear free-surface computations for arbitrary and complex hull forms. *22nd Symposium on Naval Hydrodynamics*, pages 390 – 402, 1998.

[3] Y. Liu, M. Xue, and D. K. P. Yue. Computations of fully nonlinear three-dimensional wave wave and wave body interactions. *Journal of Fluid Mechanics*, 438(01):41–66, 2001.

[4] W. Bai and R. Eatock Taylor. Fully nonlinear simulation of wave interaction with fixed and floating flared structures. *Journal of Ocean Engineering*, 36(3-4):223 – 236, 2009.

[5] P. Persson. *Mesh Generation for Implicit Geometries*. PhD thesis, MIT, 2005.

[6] Y. Liu. *Fast Multipole Boundary Element Method*. Cambrigde University Press, 2009.

[7] J.N. Newman. *Marine Hydrodinamics*. MIT Press, 1977.

[8] J.A Sethian. *Level Set Methods and Fast Marching Methods*. Cambrigde University Press, 1999.

[9] D. Peng, B. Merriman, S. Osher, H. Zaho, and M. Kang. A pde-based fast local level set method. *Journal of Computational Physics*, 155(2):410 – 438, 1999.

[10] B. Biausser, S.T. Grilli, and P. Fraunie. Numerical simulation of three-dimensional wave breaking by coupling of vof method and a boundary element. *The International Society of Offshore and Polar Engineers*, pages 333 – 339, 2003.