Investigation on the radiation and diffraction forces of a bulging tube

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Introduction

In the theoretical work of Farley [1] on the propagation of a bulge wave in the elastic wall of the Anaconda Wave Energy Converter, only the contribution of Froude Krylov terms was considered in the hydrodynamic forces. Radiation and diffraction contributions were neglected.

The aim of this study is to investigate to which extent this assumption is correct. One should notice that recent experimental work carried out by Chaplin et al. [2] indicates that free surface radiated waves associated with the propagation of a bulge wave can be significant. Hence, it seems reasonable to assume that radiation and diffraction effects should be taken into account in the numerical analysis of such a WEC.

In order to investigate this assumption, the seakeeping code Achil3D, based on classical linear potential theory has been adapted in order to be able to deal with radial deformations of a submerged tube. The first part of this paper presents the methodology. The second part shows some results of calculations of hydrodynamic coefficients and impulse response functions for the excitation pressure, including the diffraction.

Methodology

Let us consider an elastic tube of radius r_0 at rest whose axis is submerged at a distance *Im* below the free surface. The continuous problem of radial deformations of the tube is turned into a discrete problem. The tube is discretised in N sections of equal lengths. The radial deformation is supposed to be uniform on each single section. Let r_i be the radius of the section *i*. Let S_i the surface of the section *i* of the tube and A_i its area.



Figure 1: Discretisation of the tube.

Let consider a mesh of the tube composed of *M* flat panels, see figure (2). Let P_j the surface of panel *j*. Let φ_j be the potential associated with the elementary problem:

$$\begin{cases} \Delta \varphi_{j}(M,t) = 0 & \forall M \in \Omega & \forall t \ge 0 \\ \frac{\partial}{\partial z} \varphi_{j}(M,t) = 0 & \forall M \in S_{B} \cup S_{\infty} & \forall t \ge 0 \\ \frac{\partial}{\partial z} \varphi_{j}(M,t) = H_{j}(M,t) & \forall M \in S_{B} & \forall t \ge 0 \\ \frac{\partial^{2}}{\partial t^{2}} \varphi_{j}(M,t) + g \frac{\partial}{\partial z} \varphi_{j}(M,t) = 0 & \forall M \in S_{F} & \forall t \ge 0 \end{cases}$$
(1)

With $H_j(M,t)$ the step function:

$$H_{j}(M,t) = \begin{cases} 1 & if \quad M \in P_{j} \quad and \quad t \ge 0\\ 0 & else \end{cases}$$
(2)

 φ_i can be calculated with the BEM code Achil3D for example [3].



Figure 2. Mesh of the tube. The colors are for each independent section.

Knowing φ_j , one can calculate the mean pressure \overline{p}_{ij} over the surface of each section *i* of the tube associated with the elementary problem of a step function on panel *j*. Let define $CM_{ij}(t)$ and $CL_{ij}(t)$ such as:

$$CM_{ij}(t) = \frac{\rho}{A_i} \left(\int_{S_i} \varphi_i(M, t) dS \right)$$

$$CL_{ij}(t) = \frac{\rho}{A_i} \int_{S_i} \frac{\partial}{\partial t} (\varphi_i(M, t) - \varphi_i(M, 0)) dS$$
(3)

One can show that:

$$\overline{p}_{ij}(t) = -CM_{ij}(0)\delta(t) - CL_{ij}(t)$$
⁽⁴⁾

Bulge radiation pressure

Let Φ_{rk} be the potential associated with the elementary radiation problem of an impulse bulge at section *k*. Φ_{rk} is the solution of:

$$\begin{cases} \Delta \phi_{rk}(M,t) = 0 & \forall M \in \Omega & \forall t \ge 0 \\ \frac{\partial}{\partial z} \phi_{rk}(M,t) = 0 & \forall M \in S_B \cup S_\infty \cup S_B / S_i & \forall t \ge 0 \\ \frac{\partial}{\partial z} \phi_{rk}(M,t) = \delta(t) & \forall M \in S_k & \forall t \ge 0 \\ \frac{\partial}{\partial z^2} \phi_{rk}(M,t) + g \frac{\partial}{\partial z} \phi_{rk}(M,t) = 0 & \forall M \in S_F & \forall t \ge 0 \end{cases}$$
(5)

The body condition can be written in function of the step function $\delta(t) = \sum_{j \in S_{ki}} \dot{H}_j(M, t)$. One can show:

$$\overline{p}_{rad\,ik}(t) = -\mu_{\infty\,ik}\,\dot{\delta}(t) - K_{rad\,ik}(t) \tag{6}$$

With:

$$\mu_{\infty_{ik}} = \sum_{j \in S_{ki}} CM_{ik}(0)$$

$$K_{rad_{ik}}(t) = \sum_{j \in S_{ki}} CL_{ik}(t)$$
(7)

Let $(r_k(t))_{1 \le k \le N}$ be a time record of the radial deformation of the tube, which could correspond to a discretisation of a bulge wave propagating along the tube. The mean hydrodynamic pressure measured on section *i* due to the radiation of waves by the bulge reads:

$$\overline{p}_{rad\,i}(t) = -\sum_{k=1}^{N} \mu_{\infty\,ik}\,\ddot{r}(t) - \sum_{k=1}^{N} \int_{0}^{t} K_{rad\,ik}(t-\tau)\dot{r}(\tau)d\,\tau \tag{8}$$

Excitation pressure

As it was shown in [4], one can define a potential $\tilde{\phi}_0$ (M, x_{δ} , y_{δ} , θ , t) corresponding to an impulse elevation on the free surface at a location ($x_{\delta}y_{\delta}$) propagating in the direction θ . Let $\tilde{\phi}_{\gamma}(M, x_{\delta}, y_{\delta}, \theta, t)$ be the corresponding diffraction potential. Let \overline{p}_{0i} the mean pressure on section *i* associated with the potential $\tilde{\phi}_0$, and \overline{p}_{γ_i} the mean pressure on section *i* associated with the diffraction potential $\tilde{\phi}_{\gamma}$. They can be written:

$$\overline{p}_{0i}(x_{\delta}, y_{\delta}, \theta, t) = -\frac{\rho}{A_{i}} \int_{S_{i}} \frac{\partial}{\partial t} \widetilde{\phi}_{0}(M, x_{\delta}, y_{\delta}, \theta, t) dS$$

$$\overline{p}_{7i}(x_{\delta}, y_{\delta}, \theta, t) = \sum_{j=1}^{M} CM_{ij}(0) \frac{\partial}{\partial n} \dot{\widetilde{\phi}}_{0} + \sum_{j=1}^{M} \int_{0}^{t} \dot{CL}_{ij}(t-\tau) \frac{\partial}{\partial n} \dot{\widetilde{\phi}}_{0}(\tau) d\tau$$
(9)

Finally, the excitation pressure associated measured on section *i* corresponding with a wave profile η measured at $(x_{\delta}y_{\delta})$ reads:

$$\overline{p}_{ex_i}(x_{\delta}, y_{\delta}, \theta, t) = \int_{-\infty}^{+\infty} (\overline{p}_{0_i}(t-\tau) + \overline{p}_{7_i}(t-\tau)) \eta(\tau) d\tau$$
(10)

Results

A tube of 10 m length, 0.175 m radius was considered. It is fully submerged; with its axis located 0.2625m below the free surface. It was discretised in 420 flat panels and 20 sections, see figure (2).

The BEM code Achil3D was used to calculate the hydrodynamic data base of functions CM_{ij} and CL_{ij} . Added masses and impulse response functions for the radiation, diffraction and excitation mean pressure were calculated for each section using equations (7) and (9).

Figure (3) shows the comparison of the total impulse response for the excitation mean pressure on each section ($\overline{p}_{0_i} + \overline{p}_{7_i}$, in red) and the contribution of the diffraction part (green). The focalisation point of the wave is set equal to the origin $(x_{\delta}y_{\delta})=(0,0)$ and the direction of wave propagation is 0° .

By considering only the diffraction part, one can see how the diffracted wave field develops along the tube. One can see that its contribution to the total excitation pressure increases as the wave propagates along the tube. For the sections located downstream the focalisation point, it is the most part of the pressure whereas the Froude-Krylov part is dominating for sections located upstream. Therefore, in accordance with our initial conjecture, diffraction effects should be taken into account in the analysis of such deformable structure.

Bulge radiation coefficients and added masses were also calculated. They are not presented here because of lack of space. They will be presented at the Workshop. Main conclusions are:

- Radiation coefficients and functions are invariant along the tube. The important parameter is the distance between the section which bulges and the section where is measured the pressure.
- Radiated wave field generated by a particular section can still be observed at a distance of about 20 times the diameter of the tube.

Bibliography

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Figure 3: Comparison of total impulse excitation pressure and the contribution from the diffracted wave field on each section in function of time. Pressure is in Pa.