The 25th International Workshop on Water Waves and Floating Bodies, 9-12 May 2010: Harbin, China

Long Internal solitary waves induced currents in the ocean

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Abstract

We assume that the fluids are two-dimensional and incompressible with stratified density, and apply asymptotic method to the motion of the wave, an approximate theoretical formulation related to flow velocity field is presented in the case when the sea is continuously or two-layer stratified, The main point of the mathematical analysis is to clarify the structure of the induced velocity field. The results show that the horizontal components are basically uniform in each layer with a shear at the interface. In contrast, the vertical components vary monotonically in each layer with a maximum there. Although, the vertical components are generally one order of magnitude smaller than horizontal ones, they can never be neglected in the prediction of heave response of platform in gravitationally neutral balance.

1 Introduction

Internal solitary waves (ISWs), which may cause serious threat to offshore structures, such as jacket/floating platforms, are ubiquitous in the ocean [*H. Q. Zhang, J. C. Li, 2007*]. The US Navy has been investigating the motion of internal pycnocline in the ocean because of several losses of modern submarines [*Grue and Trulsen, 2006*]. In the north of the South China Sea, for instance, strong current of 2 m/s induced by a group of internal solitons has swung the tanker connected with drilling rig about 110º in a few minutes (*Ebbesmeyer et al., 1991*). Other accident reports thought that the drilling rigs have to withstand internal wave force. So much loss of oceanic structures indicates that internal waves are an important factor in the design of drill operations and production in the ocean. This greatly promotes research interest into the behavior and characteristics of ISWs (*e.g. Casagrande et al. 2010*). In contrast to the great deal of work such as wave propagation itself which has been done, there appears to be a dearth of induced current study. For the weakly nonlinear models, the wave induced fluid velocity in horizontal direction has been obtained from KdV theory *(Grue and Trulsen, 2006)*, but no detailed property analysis is available. For the corresponding vertical direction cases, few work can be obtained whether theoretical studies or numerical simulations. Considering that internal wave induced flow field is a crucial environmental factor in exploring the mechanism of interaction between internal wave and platform, particular attention in this paper is focused on the induced currents, the most important environmental parameters due to internal solitary waves with long wave length and period in ocean engineering.

2 Governing equation for induced velocity field

A two-dimensional, inviscid, and incompressible, with non-uniform density fluid where the internal wave motion takes place, is bounded above by a free surface and below by a rigid boundary. We shall suppose that the flow can be described by the spatial coordinates (*x, z*) where *x* is directed along the horizontal and *z* the vertical (see Fig. 1), so that the horizontal and vertical velocity components (u, v), the fluid density ρ as well as the pressure p in Cartesian coordinate satisfy the continuity equation (1) and the Euler equations (2)-(4),

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}=0 \tag{1}$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial z}=0 \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial z}\right)+\frac{\partial p}{\partial x}=0 \tag{3}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial z}\right)+\frac{\partial p}{\partial z}+g\rho=0 \tag{4}$$

Here g is the gravity acceleration and t the time coordinate.

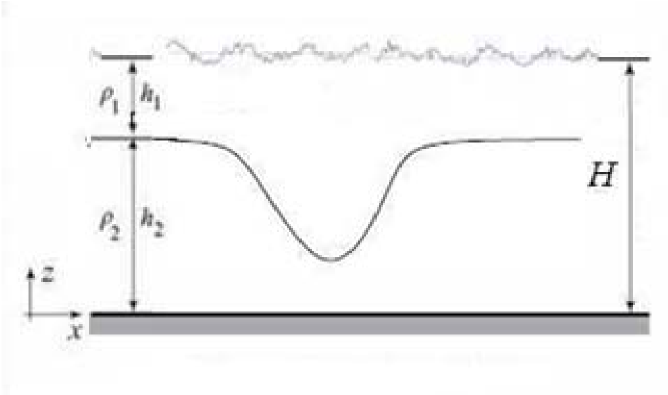

Fig. 1 Illustration of fluid coordinate system in which the upper boundary extends to the free surface and the lower to the rigid wall. Right panel: Stratification fluid. H is the undisturbed water depth; Left panel: Two-layer fluid coordinate system. Let the density be a constant ρ_1 in an upper layer of depth h_1 and $\rho_2>\rho_1$ in the lower layer of depth h_2. $h_1+h_2=H$.

When the fluid is in basic stable state, it has a density $\rho_0(z)$ satisfying $d\rho_0/dz<0$, a corresponding pressure field $p_0(z)$ such that $dp_0/dz+\rho_0 g=0$ and without flow in x- and z- direction.

Applying asymptotic method we derive the expressions of the wave induced fluid velocity,

$$u=\varepsilon V\eta\frac{d\phi}{dz} \tag{5}$$

$$v=-\varepsilon^{3/2}V\frac{\partial\eta}{\partial\xi}\phi \tag{6}$$

where $\varepsilon=O(a/H)=O((H/L)^2)$, here a and L are the wave amplitude and the wave length, respectively, and H is the undisturbed water depth; V is the nonlinear wave speed; η is the departure of the interface from the mean position; $\Phi(z)$ is the modal function.

It is readily seen from (5) and (6) that the velocity field is determined by the nonlinear wave speed V, isopycnal displacement η and modal function Φ, which satisfies an eigenvalue problem..

A useful case in practical application is that for a two-layer fluid of constant density ρ_1 and basic depth h_1 overlying another fluid of constant density ρ_2 and basic depth h_2 (see Fig. 1). That is

$$\rho_0(z)=\rho_1 H(z-h_2)+\rho_2 H(-z+h_2)$$

Here *H(z)* is the well known Heaviside function.

For this situation, the internal solitary wave induced fluid velocity structure in two-layer case is obtained, i.e. for $h_2 < z < h_1 + h_2$,

$$\begin{cases} u = \varepsilon Va \sec h^2 \beta(\xi - V\tau)(-\dfrac{1}{h_1}) \\ v = -\varepsilon^{3/2} V(-2\beta a \sec h^2 \beta(\xi - V\tau) \tanh \beta(\xi - V\tau))(\dfrac{-z + h_1 + h_2}{h_1}) \end{cases} \tag{7}$$

and for $0 < z < h_2$,

$$\begin{cases} u = \varepsilon Va \sec h^2 \beta(\xi - V\tau)(\dfrac{1}{h_2}) \\ v = -\varepsilon^{3/2} V(-2\beta a \sec h^2 \beta(\xi - V\tau) \tanh \beta(\xi - V\tau))(\dfrac{z}{h_2}) \end{cases} \tag{8}$$

where $\xi = \varepsilon^{1/2}\left(x - ct\right); \tau = \varepsilon^{3/2} t\,; \beta = \sqrt{\dfrac{\mu a}{12\lambda}}$.

Here c is linear wave speed; μ and λ are the coefficients of *KdV* equation defined by

$c = \sqrt{g \dfrac{\rho_2 - \rho_1}{\rho_2} \dfrac{h_1 h_2}{h_1 + h_2}}$, $\mu = \dfrac{3}{2} c \dfrac{h_1 - h_2}{h_1 h_2}$, $\lambda = \dfrac{c}{6} h_1 h_2$.

According to (7) and (8), we obtain the structure of the flow velocity induced by internal solitary wave for the two-fluid case. It is shown that the first of (7) and (8) follow from the fact that the induced flow velocity u in the direction of propagation has the same profile with η which is the departure of the interface from the mean position. At the same time the amplitude of horizontal component is inversely proportion to the depth of the corresponding layer and independent of the vertical variable z. While the induced vertical flow velocity v has the form of "*sech^2*tanh*"– type profile in horizontal direction and depends linearly on variable z in vertical direction within each layer. We also notice from the first term of (7) and (8) that the interface separates an upper layer with horizontal currents associated with internal wave motion from a lower layer having opposite direction currents. These analysis indicate that the horizontal velocity basically uniform in each layer with a shear at the interface, simultaneously, the vertical components vary monotonically in each layer with a maximum there.

Under some field measurements range such that the wave amplitude is taken 38 m, the upper layer 35 m and lower layer 315 m, the flow profile is illustrated in Fig. 2.We notice from Fig. 2 that the horizontal velocity component in upper layer is characterized by a pulse-like increase in current speed to a positive maximum up to 0.5 m/s at the wave peak, and the flow velocity in lower layer has the opposite direction amounted to 0.1 m/s. In contrast, the vertical velocity component has a positive pulse increase with a maximum amounting to about 0.07 m/s prior to the arrival of wave peak, then decrease sharply to zero at the wave peak, and then exhibits an equal magnitude negative pulse, finally vanishes behind the approaching wave. As shown in Fig. 2, compared to horizontal flow velocity, the vertical component is generally one order of magnitude smaller than horizontal ones, but the vertical counterpart's most prominent feature is that it changes sign in the direction of the waves which shows that the vertical component often neglected needs reconsider seriously in the prediction of heave response of platform in gravitationally neutral balance.

Acknowledgements

The authors are grateful to the support of MST of China and CAS under the Grants 863(2006AA09A103-4).

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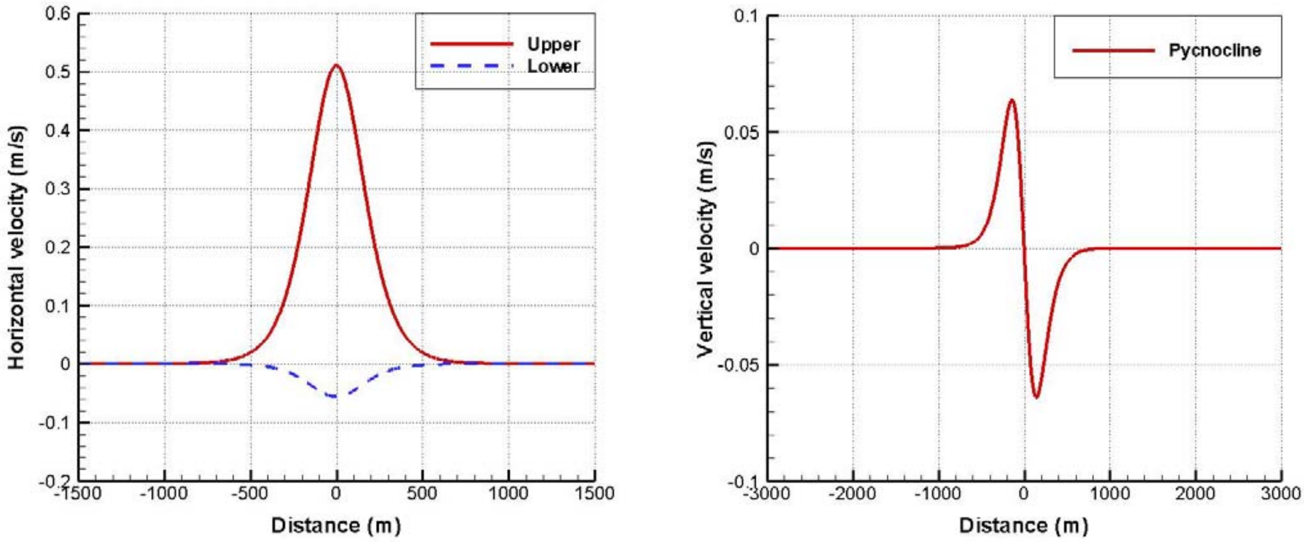

(a) (b)

Fig. 2 Velocity components profile due to internal solitary waves obtained by (7) and (8) for (a): horizontal calculated at the wave centre in upper and lower layer, respectively, and (b): vertical calculated at the pycnocline. For a two-layer model, approximation theoretical flow velocity components plot in the propagation of an internal solitary wave direction. The left panel is horizontal variable and the right panel is the corresponding vertical. The results show some obvious difference feature in flow velocity components distribution.