Investigations of Freak Waves on Uniform Current

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Introduction

Freak waves (e.g. New Wave recorded at the Draupner platform in the North Sea on 1st, January, 1995) are extremely large water waves which may pose real threat to human activities in ocean despite their low possibility of occurrence. A great deal of effort has been made to experimentally and numerically study the generation mechanisms and the physical properties of freak waves [1-2]. However, most of them are carried out in absence of currents. In fact, the wave-current interaction plays an important role in the formation of freak waves [3]. It may initiate a freak wave [4] or influence the properties of freak waves generated by other mechanisms [5-6], e.g. spatial-temporal focusing, modulation instability. Therefore, it is necessary to carry out further investigation addressing the above two issues to gain better understanding of freak waves.

In this abstract, we will numerically investigate freak waves generated by the spatial-temporal mechanism under the action of uniform collinear currents to shed some light on the second issue. The nonlinearities involved and the effects of important wave parameters, such as the wave amplitude and the spectrum bandwidth, will be studied. Apart from this, we are also concerned about the effect of the occurrence of the wave overturning. This is motivated by the fact that the wave overturning is always observed to be accompanied with freak waves but the corresponding effect is still uncertainty. A fully nonlinear investigation was undertaken using the Quasi Arbitrary Lagrangian-Eulerian finite element method (QALE-FEM) based on the fully nonlinear potential theory (FNPT), which has been proven to be the fastest FNPT-based method [8] and has been successfully applied to modeling freak waves [11-13] without current.

Mathematical Model and Summary of the QALE-FEM

The waves are generated by a wavemaker in a numerical tank. The wavemaker is mounted at the left end and a damping zone with a Sommerfeld condition [10] may be applied at the right end in order to suppress the reflection. A Cartesian coordinate system is used with the *oxy* plane on the mean free surface with the *x*-axis pointing from the wavemaker to the right end and the *z*-axis being positive upwards. The flow in the tank is governed by the FNPT model. In the present of an uniform current U_{cx}, the total velocity potential can be considered to consist of the component related to the current, i.e. xU_{cx} and the rest part of potential φ. Both the total velocity potential and φ satisfies the Laplace's equation. On the wavemaker and the free surface ($z = \eta$), corresponding fully nonlinear boundary conditions are imposed.

The QALE-FEM is developed by Ma & Yan [7-8] from the conventional FEM described by Wu & Eatock Taylor [9] and Ma, Wu and Eatock Taylor [10] but differs from the them mainly in two aspects when they are applied to modelling wave problems (without structures). One is that the computational mesh is moving in the QALE-FEM method, instead of being regenerated, at every time step during the calculation. To do so, a novel methodology has been suggested to control the motion of the nodes, in which interior nodes and nodes on the free surface (free-surface nodes) are separately considered. Different methods are employed for moving different groups of nodes. For interior nodes, a spring analogy method specially developed for nonlinear water wave problems is applied. The free-surface nodes are moved by using a arbitrary Lagrangian scheme, in which the node velocities ($v_{mx}\vec{i} + v_{my}\vec{j} + v_{mz}\vec{k}$) are given by

$$v_{mx} = \frac{\partial \phi}{\partial x} + \varepsilon U_{cx}, v_{my} = \frac{\partial \phi}{\partial y} + \varepsilon U_{cy} \text{ and } v_{mz} = \frac{\Delta \eta}{\Delta t} = (\varepsilon - 1)\left(U_{cx}\frac{\partial \eta}{\partial x} + U_{cy}\frac{\partial \eta}{\partial y}\right) + \frac{\partial \phi}{\partial z} \quad (1)$$

where $\varepsilon = 0$ for nodes near a moving boundary such as the wave maker; $\varepsilon = 1$ in the area likely with overturning waves, it linearly varies between the two. However, the free-surface nodes moving in this way may become too close or too far away from others. To prevent this from happening, they are redistributed at a specific frequency (e.g. once every 20 time steps). To redistribute free-surface nodes, they are further split into two groups: those on waterlines and those not on waterlines (inner-free-surface nodes). A principle for a self-adaptive mesh is adopted to treat the nodes on waterlines, while a spring analogy method based on local tangential-normal coordinate system is employed for the inner-free-surface nodes. The other aspect of the difference between the

QALE-FEM and conventional FEM methods is the calculation of the fluid velocity on the free surface. The technique developed in the QALE-FEM is suitable for computing the velocity when waves become very steep or even overturning. More details of these techniques will not be repeated here. Readers are referred to the papers cited above.

Numerical Results

In this paper, the QALE-FEM is used to model freak waves under currents. For convenience, in the rest of this abstract, all parameters with a length scale are nondimensionalised by the water depth d and other parameters by g and d. Both 2-D and 3-D cases have been investigated. Only 2-D investigations are considered in the abstract but some 3-D results will also be presented in the workshop.

The 2-D freak waves may be generated using a piston-type wavemaker [5,11], whose displacement (S_{wm}) is specified by

$$S_{wm}(\tau) = \sum_{n=1}^{N} \frac{a_n}{F_n} \cos(\omega_n \tau + \varepsilon_n) \text{ with } F_n = \frac{2[\cosh(2k_n) - 1]}{\sinh(2k_n) + 2k_n} \tag{3}$$

where N is the total number of components; F_n is the transfer function of the wavemaker for the n-th wave component; k_n and ω_n are the wave number and frequency of the n-th component, respectively. They are related to each other by the linear Doppler-shifted dispersion relation $(\omega_n - k_n U_{cx})^2 = k_n \tanh k_n$. a_n is the amplitude of n-th component. ε_n is the phase of the n-th component and is chosen to be $k_n x_f - \omega_n \tau_f$ with x_f and τ_f being the location and the time corresponding to the phase coherence. Due to the nonlinearity and current effects, the phase coherence may never happen [2] or the maximum wave elevation occurs at a position and time different from x_f and τ_f . Therefore, the location (x_f^*) and time (τ_f^*) corresponding to the highest crest are used to indicate the occurrence of freak waves. They are referred to as the focusing point and time, respectively. A numerical validation has been undertaken. Figure 1 illustrates the comparison between freak waves generated using Eq. (3) by the QALE-FEM and those measured in the physical wave tank [4]. The satisfactory agreement shown in this figure confirms the accuracy of the QALE-FEM in producing freak waves under uniform currents.

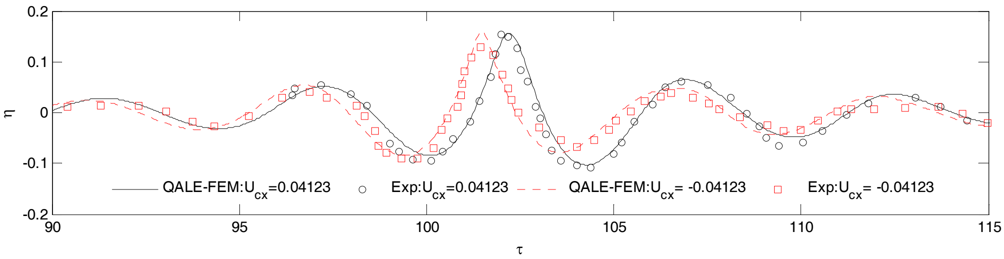

Figure 1 Wave histories recorded at focusing point (N=32, x_f = 12.5, τ_f = 102.5, ω_{min} = 1.0724; ω_{max} = 2.2846, the wave spectrum and the experimental data are duplicated from Wu and Yao [5])

As revealed by Wu et al [5], both the freak wave amplitude and the spectrum bandwidth may significantly affect the nonlinearity involved in this problem. The effect of the wave amplitude is first considered. In this investigation, different a_n varying from 0.000625 to 0.00465 (corresponding to a range of target wave amplitude A_f ranging from 0.002 to 0.1488) are used. The frequency of the wave components are equally spaced over the range [0.8683, 1.8825]. The mean group velocity (c_g) in the case with zero current is 0.396. In order to study the current effects, both following current and opposing current are considered. The current speed varies from 0 to 0.25c_g. For all the cases, N=32 is used and the linear phase coherence occurs at $x = x_f = 10$ and $\tau = \tau_f = 125$. Simulations are run in a numerical tank with length of 50. The representative mesh size (ds) is taken as $\lambda^0_{min}/20$ (i.e. ds = 0.0893) according to the convergence investigation. In this study, we mainly focus on the maximum wave elevation and local wave steepness, which are important for engineering design. The variations of the maximum wave elevation and local wave steepness in the cases with different amplitudes subject to different currents are plotted in Figure 2, in which wave overturning is observed in the cases marked by symbol '*' as illustrated in Figure 3. From this figure, it is found when the wave amplitude is small, e.g. $A_f \leq 0.037$, the maximum wave elevations and the local wave steepness decreases when U_{cx} increases in the range considered in this investigation. This is due to the energy exchange between the current and the freak wave, i.e. the opposing current transfers the energy to the freak waves and increases its wave amplitude; whilst the following current decreases the wave amplitudes. For the cases with larger wave amplitudes but without appearance of wave overturning, i.e. A_f = 0.074 and A_f = 0.1114, the trends are largely the same except that a crest is observed for both η_{max} and local wave steepness around U_{cx} = -0.2 c_g. This suggests that stronger opposing current may lead to a smaller freak waves. There may be two reasons to explain this phenomenon. The first one is that the linear phase coherence described in Eq. (3) is affected by the nonlinearity, causing that not all wave components focus at $x = x_f^*$. Another one is that the strong opposing current may partially reflect or block high-frequency components as discussed by Wu et al [5]. However, when an overturning wave occurs ($A_f \geq 0.128$ in Figure 2), η_{max}/A_f reach a crest when the freak wave is subject to a relative small opposing current, e.g. U_{cx} = -0.1c_g in the cases with A_f = 0.128 and U_{cx} = 0 for A_f =0.1488; after that, η_{max}/A_f decreases as the current speed increases up to a trough point around $U_{cx} \approx$ -0.2c_g and increases again thereafter.

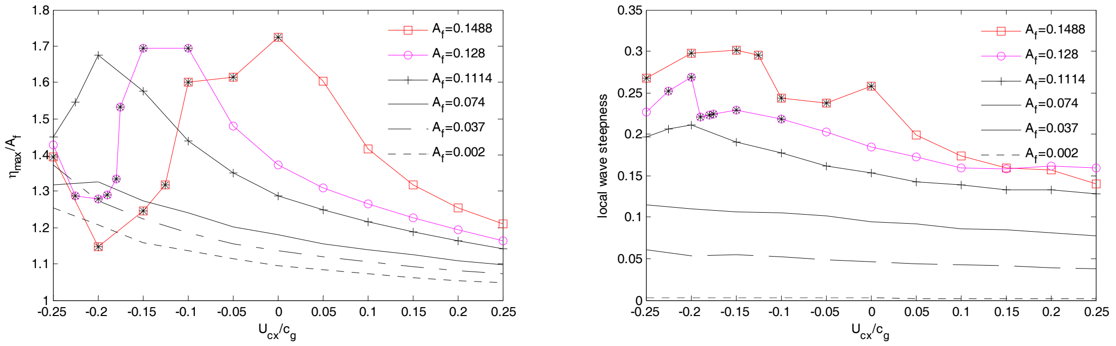

Figure 2. Maximum wave elevation and local wave steepness in the cases with different wave amplitudes (N=32, x_f = 10, τ_f = 125, ω_{min} = 0.8683; ω_{max} = 1.8825; symbols containing '*' represent occurrence of wave overturning)

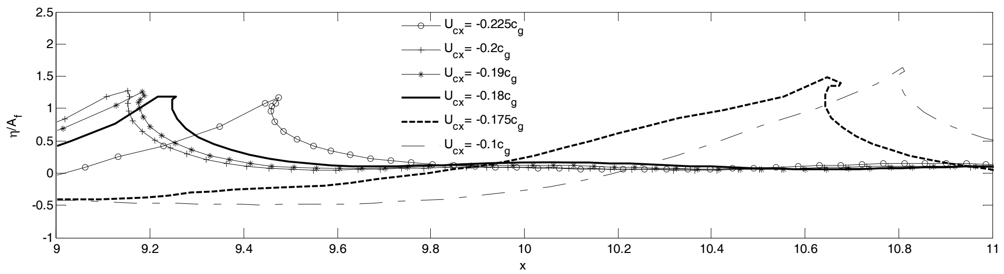

Figure 3 Free surface profiles in the cases involving wave overturning subject to different currents at (a) $\tau = \tau_f^*$ +0.25(N=32, x_f = 10, τ_f = 125, ω_{min} = 0.8683; ω_{max} = 1.8825; A_f =0.128)

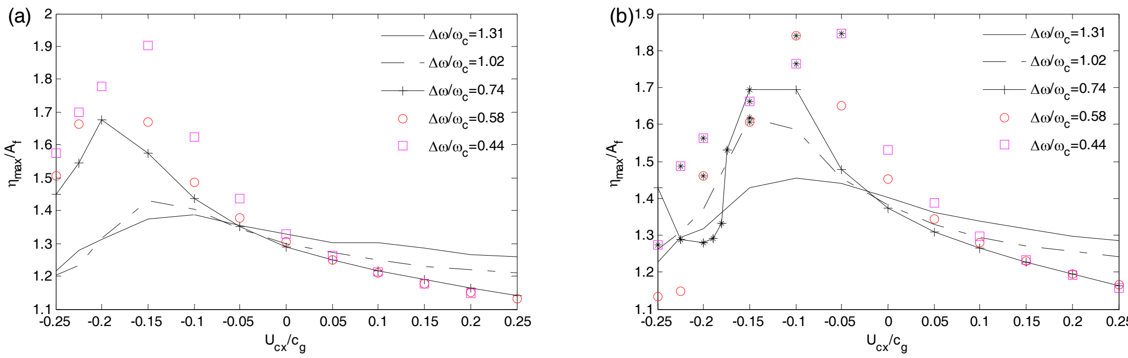

Figure 4. Maximum wave elevation in the cases with different bandwidth (a) A_f =0.1114 and (b) A_f =0.128 (N=32, x_f = 10, τ_f = 125, ω_c= 1.3774; symbols containing '*' represent occurrence of wave overturning)

Another factor that affects the behavior of the freak waves is the spectrum bandwidth. To investigate it, several cases with different frequency range are run. In these cases, we fixed the central frequency (ω_c =1.3774) to be the same as that used in Figure 2 and adjust the frequency range to yield a bandwidth ratio ($\Delta\omega/\omega_c$) ranging from 0.45 to 1.3. Two different wave amplitudes, i.e. A_f =0.1114 and A_f =0.128 are used in the investigations. Some results are shown in Figure 4. It is shown that, when A_f =0.1114 (Figure 4a), the maximum wave elevation increases as $\Delta\omega/\omega_c$ decreases in the cases without current or with relatively small opposing current (U_{cx} < -0.2c_g), which is consistent with the experimental observation [5]; whilst when following currents are involved, the trend seems to be inversed. However, for a larger wave amplitude, i.e. A_f =0.128 (Figure 4b), the trends are significantly affected by the occurrence of the overturning waves.

Summary

In this abstract, the QALE-FEM is applied to simulate the interaction between freak waves and currents. Good agreement of numerical results from the QALE-FEM with experiments is shown. For 2-D cases, the effects of the wave amplitudes and the bandwidth on the maximum wave elevation and local wave steepness are investigated; the effect of the occurrence of wave overturning in the freak wave statics is also discussed. More results will be presented in the workshop.

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