Wedge impact on liquid surface through free fall motion in three degrees of freedom

G.D. Xu[1], W.Y. Duan[1] and G.X. Wu[2]
[1]College of Shipbuilding Engineering, Harbin Engineering University,
Harbin 150001, China email: duanwenyang@hrbeu.edu.cn
[2]Department of Mechanical Engineering, University College London,
Torrington Place, London WC1E 7JE, UK email: gx_wu@meng.ucl.ac.uk

1. Introduction

One of the fluid/structure impact problems involves a body entering a free surface at high speed. The process may last for a very short period of time but it could create very large pressure and impulse force. A large body of published work has been focusing on wedge entering water with vertical velocity, or single degree of freedom. This includes a single or twin wedges at constant entry speed (e.g. Zhao & Faltinsen 1993, Wu, 2006, Xu, Duan & Wu 2008), or in free fall (e.g.Wu, Sun & He 2004, Xu, Duan & Wu 2009). The present work considers a wedge entering the water through the free fall motion in three degrees of freedom, which does not seem to have been considered previously. The stretched coordinate system method and the auxiliary function method are used together with the boundary element method for the complex potential. Some simulation results are provided and more will be presented in the workshop.

2. Mathematical model

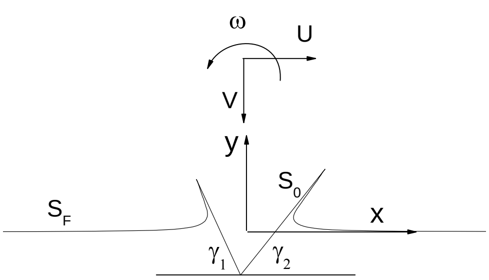


Fig.1 Sketch of the problem

We consider the free fall water entry problem of a two dimensional wedge with vertical, horizontal and rotational velocities. A Cartesian coordinate system $o-x-y$ is defined, in which x is along the undisturbed free surface and y points upwards. At time $t=0$, the tip of the wedge is at the origin of the system. Let $\vec{U}=U\vec{i}-V\vec{j}$ be the translational velocity of the centre of the mass of the wedge, and $\vec{\Omega}=\omega\vec{k}$ be the rotational velocity, where $\vec{i}$ and $\vec{j}$ are the unit vectors in the x and y directions respectively and $\vec{k}=\vec{i}\times\vec{j}$. Here minus sign has been taken before V because the vertical velocity is assumed to be positive when the body moves downwards. When the fluid is assumed to be inviscid and incompressible, and the flow is irrotational, the velocity potential ϕ can be introduced, which satisfies the Laplace equation

$$\nabla^2\phi=0 \tag{1}$$

in the fluid domain. On the wedge surface S_0 we have

$$\frac{\partial\phi}{\partial n}=(U-\omega Y)n_x+(-V+\omega X)n_y \tag{2}$$

where $\vec{n}=(n_x,n_y)$ is the normal vector on the body surface pointing out of the fluid domain and $\vec{X}=X\vec{i}+Y\vec{j}$ is the position vector from the centre of the mass of the body. The Eulerian form of the dynamic and kinematic boundary conditions on the free surface S_F or $y=\varsigma$ can be written as

$$\phi_t + \frac{1}{2}\nabla\phi\cdot\nabla\phi = 0 \quad (3)$$

$$\varsigma_t = \phi_y - \phi_x\varsigma_x \quad (4)$$

where the gravity effect has been ignored in Eq.(3).

In the Lagrangian framework, the free surface boundary conditions can be written as

$$\frac{d\phi}{dt} = \frac{1}{2}\nabla\phi\nabla\phi \quad (5)$$

$$\frac{dx}{dt} = \frac{\partial\phi}{\partial x}, \quad \frac{dy}{dt} = \frac{\partial\phi}{\partial y} \quad (6)$$

Here we use the similarity solution as the initial solution. This is then followed by the time stepping method in the stretched coordinate system. We define

$$\alpha = x/s, \quad \beta = y/s, \quad \phi = s\varphi(\alpha,\beta,t) \quad (7)$$

where $s = \int_0^t V(\tau)d\tau$. The free surface boundary conditions become

$$\frac{d(s\varphi)}{dt} = \frac{1}{2}(\varphi_\alpha^2 + \varphi_\beta^2) \quad (8)$$

$$\frac{d(s\alpha)}{dt} = \varphi_\alpha, \quad \frac{d(s\beta)}{dt} = \varphi_\beta \quad (9)$$

The introduction of the stretched system method avoids the difficulty at the early stage in the simulation in the physical domain (Zhao & Faltinsen 1993, Lu, He & Wu 2000). Another advantage of the stretched coordinate system method is that the sizes of the elements and computational domain remain more or less the same in this system, while they vary in the physical system.

For the free fall motion considered in this paper, it is important to decouple the mutual dependence between the body acceleration and fluid flow. Here we adopt the auxiliary function technique (Wu & Eatock Taylor 2003). Based on Newton's second law, we have

$$[M_b][A] = [F] + [F_e] \quad (11)$$

where

$$[M_b] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}, \quad A = \begin{bmatrix} \dot{U} \\ -\dot{V} \\ \dot{\omega} \end{bmatrix},$$

$$F = \begin{bmatrix} F_1 \\ F_2 \\ M \end{bmatrix}, F_e = \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix}$$

and the dot indicates the derivative with respect to time. m in the above equation is the mass of the two dimensional wedge of unit length, and I_{zz} is the corresponding rotational inertial about the centre of the mass. The hydrodynamic force $(F_1, F_2) = (F_x, F_y)$ and the moment M can be obtained from the integration of Bernoulli equation, or

$$F_i = \rho\int_{S_0} pn_i dS = -\rho\int_{S_0} (\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi\nabla\phi)n_i dS \quad (12)$$

$$M = -\rho\int_{S_0} (\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi\nabla\phi)(-Yn_x + Xn_y)dS \quad (13)$$

where ρ is the density of the liquid.

For ϕ_t in the above equation, we have in the fluid domain

$$\nabla^2\phi_t = 0 \quad (14)$$

The Bernoulli equation gives

$$\phi_t = -\frac{1}{2}\nabla\phi\nabla\phi \quad (15)$$

on the free surface. We also have (Wu 1998)

$$\frac{\partial\phi_t}{\partial n} = (\dot{\vec{U}} + \dot{\vec{\Omega}}\times\vec{X})\cdot\vec{n} - \vec{U}\cdot\frac{\partial\nabla\phi}{\partial n} + \vec{\Omega}\cdot\frac{\partial}{\partial n}[\vec{X}\times(\vec{U} - \nabla\phi)] \quad (16)$$

on the body surface. The direct solution of ϕ_t is not straightforward because of the acceleration term on the right hand side of

Eq.(16) is still unknown. To decouple the mutual dependence of the body motion and the fluid flow, we adopt the auxiliary function method (Wu & Eatock Taylor 1996, 2003). Thus we introduce functions $\chi_i \ (i = 1,2,3)$ which satisfy the Laplace equation

$$\nabla^2 \chi_i = 0 \tag{17}$$

in the fluid domain. We require that on the body surface

$$\frac{\partial \chi_1}{\partial n} = n_x, \frac{\partial \chi_2}{\partial n} = n_y, \frac{\partial \chi_3}{\partial n} = Xn_y - Yn_x \tag{18}$$

and on the free surface

$$\chi_i = 0 \tag{19}$$

On other rigid boundaries, we impose

$$\frac{\partial \chi_i}{\partial n} = 0 \tag{20}$$

Making use of χ_i and Green's identity, we have

$$\int_{S_0} \phi_t n_i dS = \int_{S_0} \chi_i \frac{\partial \phi_t}{\partial n} dS + \int_{S_F} (\frac{1}{2} \nabla \phi \nabla \phi) \frac{\partial \chi_i}{\partial n} dS \tag{21}$$

Substituting Eqs (12), (13), (16) and (21) into (11), we have (Wu & Eatock Taylor, 1996 & 2003)

$$[M + C][A] = [Q] + [F_e] \tag{22}$$

where $[C]$ is a matrix with

$$C_{ij} = \rho \int_{S_0} \chi_i n_j dS ,$$

$$(n_1, n_2, n_3) = (n_x, n_y, Xn_y - Yn_x)$$

and

$$[Q] = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

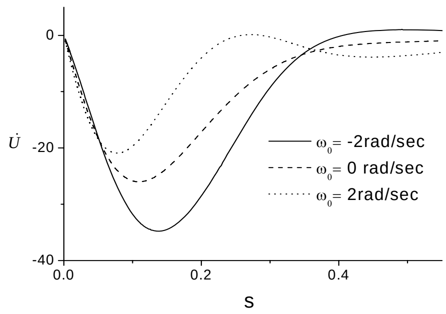
$$Q_i = \int_{s_0} \nabla \chi_i [(\vec{U} + \vec{\Omega} \times \vec{X}) \cdot \vec{n}][\nabla \phi - (\vec{U} + \vec{\Omega} \times \vec{X})] ds$$

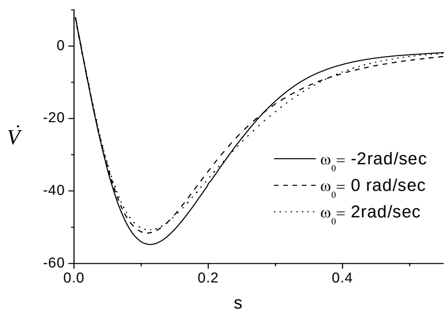
$$+ \int_{s_0} \chi_i (\vec{\Omega} \times \vec{U}) \cdot \vec{n} ds - \int_{S_F + S_0} (\frac{1}{2} \nabla \phi \nabla \phi) \frac{\partial \chi_i}{\partial n} ds$$

This equation means that the acceleration can be found before the pressure distribution.

3. Numerical results

We have verified the numerical method using the experiment data in Wu, Sun & He (2004) and the good agreement has been found. The method is then applied to various wedges in three degrees of freedom through free fall motion. Fig.2 gives the results for a wedge of $\gamma_1 = \pi/4$ and $\gamma_2 = \pi/4$ entering water in free fall motion. The initial entry velocity is $U_0 = 5m/s$, $V_0 = 5m/s$. The mass of the wedge is set as $m = 100kg/m$, and $I_{zz} = 45kg \cdot m^2$, $l = 0.25m$, where l is the distance between the mass centre and the apex of the wedge. Further discussions will be given in the workshop.





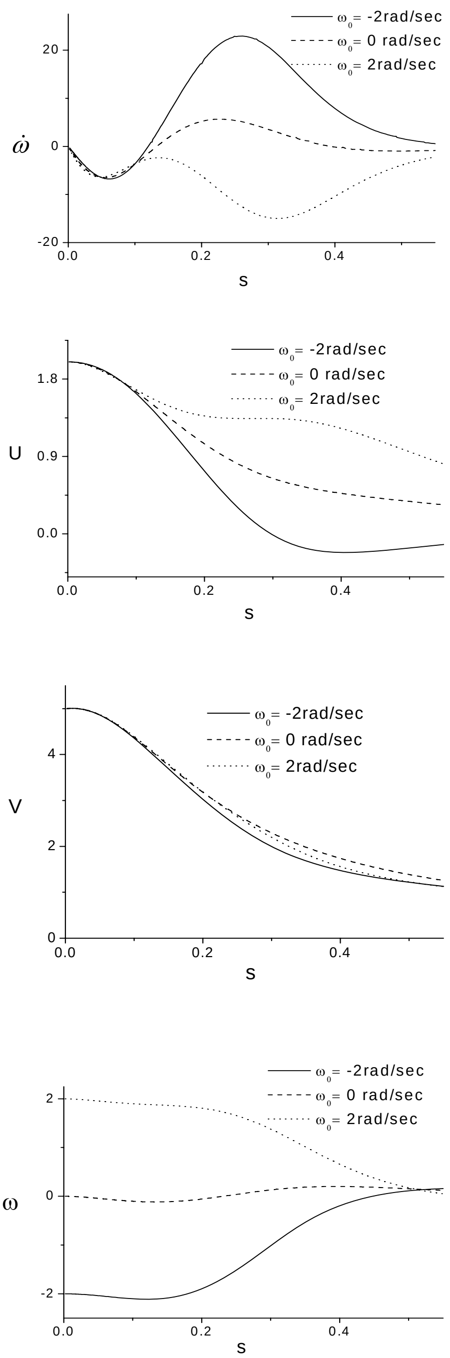


Fig.2 Motion of a wedge: accelerations and velocities in three freedoms degree

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