

A coupled Rankine - Green function method applied to the forward-speed seakeeping problem

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Introduction

In this work we try to combine the Green function, using the Green function which satisfies the free boundary condition, and Rankine methods, using the function $-1/(4\pi r) + 1/(4\pi r^*)$ for unbounded fluid, where r is the distance between the field point and the source point; r^* is that between the field point and the mirror of the source point, to solve the problem of a ship with non-zero forward speed.

Solving such a problem by only one of these two methods may arise several difficulties at the numerical stage. Among them we note here that the ship-motion Green function has the singularity and high oscillation when a field point and a source points both tend to the free surface. In this case the difficulties are to calculate the wave term of the Green function due to the integration along the dispersion curves which go to infinity, because of its slow convergence property [1].

Comparing with the Green function method for the ship with non-zero forward speed, the Rankine method is simpler in application due to the simplicity of the Green function for Rankine, but needs to introduce a damping zone and large amount of cells to discretize the free surface.

In this work we suggest to divide the fluid domain onto two parts by a control surface of specific shape, semi-spheroid. This surface separates the problem into two problems: 1) the internal one - the ship is of any form, the Green function is that for Rankine, domain is finite and all normal derivatives of velocity potential Φ are known on the ship hull and on the control surface as the solution of the external problem; 2) the external one - the shape of the control surface is known, semi-spheroid, and velocity potential is assumed to be known. Across the control surface two additional conditions must be satisfied: both the velocity potential and its normal derivative are continuous function. The second problem provides us the Dirichlet-Neumann map which is used to solve the first the problem, and as result the original one, by Rankine panel method.

The combination of these two methods keeps their advantages and brings some benefits: area to be discretize becomes smaller, no need to introduce the damping zone, the solution of the problem is that for unbound fluid domain. The calculation of the Green function for ship motion may be done only once for large set of the different ships for the one particular velocity U , incoming wave heading β and frequency ω_0 , and an additional parameter, which describes the spheroid.

Mathematical formulation

The reference system moving with the ship at the mean forward speed U along the positive x -axis is defined by letting (x, y) plane coincide with the mean free surface and z -axis be positive upward. It is assumed that the fluid is ideal with irrotational flow, the wave steepness is small and the depth is infinite.

The problem is solved by decomposing the fluid domain onto two parts (exterior and interior domains) by a control surface C which is of known predefined shape, see fig 1. Across this surface two boundary conditions are required to be satisfied: velocity potential $\Phi(x, y, z)$ and its normal derivative $\Phi_n(x, y, z)$ are continuous functions. In the outer region the solution is sought with a help of the Green function method described in [2] and the Dirichlet-Neumann map DN is computed in order to express normal derivatives of the velocity potential $\Phi_n(x, y, z)$ using the velocity potential itself: $\Phi_n(x, y, z) = DN\Phi(x, y, z)$. These derivatives are used to find the solution in the inner domain, where it is sought numerically by the Rankine method.

To simplify and reduce the domain of computation for Rankine method we define the control surface C as prolate semi-spheroid and it is given as

$$x = c \cosh \alpha \cos \beta, \quad y = c \sinh \alpha \sin \beta \cos \varphi, \quad z = c \sinh \alpha \sin \beta \sin \varphi, \quad (1)$$

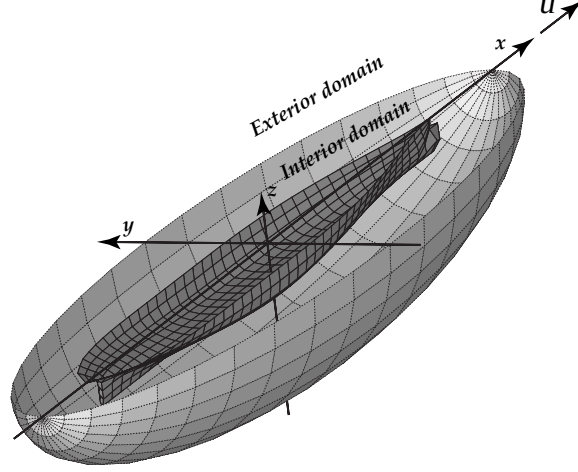


Figure 1. Formulation of the problem.

where $0 \leq \alpha < \infty$, $0 \leq \beta < \pi$, and $-\pi < \varphi \leq 0$. The constant c is a scaling factor, the angles φ and β change in the planes yOz and xOy , respectively. By setting parameters c and α we obtain a semi-spheroid with two main radii $R_x = c \cosh \alpha$ and $R_y = R_z = c \sinh \alpha$.

External problem. Application of the Green's second identity to the exterior domain provides

$$\begin{aligned} C_0 \Phi^e(P) &= \int_C [G(P; Q) \Phi_n^e(Q) - G_n(P; Q) \Phi^e(Q)] ds(Q) \\ &+ 2i\tau \int_L \Phi^e G t_y dl + \frac{U^2}{g} \int_L (G \Phi_\xi^e - \Phi^e G_\xi) t_y dl, \end{aligned} \quad (2)$$

where $G(P, Q)$ is the Green function for the source with forward speed U , e.g. see [2], $\Phi^e(P)$ is the velocity potential in the exterior region, $P = (x, y, z)$ is a field point, $Q = (\xi, \eta, \zeta)$ is a source point, L is the ellipse formed by intersection of the control surface C with the free surface (waterline for the spheroid), n is normal vector directed toward to the external region, g is the acceleration due to the gravity, $\tau = U\omega/g$ is the Brard number characterizing the flow, ω is encounter frequency defined by the frequency ω_0 and heading β of incoming wave, and the forward speed U through $\omega = \omega_0[1 - (U\omega_0/g) \cos \beta]$, and t_y is y -component of the unit vector t tangent to the L and oriented clockwise. The derivatives with respect to ξ are expressed as the following $f_\xi = c_\beta f_\beta + c_\varphi f_\varphi + c_n f_n$, where the coefficients c_n, c_β, c_φ and product $t_y dl$ for the spheroid are

$$c_n = c_\alpha = \frac{1}{c} \frac{\sinh \alpha \cos \beta}{\cosh^2 \alpha - \cos^2 \beta}, \quad c_\beta = -\frac{1}{c} \frac{\cosh \alpha \sin \beta}{\cosh^2 \alpha - \cos^2 \beta}, \quad c_\varphi = 0, \quad t_y dl = -c \sinh \alpha \cos \beta d\beta. \quad (3)$$

From the theory of spherical functions [3],[4] we know that any harmonic function on a spheroid can be presented as infinite series with respect to product of associated Legendre functions $P_n^m(\cos \beta)$ and $\sin m\varphi$ or $\cos m\varphi$:

$$u(\alpha, \beta, \varphi) = \sum_{m=0}^{\infty} \sum_{n=m+1}^{\infty} \sum_{s=1}^2 a_{nm}^{(s)}(\alpha) \mathcal{S}_{nm}^{(s)}(\beta, \varphi) = \sum_i a_i(\alpha) \mathcal{S}_i(\beta, \varphi), \quad (4)$$

where $\mathcal{S}_{nm}^{(1)}(\beta, \varphi) = P_n^m(\cos \beta) \sin m\varphi$, $\mathcal{S}_{nm}^{(2)}(\beta, \varphi) = P_n^m(\cos \beta) \cos m\varphi$, and the new index function $i = i(m, n, s)$, $m = 0, 1, 2, \dots$, $n = m + 1, m + 2, \dots$, $s = 1, 2$.

So, we can present velocity potential, the Green function and their derivatives on C as follows

$$\begin{aligned} \Phi^e(P) &= \sum_i \phi_i(\alpha) \mathcal{S}_i(\beta_P, \varphi_P), \quad \Phi_n^e(P) = \sum_i \psi_i(\alpha) \mathcal{S}_i(\beta_P, \varphi_P), \\ G(P, Q) &= \sum_{i,j} g_{i,j}(\alpha) \mathcal{S}_i(\beta_P, \varphi_P) \mathcal{S}_j(\beta_Q, \varphi_Q), \\ G_n(P, Q) &= \sum_{i,j} h_{i,j}(\alpha) \mathcal{S}_i(\beta_P, \varphi_P) \mathcal{S}_j(\beta_Q, \varphi_Q), \end{aligned} \quad (5)$$

where $\beta_P = \beta(P)$, $\beta_Q = \beta(Q)$, $\varphi_P = \varphi(P)$ and $\varphi_Q = \varphi(Q)$. Substituting (5) into (2) and then, after some algebra, we obtain the system of algebraic linear equations

$$(\psi_1, \psi_2, \dots, \psi_k, \dots)^T = DN (\phi_1, \phi_2, \dots, \phi_k, \dots)^T, \quad DN = M^{-1}K, \quad (6)$$

where the elements of the matrices K and M are

$$\begin{aligned} M_{mk} &= \sum_{i,j} g_{i,j}(\alpha) \mathcal{I}_{im} \left[\mathcal{I}_{jk} + \frac{U^2}{g} \mathcal{J}_{jk,\alpha} \right] \\ K_{mk} &= C_0 \mathcal{I}_{km} + \sum_{i,j} \mathcal{I}_{im} \left[h_{i,j}(\alpha) \left\{ \mathcal{I}_{jk} + \frac{U^2}{g} \mathcal{J}_{jk,\alpha} \right\} - g_{i,j}(\alpha) \left\{ 2i\tau \mathcal{J}_{jk} + \frac{U^2}{g} \hat{\mathcal{J}}_{jk,\beta} \right\} \right] \end{aligned} \quad (7)$$

with

$$\begin{aligned} \mathcal{I}_{jk} &= \int_C \mathcal{S}_j(\beta, \varphi) \mathcal{S}_k(\beta, \varphi) dS, \\ \mathcal{J}_{jk} &= \int_L \mathcal{S}_j(\beta, \varphi) \mathcal{S}_k(\beta, \varphi) t_y dl, \quad \mathcal{J}_{jk,\alpha} = \int_L c_\alpha \mathcal{S}_j(\beta, \varphi) \mathcal{S}_k(\beta, \varphi) t_y dl, \\ \hat{\mathcal{J}}_{jk,\beta} &= \mathcal{J}_{jk,\beta} - \mathcal{J}_{kj,\beta}, \quad \mathcal{J}_{jk,\beta} = \int_L c_\beta \mathcal{S}_j(\beta, \varphi) \frac{\partial \mathcal{S}_k}{\partial \beta}(\beta, \varphi) t_y dl. \end{aligned} \quad (8)$$

Internal problem. Application of the Green's second identity to the interior domain provides

$$\begin{aligned} C_0 \Phi^i &= \int_C (G^R \Phi_n^i - \Phi^i G_n^R) dS - \int_H (G^R \Phi_n^i - \Phi^i G_n^R) dS - \int_F \Phi^i (kG^R - G_\zeta^R) dS \\ &+ 2i\tau \int_W \Phi^i G^R t_y dl - 2i\tau \int_L \Phi^i G^R t_y dl - \frac{U^2}{g} \int_L (c_\beta \Phi_\beta^i G^R + c_n \Phi_n^i G^R) t_y dl \\ &+ \frac{U^2}{g} \int_W (c_u \Phi_u^i G^R + c_v \Phi_v^i G^R + c_n \Phi_n^i G^R) t_y dl. \end{aligned} \quad (9)$$

Here $4\pi G^R = -1/r + 1/r^*$, $r^2 = (x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2$, $r^{*2} = (x - \xi)^2 + (y - \eta)^2 + (z + \zeta)^2$, Φ^i is the velocity potential in the inner region, n is normal vector directed toward to the inner fluid domain, H and F are the ship hull and the free surface, respectively, $k = \omega^2/g$, W is the ship waterline and the rest notations are as same as for the external problem.

In (9) the first, fifth and sixth integrals on the right-hand side can be transformed using (5) and conditions of continuity of velocity potential and its normal derivatives across the control surface C . We should note that for two problems the normal vectors of C are oppositely directed and, thus, $\Phi_{n_{in}}^i = -\Phi_{n_{ex}}^i$ and $G_{n_{in}}^R = -G_{n_{ex}}^R$, where $n_{in} = n$ for the internal problem and $n_{ex} = n$ for the external one.

For the point $Q \in C$ we can use (4) to present $G^R(P; Q)$ and $G_n^R(P; Q)$ in the form of infinite series

$$G^R(P; Q) = \sum_i g_{,i}^R(P, \alpha) \mathcal{S}_i(\beta_Q, \varphi_Q), \quad G_n^R(P; Q) = - \sum_i h_{,i}^R(P, \alpha) \mathcal{S}_i(\beta_Q, \varphi_Q). \quad (10)$$

Substitution (5) and (10) into the first, fifth and sixth integrals on the right-hand side of equation (9) yields

$$\sum_k \phi_k(\alpha) \mathcal{L}_k, \quad (11)$$

where

$$\begin{aligned} \mathcal{L}_k(P, \alpha) &= \sum_i h_{,i}^R(P, \alpha) \mathcal{I}_{ik} - \sum_{i,m} g_{,i}^R(P, \alpha) DN_{mk} \left[\mathcal{I}_{im} + \frac{U^2}{g} \mathcal{J}_{im,\alpha} \right] \\ &- \sum_i g_{,i}^R(P, \alpha) \left[2i\tau \mathcal{J}_{ik} + \frac{U^2}{g} \mathcal{J}_{ik,\beta} \right]. \end{aligned} \quad (12)$$

Next, we should divide the free surface F and the ship hull H onto small panels, F_q , and H_p , respectively, on which the velocity potential Φ_r^S , $S = F$ or $S = H$, can be calculated by using N_r shape functions

$$\Phi^i(P) = \Phi_r^S = \sum_{b=1}^{N_r} \phi_{br}^S \mathcal{N}_{br}^S(u, v), \quad \text{if } P \in S_r \subset S, \quad (13)$$

where (u, v) are local coordinates. After substituting (13) into (9) and, then, regrouping the terms, we obtain the following equation

$$\begin{aligned}
C_0 \Phi^i - \sum_k \phi_k(\alpha) \mathcal{L}_k(P, \alpha) - 2i\tau \sum_{s'} \phi_{s'}^W \int_{W_s} \mathcal{N}_{s'}^W G^R t_y dl - \sum_{p'} \phi_{p'}^H \int_{H_p} \mathcal{N}_{p'}^H G_n^R dS \\
+ \sum_{q'} \phi_{q'}^F \int_{F_q} \mathcal{N}_{q'}^F (kG^R - G_\zeta^R) dS - \frac{U^2}{g} \sum_{s'} \phi_{s'}^W \int_{W_s} \left(c_u \frac{\partial \mathcal{N}_{s'}^W}{\partial u} G^R + c_v \frac{\partial \mathcal{N}_{s'}^W}{\partial v} G^R \right) t_y dl \\
= \frac{U^2}{g} \int_W c_n \Phi_n^i G^R t_y dl - \int_{S_H} G^R \Phi_n^i dS.
\end{aligned} \tag{14}$$

Here we used the notation

$$\sum_{s'} \phi_{s'}^S \mathcal{N}_{s'}^S \equiv \sum_{s=1}^{N^S} \sum_{b=1}^{N_s} \phi_{bs}^S \mathcal{N}_{bs}^S(u, v). \tag{15}$$

Then, depending on the location of the point P we consider three cases: 1) $P \in C$, 2) $P \in H$, 3) $P \in F$. For each case, using the appropriate presentation of the first term in (14) and G^R , as function of P , multiplying (14) by the function \mathcal{F} , $\mathcal{F} = \mathcal{S}_m$ for the first case, $\mathcal{F} = \mathcal{N}_{k'}^H$ - the second, and $\mathcal{F} = \mathcal{N}_{n'}^F$ - the third, and integrating the obtained product, we can transform (14) into a matrix form

$$\vec{\phi} \mathcal{M} = \vec{\mathcal{F}}, \tag{16}$$

where $\vec{\phi} = (\phi_1, \phi_2, \dots, \phi_1^{S_H}, \phi_2^{S_H}, \dots, \phi_1^{S_F}, \phi_2^{S_F}, \dots)^T$ is unknown vector; the matrix \mathcal{M} and the vector $\vec{\mathcal{F}}$ are known and should be evaluated numerically.

Conclusions

The most complicated part is to compute the DN matrix due to the complexity of the Green function $G(P; Q)$ for the moving source. On the other hand, for one velocity U , incoming wave heading β and frequency ω_0 , and ratio of the main radii of the spheroid (or parameter α) this computation should be done only once and the obtained matrix DN can be used for the large amount of bodies of different shapes which can be surrounded by the spheroid of the same ratio of the main radii. The one of advantages of presented method is that the solution does not depend on the size of the computational domain and we do not need to present artificial damping zones to avoid reflected waves.

References

- [1] CHEN, X.B. AND WU, G.X., 2001. "On singular and highly oscillatory properties of the Green function for ship motions", J. Fluid Mech., vol 445, pp 77-91.
- [2] CHEN, X.B., DIEBOLD, L. AND DOUTRELEAU, Y., 2000. "New Green-Function Method to Predict Wave-Induced Ship Motions and Loads", Proc. 23rd ONR Symp. on Naval Hydrodynamics, Val de Reuil, France, Sept. 17-22, pp. 66-81
- [3] HOBSON E.W., "The theory of spherical and ellipsoidal harmonics", Cambridge University press, 1931
- [4] LEBEDEV N.N., "Special functions and their applications", Dover publications Inc, New York