

AN EFFICIENT NUMERICAL METHOD FOR THE THREE-DIMENSIONAL WAGNER PROBLEM

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1 INTRODUCTION

Some bulbous bows, also used as sonar domes, are made of fibre-reinforced plastic and must be sized to sustain slamming loads. This problem involves dealing with the three-dimensional (3D) water impact of non-slender bodies for which classical strip methods are inaccurate [3]. CFD codes using elaborated models lead to accurate results [2] but still suffer from high computation costs. As a result, simple practical tools are required for industrial applications. The Wagner theory of water impact provides an interesting model that describes accurately the initial stage of slamming. The Wagner problem has been extensively studied in the two-dimensional and axisymmetric cases but there are still needs for simple solution methods for the general 3D geometries.

This work presents an efficient numerical method to deal with three-dimensional water impact problems based on the Wagner approach [6] and the Boundary Element Method (BEM) [5]. Indeed, it is proposed to derive an approximated solution of the 3D Wagner problem (the intersection line) in the form of a truncated Fourier series. The intersection line is determined by an iterative algorithm including two steps: the first step consists in computing the displacement at the free surface using the BEM for a guessed intersection line, and the second step consists in updating the terms of the Fourier series according to the discontinuities of the displacement between the free surface and the wetted surface.

2 THREE-DIMENSIONAL WAGNER PROBLEM

The theory of water impact developed by Wagner consists in linearizing the boundary conditions of the flow on the body and on the free surface (see Figure 1 below and [1]). Within the Wagner approximation, the displacement potential φ is governed by the following system:

$$\left\{ \begin{array}{ll} \Delta\varphi = 0 & z \leq 0 \\ \frac{\partial\varphi}{\partial z} = f(x, y) - d(t) & WS \\ \varphi = 0 & FS \\ \varphi \rightarrow 0 & |x, y| \rightarrow \infty \end{array} \right. , \quad (1)$$

where $f(x, y)$ is the body shape function and $d(t)$ the penetration depth at instant t .

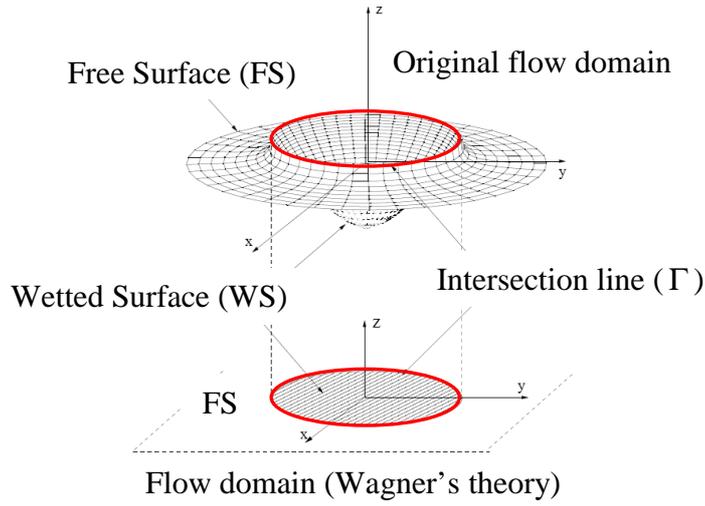


Figure 1: Description of the flow domains in the Wagner theory (from Scolan and Korobkin [1]).

The Wagner problem consists in determining the intersection line that respects the Wagner condition, which requires that the vertical displacement is continuous at the intersection line:

$$\partial\varphi/\partial z = f(x, y) - d(t) \quad (2)$$

3 GENERAL OUTLINE OF THE METHOD

Solving equation system (1) for a “wrong” intersection line leads to a discontinuous displacement of the surface $z=0$. Moreover, the discontinuity increases as the “error” on the wetted surface increases and therefore it provides a measure of the mismatch between the considered (“wrong”) intersection line and the correct one (satisfying the Wagner condition). Figure 2 displays the free surface displacement for several incorrect values of the wetted surface radius (dotted blue lines) for a cone together with the theoretical free surface displacement (red line):

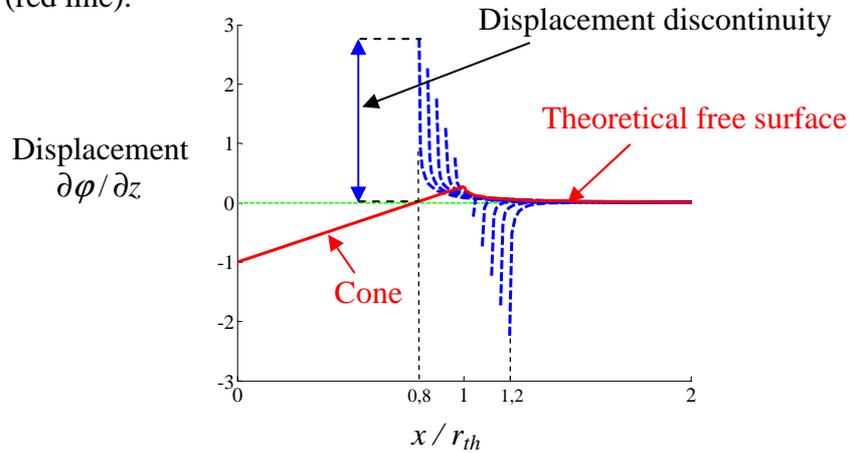


Figure 2: Free surface displacement as a function of the normalised wetted surface radius. x denotes the radial coordinate and r_{th} is the correct radius of the free surface.

In the present method, it is proposed to find an approximated intersection line Γ in the form of a truncated Fourier series. Restricting the attention to symmetric impacting bodies, the radial coordinate $a(\theta)$ of the intersection line is written in the following form:

$$a(\theta) = \sum_{i=0}^{Nha-1} a_i \cos(i\theta), \quad (3)$$

where Nha is the number of terms in the harmonic decomposition. The coefficients a_i are

determined by adjusting them in order to avoid the displacement discontinuities at Γ . For this purpose, the error of continuity is expanded in a truncated Fourier series:

$$e_i = \int_{\Gamma} (\partial\varphi/\partial z - f + d) \cos(i\theta) d\theta. \quad (4)$$

Thus, each component e_i of the error of continuity provides a measure of the error on component a_i . Therefore, in order to determine the intersection line, the nonlinear system $e_i(a_j)=0$ has to be solved. This is achieved by a fixed-point algorithm with dynamic relaxation based on the Aitken method [4]. At each step n , the intersection line is updated as follows:

$$\begin{cases} a_i^{(n+1)} = a_i^{(n)} + \alpha^{(n)} e_i^{(n)} \\ \alpha^{(n+1)} = -\alpha^{(n)} \frac{\sum_{i=0}^{Nha-1} [e_i^{(n)} e_i^{(n+1)} - (e_i^{(n)})^2]}{\sum_{i=0}^{Nha-1} (e_i^{(n+1)} - e_i^{(n)})^2} \end{cases} \quad (5)$$

With the proposed algorithm, it is necessary to compute the displacement of the free surface at each iteration. This is achieved using a dedicated Boundary Element model.

4 COMPUTATION OF THE DISPLACEMENT OF THE FREE SURFACE USING A BOUNDARY ELEMENT MODEL

Equation system (1), governing the displacement potential, can be transformed into a surface integral governing the displacement q ($q = \frac{\partial\varphi}{\partial z}$) at plane $z=0$:

$$\iint_{S(z=0)} \frac{q(x_2, y_2)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} dx_2 dy_2 = 0, \quad (6)$$

where (x_1, y_1) is a collocation point on the free surface ($(x_1, y_1) \notin WS$).

The displacement of the free surface involved in (6) is also expanded in Fourier series in order to reduce the number of degrees of freedom:

$$q(\xi, \theta) = \sum_{i=0}^{Nhq-1} q_i(\xi) \cos(i\theta), \quad (7)$$

where $\xi = \sqrt{x^2 + y^2} / a(\theta)$ is the nondimensional radial coordinate and Nhq is the number of terms in the harmonic decomposition. The evolution of the Fourier coefficients $q_i(\xi)$ of the displacement (as a function of ξ) is approximated by linear shape functions.

5 RESULTS

The method has been implemented and tested on reference geometries such as a cone and an elliptic paraboloid. Figure 3 below compares the shape of the wetted surfaces obtained for several values of Nha with the analytical solution derived by [1] for an elliptic paraboloid. It can be seen that a good approximation of the intersection line is achieved with $Nha=5$. The proposed solution method for the 3D Wagner problem makes it possible to compute the hydrodynamic pressure acting on a 3D body using the Modified Logvinovich Model (MLM) [6] and then to compute the hydrodynamic force F by integrating the pressure. Figure 4 below depicts the slamming coefficient C_S ($C_S = 2F / (\rho V^2)$) during the water impact at constant

speed of an elliptic paraboloid ($f(x, y) \approx 1.418x^2 + 0.517y^2$). The MLM results are compared to Finite Element results performed by ABAQUS/Explicit using the VOF method and experimental data carried out using a hydraulic shock machine at a constant speed V of 12m/s (see [7] for more details on the experimental set-up). Results are in very good agreement until the total immersion of the mock-up is reached ($d(t) \approx 0.023$). Oscillations can be observed on the experimental data because of impact induced vibrations of the mock-up.

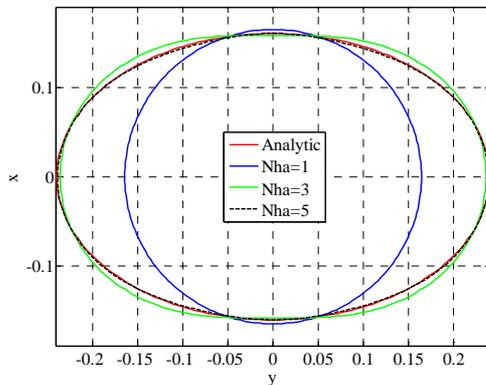


Figure 3: Intersection line for an elliptic paraboloid.

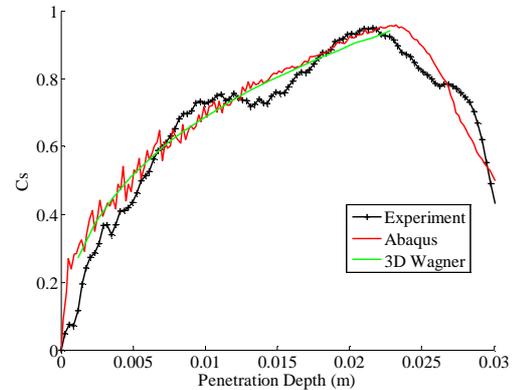


Figure 4: Slamming coefficient as a function of the penetration depth.

6 CONCLUSION

A fast numerical method for three-dimensional water impact problems is proposed. This method is based on the Wagner approach and the displacement potential formulation. With the proposed approach, the solution at a particular instant of time is independent of the previous instants of time. As a result, it can be computed very quickly. Moreover, the accuracy of the solution remains constant as the time increases.

The prediction of the wetted surface has been first validated by analytical results for an elliptic paraboloid. Then, the prediction of the slamming force has been shown to be in good agreement with numerical and experimental reference results. Other comparative studies have been carried out on several three-dimensional geometries. The proposed method was shown to be very promising. Further work will consist in coupling the fluid model to structural calculations. Then, industrial applications will be considered.

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