Rogue Waves Due To Nonlinear Broadband Wave Interactions

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Abstract

We investigate the generation mechanisms and evolution dynamics of rogue (or freak) waves using direct phase-resolved nonlinear wave-field simulations. The focus is on the understanding of the effects of nonlinear broadband wave-wave interactions on rogue wave development and characteristics. Large-scale long-time simulations of nonlinear long-crested wave-fields with various wave spectrum parameters are obtained. Based on these simulations, we find that nonlinear self-focusing of wave groups and nonlinear wave group interactions play a critical role in the formation and development of rogue waves in deep seas. Occurrence of rogue waves is closely correlated to kurtosis of the wave-field, with larger kurtosis associated with rogue waves of higher height. Moreover, occurrence of rogue waves (especially of large height) is usually correlated with broadband wave spectra.

Background

The occurrence of rogue/freak waves in deep seas has been observed (e.g. Haver 2000). Due to exceptionally large amplitudes, rogue waves may cause catastrophic damage to offshore structures and surface vessels (e.g. Lawton 2001). Despite its importance and significant number of previous studies, the key mechanisms underlying the occurrence and generation of rogue waves remain inconclusive (Aha'05).

The occurrence of rogue waves is a rare event, and it is generally difficult to obtain systematic measurements in the field or in the laboratory. In view of this, numerical simulations are particularly useful. Model equations based on nonlinear Schrödinger (NLS) equation and its modifications (e.g. Trulsen & Dysthe 1997) have been used to study the occurrence of rogue waves. These methods are however limited to narrow-banded waves and to relatively short evolution times.

In this work, we apply direct large-scale phase-resolved simulations of long-crested nonlinear wave-field evolution to investigate the dynamics and statistics of rogue waves in deep seas (Wu, Liu & Yue 2005). The present simulations overcome all of the shortcomings of model-equation-based approaches. The main objectives are to find and verify the validity and limitations of existing theories and models for the prediction of rogue wave events; to understand and characterize the mechanisms for the generation of rogue waves associated with nonlinear broadband wave interactions; and to quantify the statistics of rogue wave events in realistic ocean environments.

Approach

The objectives stated above are achieved by (I) development of a significant number of large-scale computations and datasets for nonlinear evolution of wave-fields for different initial wave (spectral) parameters; (II) use of direct computations to quantitatively verify and validate existing theories and models for wave-field statistics and the hypotheses on rogue wave formation; and (III) use of these computations to investigate the stochastic and deterministic mechanisms underlying the occurrence of rogue wave events and to characterize the statistical and physical properties of such events.

The highly efficient high-order spectral (HOS) method is applied for direct phase-resolved simulation of nonlinear ocean wave-field evolution (Dommermuth & Yue 1987; Wu 2004). HOS resolves the phase of a large number of wave modes and accounts for their nonlinear interactions up to an arbitrary high order M including broadband non-resonant and resonant interactions up to any specified order. HOS achieves an exponential convergence and a (near) linear computational effort with respect to the number of wave modes N and the interaction order M, and has (near) linear scalability on high-performance parallel computing platforms. Unlike the phase-averaged and model-equation-based approaches, HOS accounts for physical phase-sensitive effects in a direct way. These include the initial distribution of wave phases in the wave-field specified by wave spectrum and energy dissipation due to wave breaking.

From the direct HOS simulation, we obtain datasets of wave elevation and kinematics of the complete wave-field during its time evolution. By analyzing these datasets, we determine the statistics of the wave-field, identify rogue wave events, compute the statistics of rogue waves, study the development of rogue waves and groups, and understand details of the rogue wave dynamics.

Key Results

Based on the direct phase-resolved HOS simulations of long-crested nonlinear wave-field evolutions, we confirm that nonlinear self-focusing of wave groups and nonlinear wave group interactions are the major factors causing the formation and development of rogue waves in deep seas. In particular, the occurrence of large amplitude rogue waves is closely correlated to the presence of broad band wave components. Specifically, the inclusion of higher-order broad-band nonlinear wave interactions in longer-time evolution generally leads to the generation of larger rogue waves.

(I) Very long-time nonlinear evolution of a modulated Stokes wave train: To elucidate the effects of nonlinear broadband wave interactions in rogue wave development, we consider a very long-time evolution of a modulated Stokes wave train, which is obtained using nonlinear HOS simulation (with order M =4). . The steepness of the initial wave train is ε_0 = 0.06. Small disturbances at a broad band of wavenumbers around the primary wavenumber are added at the initial time t = 0. Figure 1a displays the time variation of maximum wave crest/trough elevation in the wave-field during the evolution. In the initial period of evolution, modulational instability causes spreading of wave energy from the primary wave to its dominant sidebands. As a result, large waves are developed with wave peak height approaching ~ 3.0a_0 where a_0 is the wave amplitude of the initial wave train. For t/T_0 < ~1000 where T_0 is the period of the carrier wave, the phenomenon of recurrence is observed. For t/T_0 > ~1000, as broader and broader sidebands are developed, the recurrence is lost, and the wave-field becomes irregular. Importantly, the wave peak height of the developed extreme waves becomes much larger and could reach a value of ~ 5.0a_0 as shown in figure 1a. Figure 1b compares the two representative rogue wave profiles at t/T_0 = 110 and 1657. It seems that as broader band wave interactions are involved, the peak of the rogue wave increases significantly while the trough remains almost unchanged. The inspection of the wave spectrum of the wave-field reveals that larger rogue waves are always correlated with broadband wave spectra of the wave-field, as shown in figure 1c. These results indicate that nonlinear focusing of wave groups together with nonlinear broadband wave-wave interactions can cause the development of rogue waves. Low order narrowband models (such as those based on NLS equation) may not be able to properly predict and describe rogue waves (especially of large height and in very long time evolution).

(II) Rogue wave occurrences in long-time evolution of nonlinear long-crested wave-fields: Using the direct phase-resolved HOS simulations, we obtain a number of datasets of long-time nonlinear wave-field evolution corresponding to various (JONSWAP) wave spectrum parameters. From these simulated nonlinear wave-fields, we obtain statistics of rogue wave events and its dependence on wave-field parameters. Figure 2 shows the probability of rouge waves with wave height greater than $2H_s$ as a function of the Benjamin-Feir index BFI, where H_s is the significant wave height. BFI is the ratio of the wave peak steepness to the (normalized) bandwidth of a spectrum (Janssen 2003), defined by BFI = $2^{1/2}\ \varepsilon_p\ /\ (\Delta k/k_p)$. The results are obtained from a total of 21 different wave spectra. Nonlinearity order M = 4 is used for all the simulations. The simulation for each case lasts 2000 T_p where T_p is the peak period. The results in figure 2 show that for relatively large BFI (BFI >~ 0.7), the occurrence of rogue waves almost linearly increases with BFI and the linear Rayleigh distribution significantly underestimates the rogue wave occurrence (Janssen 2003). For relatively small BFI (BFI < ~ 0.5), rogue wave occurrence is almost independent of BFI, and Rayleigh distribution overestimate the probability of rogue waves.

To assist in the prediction of the occurrence of rogue wave events in realistic ocean environments, the correlations between rogue wave occurrence (and characteristics) with wave-field statistics such as Kurtosis are investigated in details. Nonlinear wave-field statistics and comparisons with the existing linear and second-order theories are also obtained. These results will be discussed in the presentation.

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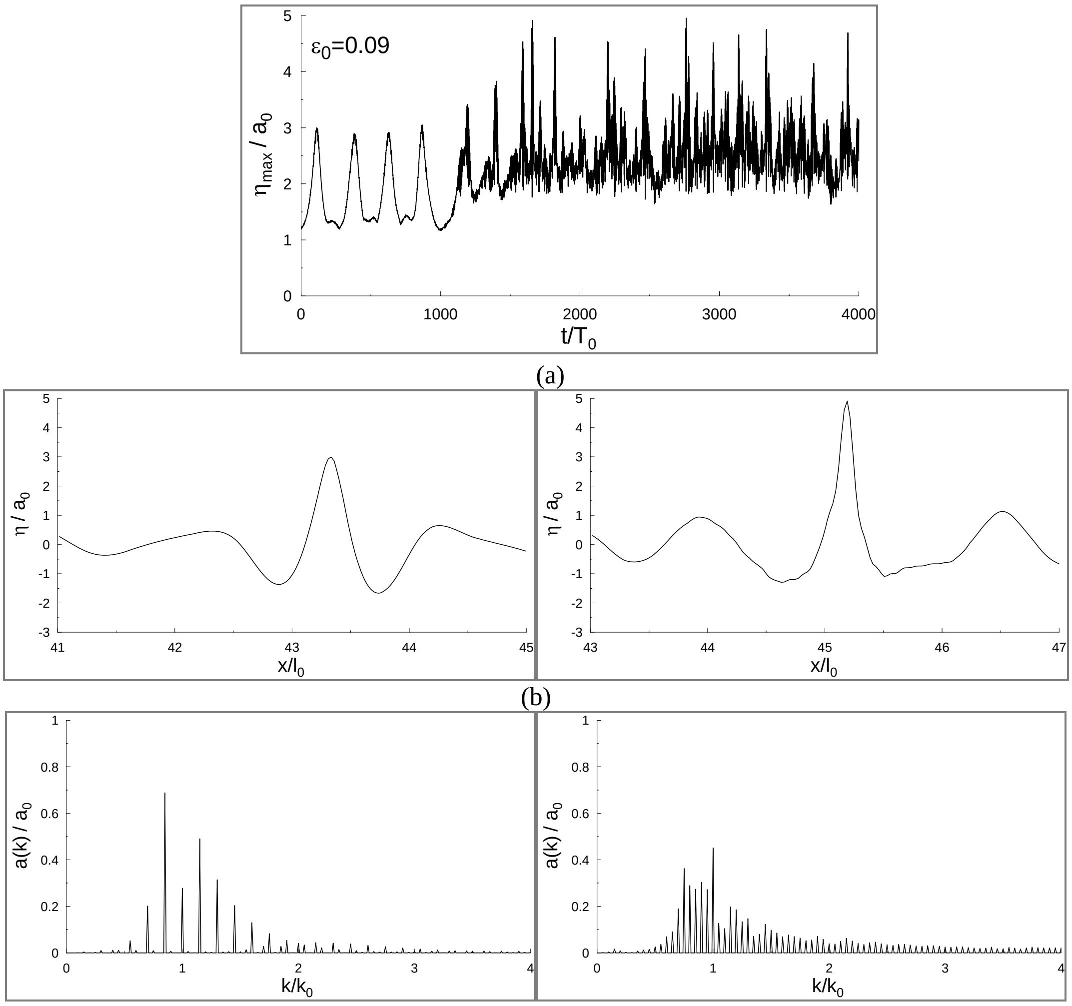

Figure 1: (a) Time variation of maximum crest/trough height (normalized by the initial wave amplitude a_0); (b) rogue wave profile at time t/T_0=110 (left) and t/T_0=1657 (right); and (c) wave-number spectra of rogue wave profiles at t/T_0=110 (left) and t/T_0=1657 (right).

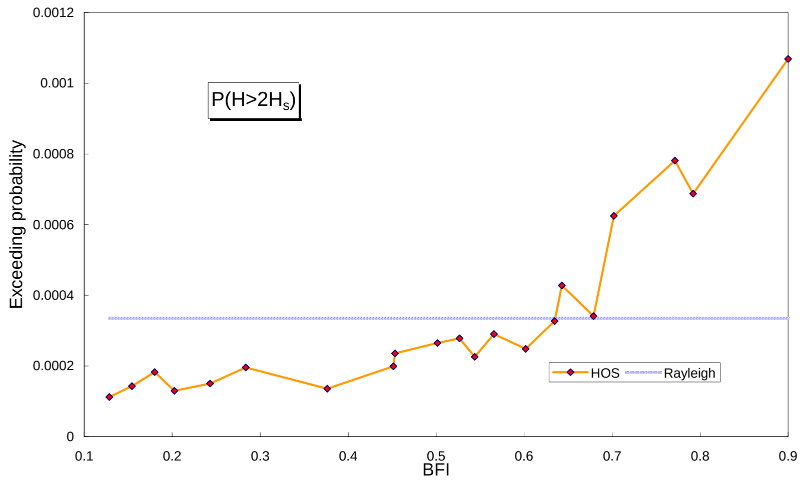

Figure 2. Probability of rogue waves as a function of Benjamin-Feir index. The dot line represents the prediction from the Rayleigh theory, and the symbol is the result from direct HOS simulations.