Numerical Study on the Second-Order Radiation/Diffraction of Floating Bodies with/without Forward Speed

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INTRODUCTION

Second-order wave-body problem with/without a small forward speed is considered. One of the difficulties of the formulation of the second-order wave-problem in the inertial coordinate system is the higher-order derivatives on the right-hand side of the body boundary conditions. It is difficult to get accurate results for the higher-order derivatives on the mean body surface with high curvatures. For a body with sharp corners, if the body boundary conditions are obtained by Taylor expanding about the mean body surface, the resulting integral equation involving the higher-order derivatives of the velocity potential in the integrand is not integrable.

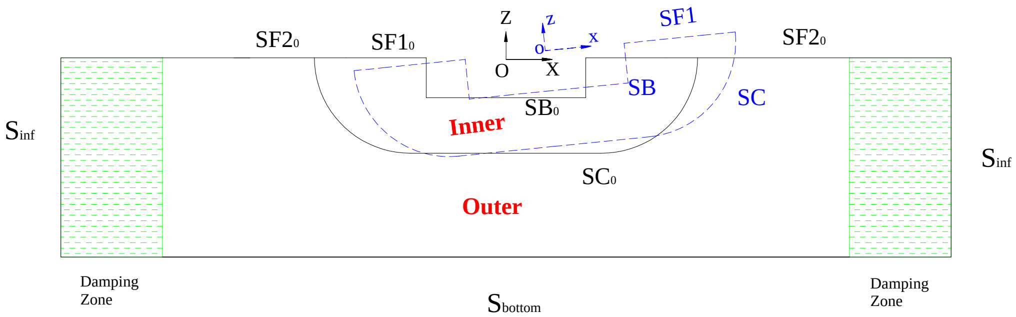


Fig.1 Definition of coordinate systems and the illustration of fluid domain, body boundary, free surfaces, control surface, bottom surface and damping zone.

A method based on domain decomposition and body-fixed coordinate system in the inner domain (near-field) is proposed to avoid derivatives on the right hand side of the body boundary conditions of weakly-nonlinear wave-body problems [1, 2]. An inertial coordinate system is used in the outer domain. As shown in Fig. 1, a control surface (SC) is introduced to divide the computational domain into two parts, i.e. the inner domain and the outer domain. The inner domain is enclosed by a projection of the free surface on the oxy-plane near the body (SF1), the body surface (SB) and the control surface (SC). The outer domain contains the mean free surface away from the body ($SF2_0$), the mean position of the control surface SC_0, the sea bottom S_{bottom} and the vertical surface connecting the free surface and the sea bottom. In the inner domain close to the body, the problem was solved in a body-fixed coordinate system, while the solution in outer domain was obtained in an inertial coordinate system. The solutions of the inner and outer domains are then matched at the control surface. The body boundary condition based on this formulation is 'body exact' and the free-surface condition remains as a second-order approximation. Because only the mean wetted body surface area on the instantaneous surface position is considered, the body-boundary condition is strictly speaking not body-exact. That means the effect of small variation of the wetted body surface due to the wave elevation and body motion will be handled by the Taylor expansion about the oxy-plane of the body-fixed coordinate system, see Fig.1 for definitions. The boundary conditions in the body-fixed coordinate system can be written in the form of [2]

$$\tilde{\eta}_t^{(m)} = \phi_z^{(m)} + f_1^{(m)} \quad \text{, on z=0,} \tag{1}$$

$$\phi_t^{(m)} = -g\tilde{\eta}^{(m)} + f_2^{(m)}, \text{ on z=0} \tag{2}$$

$$\phi_n^{(m)} = b^{(m)} \quad \text{on } SB_0 \tag{3}$$

Here $f_1^{(m)}$, $f_2^{(m)}$ and $b^{(m)}$ with m=1, 2 are the forcing terms, which can be found in [2].

The description of the incident wave field in the body-fixed coordinate system is difficult. However, it is not necessary to separate the velocity potential and wave elevation into the known incident part and the unknown scattered part, as it was done by for instance Büchmann [3]. In the new method based on domain decomposition, the incident wave is only specified in the outer domain which has a formulation in the inertial coordinate system. In the inner domain, the total velocity potential and the total wave elevation are solved through the free surface conditions. Physically the free surface conditions in the outer domain act as a wave generator. The generated incident wave enters the inner domain and is then diffracted by the body. A time-domain higher-order BEM using the cubic shape functions is adopted in the present study. A damping zone in the outer layer of the free surface in the outer domain is used to enforce the radiation conditions. The damping zone applies only for the scattered part of the waves and its nonlinear interactions with the incident waves. The fourth-order Adams-Bashforth-Moulton (ABM4) predictor-corrector method is used for the time-marching of the free surface. The increment of the time step is taken as $\Delta t = T/200$. Here T is the linear wave period. The ABM4 scheme has accuracy of O(Δt^4).In this article, the low-pass filter used by Büchmann [3] in the linear BEM based numerical wave tank(NWT) is adopted. Büchmann [3] has successfully applied this low-pass filter in the second-order wave diffraction of a bottom mounted vertical cylinder with the presence of a weak current.

CASE STUDIES

1. Diffraction of a bottom-mounted vertical cylinder in bichromatic waves, F_r=0.0, h=4R

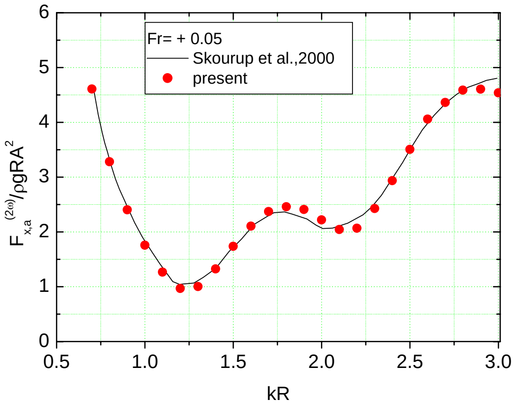
The diffraction of a bottom-mounted cylinder with h=4R in bichromatic waves without forward speed is studied. h is the water depth and R is the radius of the cylinder. In table 1, Comparisons are made for the nondimensional amplitude of the sum-frequency in-line forces and the difference-frequency in-line forces.

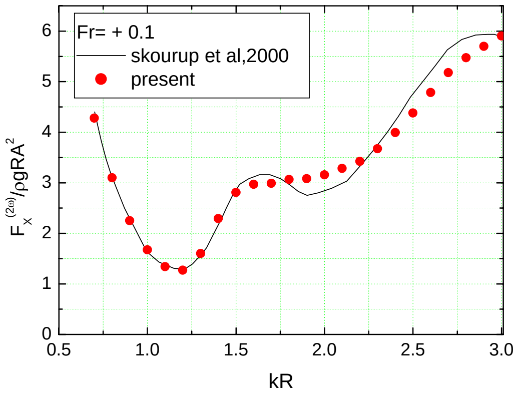
Table 1. The nondimensional amplitude of the sum-frequency forces and difference-frequency in-line forces on a bottom-mounted circular cylinder. υ_i is defined as $\omega_i^2 R/g$. At each frequency pair, four numbers are given, corresponding successively from the top line to the present results, those of Kim and Yue [4], Eatock Taylor and Huang [5] and Moubayed and Williams [6]. h=4R.

$\upsilon_i=1.0$, $\upsilon_j=1.6$			$\upsilon_i=1.2$, $\upsilon_j=1.8$			$\upsilon_i=1.4$, $\upsilon_j=2.0$	
sum-freq.	diff-freq.		sum-freq.	diff-freq.		sum-freq.	diff-freq.
1.868	0.861		2.190	0.788		2.088	0.759
1.853	0.856		2.182	0.788		2.094	0.765
1.883	0.849		2.294	0.769		2.114	0.777
1.783	0.840		2.091	0.761		1.998	0.734

2. Diffraction of a bottom-mounted vertical cylinder in monochromatic waves, $Fr \neq 0$, h=R

The diffraction of a bottom-mounted vertical cylinder with h=R in regular waves with the presence of a small forward speed is studied. Fig.2 shows the nondimensional amplitude of the sum-frequency force for different nondimensional wave numbers. $Fr = U/\sqrt{gR}$=0.05 and $Fr = U/\sqrt{gR}$=0.1 were used. Comparison is made between the present results and that of Skourup et al. [7].





(a) (b)

Fig 2 Nondimensional amplitude of the sum-frequency in-line force versus kR with h=R. (a) Fr=0.05, (b) Fr=0.1.

3. Forced oscillation of a truncated vertical cylinder with sharp corner, Fr=0.0

A truncated vertical cylinder under forced sinusoidal surge and heave motion is studied. The radius of the cylinder is chosen as R=1.0 with draft D=0.5R. The water depth considered here is h=1.5R. This problem has been solved to second order by Isaacson and Ng [8] by a time-domain lower-order panel method based on the formulation in the Earth-fixed coordinate system. Later Teng [9] studied the same problem in the frequency domain by a higher-order BEM, while Bai [10] and Teng et al. [11] applied a time-domain B-spline based BEM. The formulation in the Earth-fixed coordinate system was used. However, they avoid calculating the second-order derivatives in the second-order body boundary condition by using a Stokes-like theorem. The cost of doing so is an additional integral on the mean waterline and the evaluation of integrals involving the first-order derivatives and the normal derivative of the first-order derivatives of the Green's function. The results by Teng et al. [11] show large difference when compared with those of Isaacson and Ng [8], but are much closer to Teng's [11] results.

In this study, we re-investigated the same problem by using the domain decomposition based method. Consistent results with that of Teng et al. [11] and Bai [10] were obtained. We have also studied the problem based on a formulation in the Earth-fixed coordinate system with the second-order derivatives calculated by the cubic shape functions. No convergent results have been achieved with very fine mesh resolution. Increasing the element number does not show any trend that the result is going to be closer to that of the new method (or results by Bai [10] and Teng et al.'s [11]). The results will be presented in the workshop.

The reason of the differences between different results is likely to be due to the singular behavior of the flow velocity at the sharp corner. The second-order derivatives of the linear velocity potential on the mean body position are not integrable. The reason why the integrals are not integrable when the body boundary condition is satisfied on the mean position of the body boundary is that the formulation of the body boundary condition is wrong with the presence of the sharp corner. The second-order derivatives have been derived by a Taylor expansion about the mean body surface. This is not valid at a sharp corner. However, the body boundary conditions of the new method are formulated on the exact body position, and no Taylor expansion is needed. Therefore the integral equations of the new method are valid for cases with and without sharp corners.

Another way of handling the sharp corner cases may be that one introduces a finite bilge radius R at the corner. Then one can use a Stokes-like theorem to reduce the second-order derivatives to first-order derivatives. Afterwards one let $R \rightarrow 0$. This may explain that the results by Bai[10] and Teng et al.[11] are consistent with our results by the new method.

4. Forced oscillation of a vertical axisymmetric body without sharp corner, $Fr \neq 0$

The forced oscillations of a vertical axisymmetric body without sharp corner are also studied. The dimensions of a cross section of the body in the oxz-plane are shown in Fig.3. In this case, the traditional method using the formulation in the inertial coordinate system is expected to give consistent results with the new method using body-fixed coordinate system in the near field. Our numerical results have confirmed this. Results will be shown in the workshop.

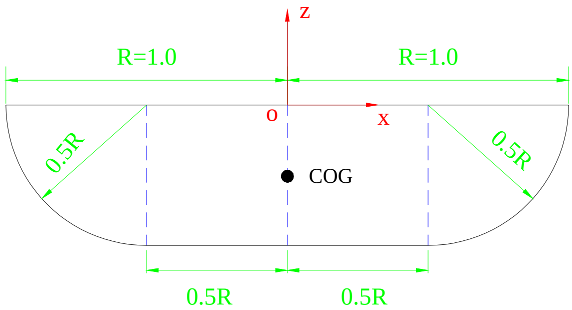


Fig.3 Sketch of the cross-section of the axisymmetric body in the oxz-plane.

Acknowledgements

This work is supported financially by the Research Council of Norway through Centre for Ships and Ocean Structures.

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