

# Influence of wind on focusing waves packet using a Boussinesq-type model

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## 1 Introduction

Regular or irregular waves formation and propagation as well as the freak wave formation due to the dispersive focusing mechanism are investigated numerically in the presence of wind. The formalism used is based on the method developed by Bingham *et al* [2]. This model deals with the fully nonlinear equations in term of the velocity potential and can simulate water waves propagation on variable sea bottom from deep water to shallow water.

We introduce in this Boussinesq-type model a forcing term due to wind effect: the Miles' theory and the modified Jeffreys' sheltering mechanism. To describe the whole process of the focusing mechanism with wind a roller model developed by Madsen *et al* [6] is included to take into account the dissipation due to breaking. The present numerical results are compared with experiments conducted in the large air/sea facility of IRPHE at Marseille Luminy.

## 2 Model

We consider the flow of an incompressible inviscid fluid on a free surface to refer by a cartesian coordinate system. The horizontal axes  $x$  and  $y$  are located on the still-water plane and the  $z$  axis pointing vertically upwards. The fluid domain is bounded by the sea bed at  $z = -h(x, y)$  and the free surface at  $z = \eta(x, y, t)$ . According to the hypothesis of irrotational flow, we can introduce the velocity potential  $\phi$ :  $\mathbf{u} = \nabla\phi$  where  $\mathbf{u}$  is the velocity and  $\nabla = [\frac{\partial}{\partial x}, \frac{\partial}{\partial y}]$  is the horizontal gradient operator.

The Boussinesq hypothesis assumes that the velocity potential has a vertical variation in the form of polynomial. The free surface boundary conditions are written in terms of the velocity potential  $\tilde{\phi} = (\phi)_{z=\eta}$  and the vertical velocity  $\tilde{w} = (\phi_z)_{z=\eta}$  defined directly on free surface. The dynamics and kinematics conditions on the free surface are written allowing to advance in time

$$\eta_t + \nabla\eta \cdot \nabla\tilde{\phi} - \tilde{w}(1 + \nabla\eta \cdot \nabla\eta) = 0$$

$$\tilde{\phi}_t + g\eta + \frac{1}{2} \left( \nabla\tilde{\phi} \right)^2 - \frac{1}{2} \tilde{w}^2 (1 + \nabla\eta \cdot \nabla\eta) = 0$$

where  $g$  the gravitational acceleration. The numerical integration is computed with a classical fourth-order runge-Kutta scheme.

The problem of modelling the interaction of wind and sea waves has been widely studied during the last decades. Many theories have been proposed to describe the phenomenon. None of them were completely satisfying before the theory derived by Miles (1957) [7] and the modified Jeffreys' sheltering mechanism. We will describe these two mechanisms, the first one on regular waves and the second one on focusing group waves. In the two cases, the wind is introduced in the dynamic condition as the term of pressure.

### 2.1 Miles' theory

#### 2.1.1 Description

Miles' theory is based on the modelling of resonant interaction of a sheared air flow with weakly nonlinear periodic wave field. This theory, complementary to the theory proposed by Philips (1957) [9], was the first to predict an exponential growth of waves corresponding to the growth observed, using the same set parameter. This principal parameter controlling growth rate was the curvature of the mean wind vertical profile at the

critical height. This growth rate was then parameterized by using the wave age ( $\frac{c}{u^*}$ ),  $c$  being the wave phase velocity and  $u^*$  the wind friction velocity. Since then, this mechanism has been widely studied and improved, and it is recognized as an excellent description of the wind-wave interaction (see Janssen (2004)[3]). The wind effect is described by the pressure term  $p(x, t)$  applied at the interface and is assumed to be in phase with the wave slope:

$$p(x, t) = \alpha \rho_a u^{*2} \frac{\partial \eta}{\partial x}$$

where  $\rho_a$  is the air density and  $\alpha = \frac{2\beta}{\kappa^2}$ ,  $\beta$  is the energy-transfer parameter of Miles,  $\kappa = 0.4$  is the Von Karman constant,  $u^* = \sqrt{C_d}U$  is the friction velocity,  $C_d = 0.004$  is the drag coefficient and  $U$  is the wind velocity.

### 2.1.2 Illustration

To illustrate the Miles' mechanism an example is displayed in figure 1. It corresponds to a strong wind blowing on a regular sea state in a tank of 180 meters long and 2 meters deep. The following parameters are used for wave and wind : wave period: 2.2s, wave amplitude: 0.1m, Mile's parameter:  $\beta = 3$  and inverse wave age: 0.2. In presence of wind, as expected, an increase of the wave height, an asymmetry of the wave profile and a

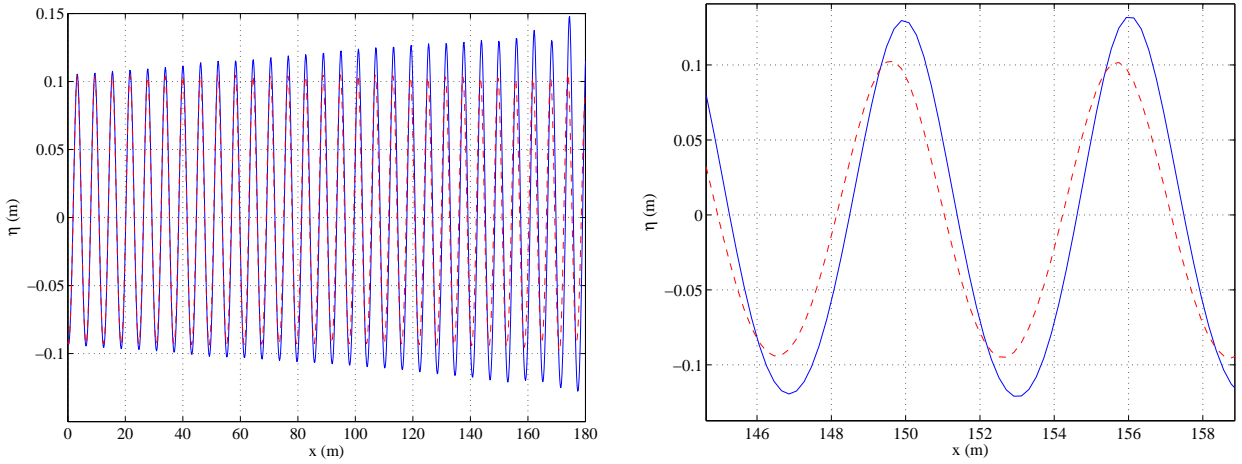


Figure 1: Surface elevation (in m) as function of fetch  $x$  (in m). (right) is zoom of (left). (blue solid line): with wind, (red dashed line): without wind.

frequency downshift are observed.

## 2.2 Jeffreys' sheltering mechanism

### 2.2.1 Description

This mechanism, first introduced by Jeffreys (1925) [4], is based on the difference of pressure between the leeward and windward faces of the waves induced by air flow separation over waves crest. Jeffreys suggest that the air flow pressure at the interface,  $z = \eta(x, t)$ , is related to the local wave slope, according to the following expression:

$$p = \rho_a s (U - c_\varphi)^2 \frac{\partial \eta}{\partial x}$$

Or the modern experiment (Banner and Melville (1976)[1], Reul *et al* (1999)[8]), showed that this phenomenon, does not occur over all waves crest but only over the higher wave crests. Following Touboul *et al* [10] a critical value of the local slope  $\eta_{xc}$ , above which an energy transfer from wind to the waves occurs is introduced. For each wave, the maximal local slope is computed and the pressure distribution on the surface of the wave is given by:

$$p(x) = \begin{cases} 0 & \text{if } \eta_{xmax} < \eta_{xc} \\ \rho_a s (U - c)^2 \frac{\partial \eta}{\partial x}(x) & \text{if } \eta_{xmax} \geq \eta_{xc} \end{cases}$$

where  $s$  is the Jeffreys' sheltering coefficient.

## 2.2.2 Comparison with experimental study

We have compared the present numerical results with experimental study developed by Kharif *et al* [5], in the large air/sea facility of IRPHE at Marseille Luminy. It is constituted of a closed loop wind tunnel (40m long, 3.2m wide, 1.6m high) located over a water tank 40m long, 0.9m deep and 2.6m wide. The blower allows to produce wind speed up to 14m/s and the paddle submerged under the upstream beach can generate regular or random waves in a frequency range from 0.5Hz to 2Hz. The water surface elevation is determined by using capacitive wave gauges located at different fetches.

Freak waves are generated by means of a spatio-temporal focusing mechanism based upon the dispersive behavior of water waves. A linear approach of the problem would lead to consider sea surface as a superposition of linear waves of frequencies  $\omega(x, t)$  imposed to the paddle for focusing all the waves in one point. We take the signal of the wave gauge at fetch 1m for the initial condition in our model (figure 2 top). In this case, The frequency of the wavemaker is varied linearly from  $f_{max} = 1.85Hz$  to  $f_{min} = 0.8Hz$  during  $\Delta T = 23.5s$ . The difficulty of this simulation comes from the high frequency content that requires a very accurate discretization. In order to estimate the accuracy of the model, we first compare the time evolution of the free surface for

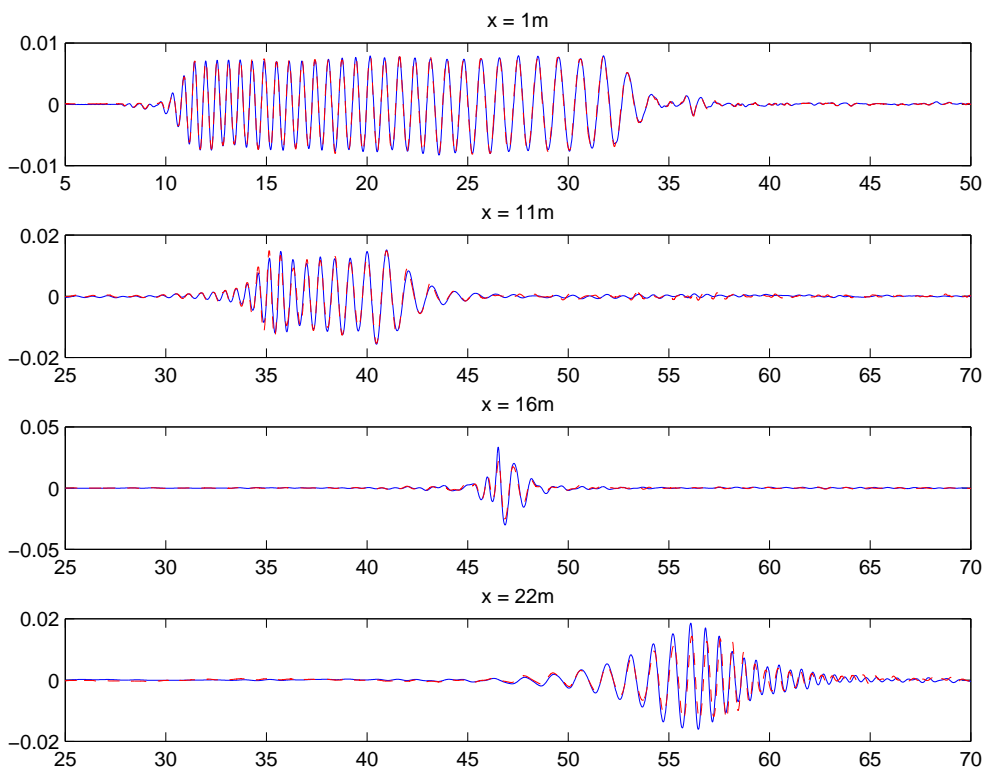


Figure 2: (Left) Comparison between numerical simulations (blue solid line) and experiment (red dashed line) for free surface elevation at different fetches.

different fetches for the case without wind. The results are displayed in figure (2) and show the good agreement between experiments and numerics. To examine how the wave envelope amplitude changes along the direction of the wave propagation, we compute the amplification factor  $A$ , defined as:

$$A(x, U) = \frac{H_{max}(x, t)}{H_{ref}}$$

where  $H_{max}(x, t)$  is the maximal height between two consecutive crests and through at different fetches  $x$  for a fixed wind speed and  $H_{ref}$  is the initial wave height. Results are displayed in figure (3) for both wind case ( $U=6$  m/s) and no wind case. We observe that the numerical results are in good agreement with experimental results for both cases, and as it was expected, for the windy case, the coalescence of the group is maintained after the expected focal point. We have to notice that breaking dissipation plays an important role as we can observed in figure (3,Right).

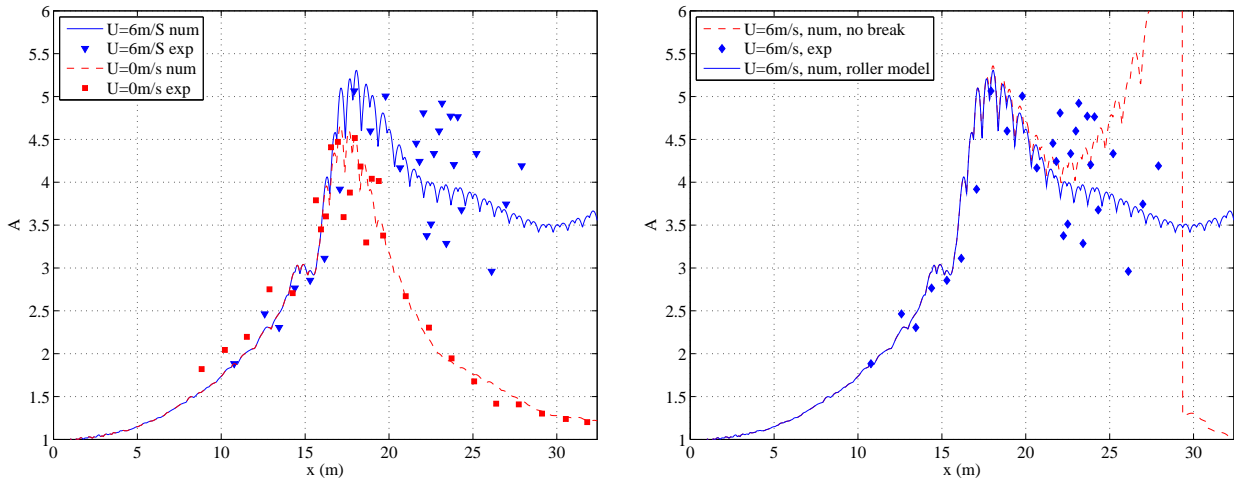


Figure 3: (Left) Comparison between the numerical ( $U=0\text{m/s}$  blue solid line,  $U=6\text{m/s}$  red dashed line) and the experimental ( $U=0\text{m/s}$  red square,  $U=6\text{m/s}$  blue triangle) results for the evolution of the amplification factor  $A(x,t)$  as function of the distance (in m). (Right) Influence of the breaking dissipation.

### 3 Conclusion

We have first shown the ability of this model to describe correctly the formation of extreme wave events even in deep water. Secondly, the addition of a wind forcing term in the model allow us to describe the wind influence on the mechanism of coalescence. Our results are in good agreement with experimental results of Kharif *et al* [5]. The next step of our study, is the behavior of this kind of wave group in shallow water and in variable bathymetry, including interaction with strong wind. An experimental campaign is currently performed for the case of a sloping beach for the cases of regular and irregular waves and waves packet in presence of wind.

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