

Wave drift forces on a rectangular barge in varying bathymetry

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There has been much interest recently for station-keeping in variable waterdepth. This interest is related to near-shore marine operations like pipe landing and to the expected development of LNG terminals. Some ongoing Joint Industry Projects (HAWAI, CHEEPP) are addressing associated issues like low-frequency wave excitation.

In the present paper we greatly simplify the problem by considering the two-dimensional case of a rectangular barge floating over a variable depth zone confined in-between two semi-infinite domains, with constant depths, to the left and to the right. This is the same kind of bathymetry as assumed by Belibassakis (2008) who uses the "coupled-mode" theory of Athanassoulis & Belibassakis (1999) and presents hydrodynamic coefficients for a rectangular barge over a sloping bottom. Our main interest is for the drift force and for the way it is being affected by the varying bathymetry. Some tests in our canal with a rectangular barge floating over a rectilinear beach have revealed that the mean hydrodynamic force can be acting "in the wrong direction", that is toward the incoming waves. The question is whether that was a non potential effect, due to some viscous contribution, or to basin related spurious problems like the return current, or whether the drift force, as computed from potential flow theory, can actually be negative.

The technique that we use to solve the linearized potential flow problem consists in representing the variable bottom as a succession of steps, thereby dividing the fluid domain in a series of rectangular sub-domains where eigen-function expansions can be used to express the velocity potential. This technique has been used by many people interested by wave transformation over varying bathymetry and related phenomena like Bragg scattering (e.g. see Rey *et al.* 1992). We have chosen this method because we wanted to be able to tackle the cases of vertical submarine cliffs or dredged channels, where methods that assume a mild slope would fail. Another reason is that we aim at treating the transformation of the second-order long wave associated to a bichromatic wave system: a mild slope for the first-order waves can be very steep for the accompanying long waves.

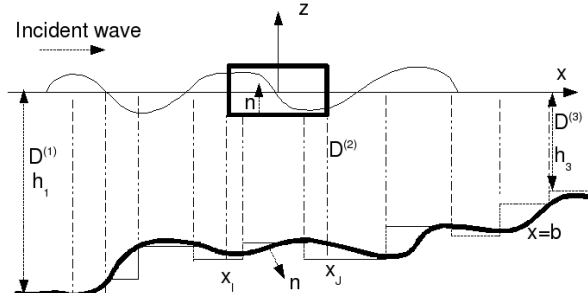


Figure 1: Geometry.

Figure 1 illustrates the geometry and figure 2 presents our results for the same case as considered by Belibassakis (2008) (figure 2 of his paper), with good agreement.

Coming to the calculation of the drift force, it is known that basically two methods are available: the direct pressure integration method, and the momentum method, with a control contour around the body. When this control contour consists in two vertical cuts away from the body, in the two constant waterdepth regions, the drift force is easily obtained as

$$F_d = \frac{1}{2} \rho g A^2 \left\{ \frac{C_{G1}}{C_{P1}} (1 + R_1 R_1^*) - \frac{C_{G2}}{C_{P2}} T_2 T_2^* \right\} \quad (1)$$

with A the incoming wave amplitude, C_{P1} and C_{P2} the phase velocities in the upwave and downwave regions with waterdepths h_1 and h_2 , C_{G1} and C_{G2} the group velocities, R_1 the reflection coefficient and T_2 the transmission coefficient. But this is the mean force acting both on the body and the sea-floor! To differentiate the body and sea-floor, a contour closer to the body must be taken, possibly its mean position, or the pressure integration method must be used.

Both the pressure integration method and the momentum method with control contour at the body involve the calculation of $\int_C (\nabla \Phi)^2 \vec{n} dl$, with the problem that the velocity $\nabla \Phi$ is singular at the square bilges. As a

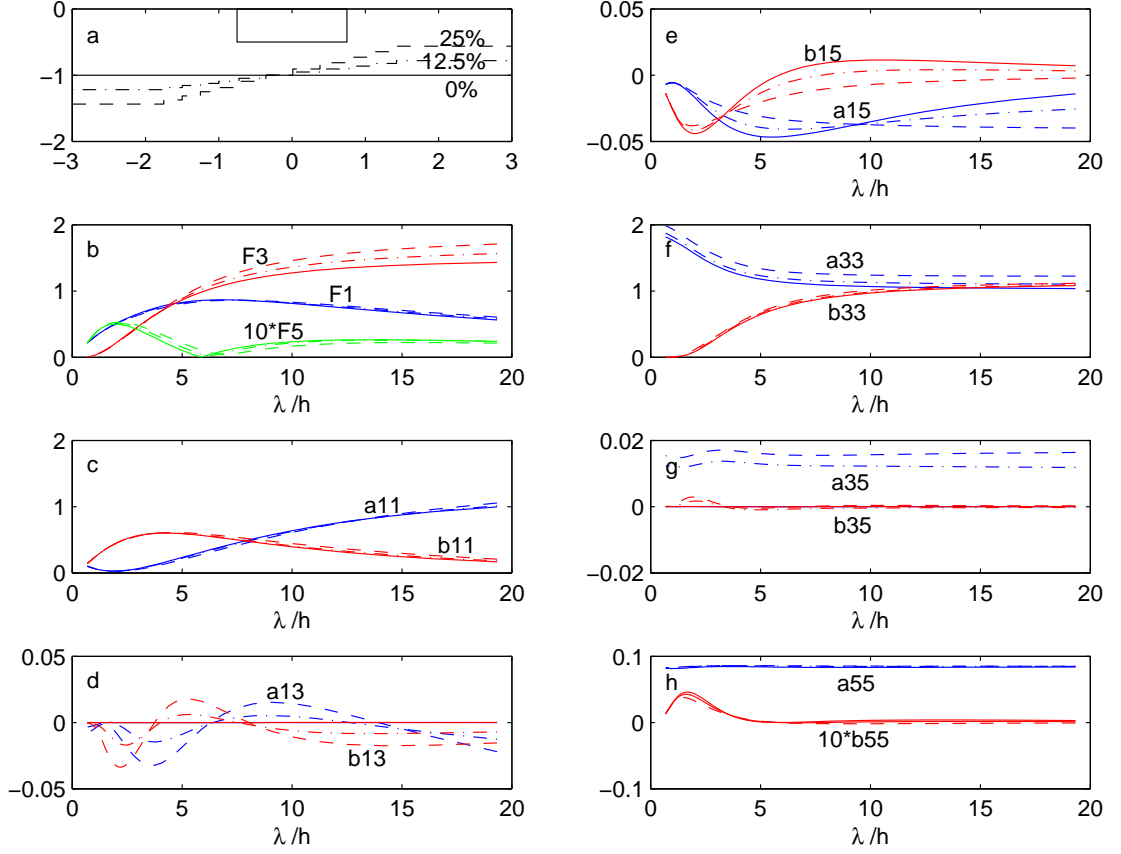


Figure 2: Hydrodynamic coefficients for Belibassakis' case.

result the numerical accuracy is poor due to slow convergence of the series representation of the potential. In the paragraph below we present an attempt, not fully successful, at extracting the singular behavior from the series representation.

For the sake of simplicity we do not consider a wave flow but a channel flow, with the free surface turned into a rigid lid and an abrupt change of depth from h_1 to h_2 at $x = 0$. This problem can easily be solved by conformal mapping (e.g. see Milne-Thomson, 1969, 10.7). Using the eigen-function expansion method the velocity potentials in the upstream and downstream regions write:

$$\varphi_1 = U x + \sum_{n=1}^{\infty} A_n \cos \lambda_n z e^{\lambda_n x} \quad \varphi_2 = U \frac{h_1}{h_2} x + B_0 + \sum_{n=1}^{\infty} B_n \cos \mu_n z e^{-\mu_n x} \quad (2)$$

with $\lambda_n = n \pi / h_1$ and $\mu_n = n \pi / h_2$, while it can easily be shown that $I = \int_{-h_1}^{-h_2} \varphi_z^2(0, z) dz$ is theoretically equal to $U^2 h_1 / h_2 (h_1 - h_2)$ (U being the current velocity in the upstream region).

By matching the velocity potentials and their x -derivatives at the common boundary $x = 0$, and taking advantage of the orthogonality of the $\cos \lambda_n z$ (resp. $\cos \mu_n z$) functions over $[-h_1, 0]$ (resp. $[-h_2, 0]$), the coefficients A_n and B_n can be obtained to any degree of accuracy, the series being here truncated at the same order N_1 . Figure 3 shows the results obtained for $h_2 I / h_1 / (h_1 - h_2) / U^2$, theoretically equal to 1, when the calculation of I is analytically performed from the series representations. It can be seen that the numerical convergence is very bad, more particularly in the case of small and high steps.

Since the singularity of the potential at the square corner is known ($z^{2/3}$), we decompose the potential $\varphi_1(0, z)$ into an analytical part with the proper singularity and a regular part expressed as a series

$$\varphi_1(0, z) = \alpha F(z) + \sum_{n=1}^{N_\alpha} \tilde{A}_n \cos \lambda_n z \quad (3)$$

with

$$F(z) = |z + h_2|^{2/3} z^2 (z + h_1)^2 \quad -h_1 \leq z \leq -h_2 \quad (4)$$

$$F(z) = -\frac{1}{2} |z + h_2|^{2/3} z^2 (z + h_1)^2 \quad -h_2 \leq z \leq 0 \quad (5)$$

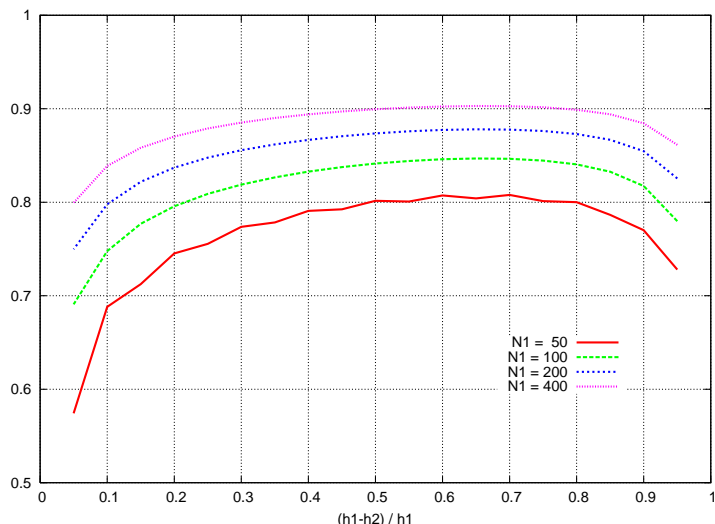
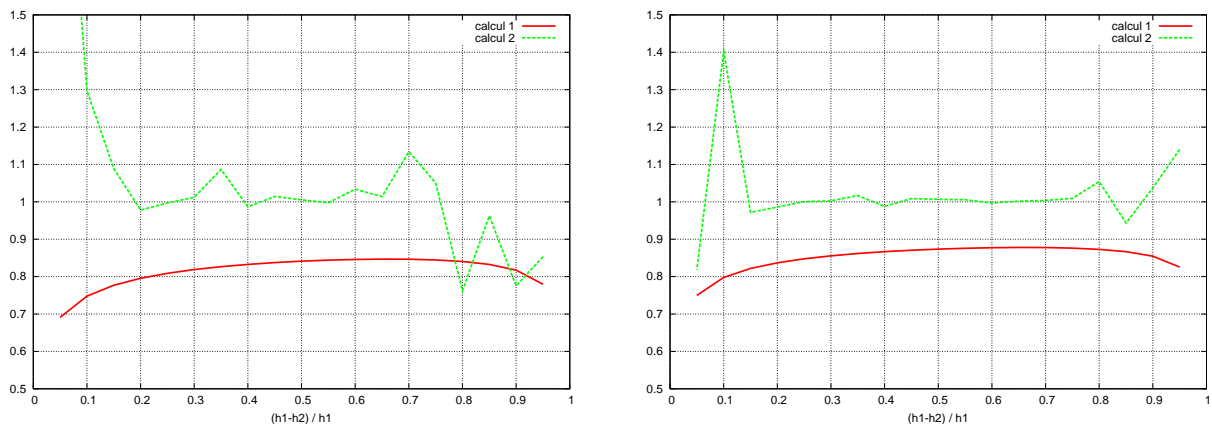


Figure 3: $h_2 I/h_1/(h_1 - h_2)/U^2$ vs. $(h_1 - h_2)/h_1$ for truncation orders from 50 to 400.

where the somewhat arbitrary $F(z)$ function has been chosen so as to fulfil the no-flow condition $F' \equiv 0$ in $z = 0$ and $z = -h_1$. When $F(z)$ is written as a series $F(z) = \sum_{n=0}^{\infty} C_n \cos \lambda_n z$, since the \tilde{A}_n are expected to decrease quickly with n , the coefficient α should turn out to be the limit of the ratio A_n/C_n as n increases. Actually this ratio does not evolve monotonically when n increases and some averaging/filtering over the observed oscillations must be made. This being done more or less empirically to derive the α coefficient, and integrating analytically again the velocity squared, the following result is obtained for $N_1 = 100$ (left) and $N_1 = 200$ (right), where it



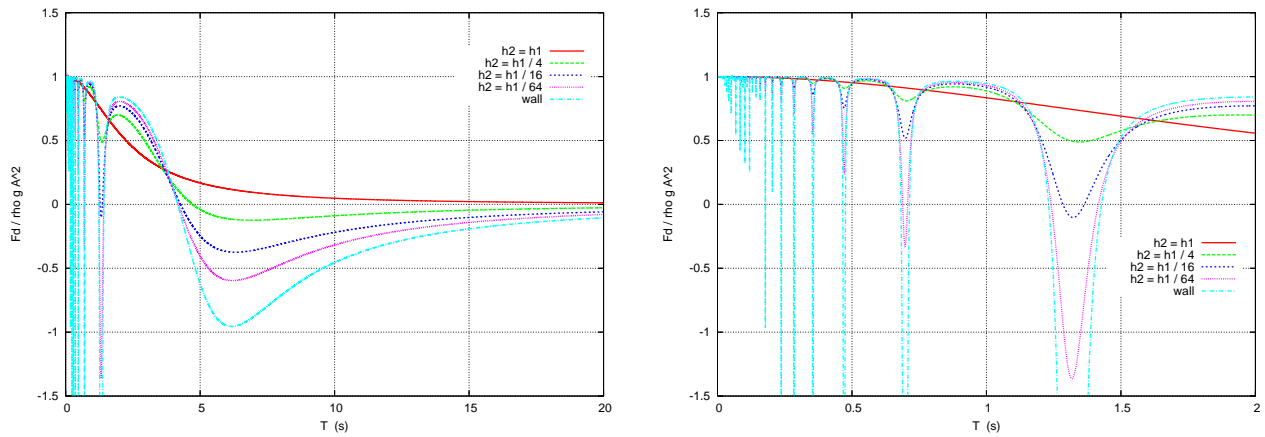
can be seen that the truncation order must be taken very high to see some improvement and that the case of small and high steps is still not properly solved.

Putting this problem aside for the moment, we move on to the question whether the drift force can be negative. We consider a very simple geometry where the motionless barge sits some distance on the weather side of an abrupt transition from depth h_1 to h_2 at $x = 0$. Since the sea-floor is horizontal below the barge there is no problem with calculating the drift force with the momentum method. We even simplify further the matter by making use of long wave theory, assuming the waterdepth to be much smaller than the wavelengths. The fluid domain divides into 4 sub-domains: on the left-hand side of the barge, below the barge, in-between the barge and the cliff, and the semi-infinite region with depth h_2 . In the successive sub-domains the velocity potential takes the form (the common factor $A g/\omega$ being omitted):

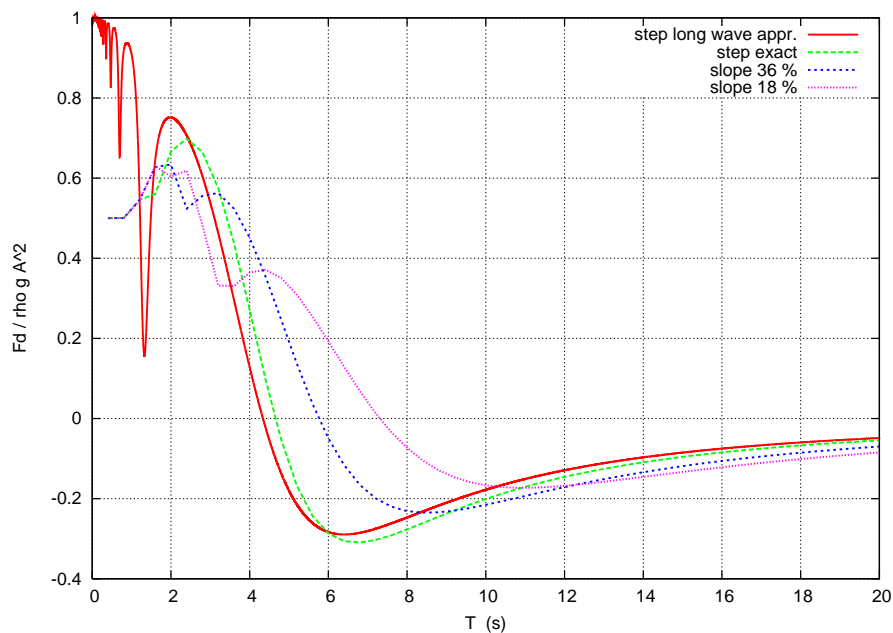
$$\varphi_1 = e^{i k_1 x} + R_1 e^{-i k_1 x} \quad \varphi_2 = C_2 + D_2 x \quad \varphi_3 = T_3 e^{i k_1 x} + R_3 e^{-i k_1 x} \quad \varphi_4 = T_4 e^{i k_2 x} \quad (6)$$

with $\omega = k_1 \sqrt{g h_1} = k_2 \sqrt{g h_2}$, b the length of the barge and l the distance from the barge to the cliff. The 6 unknowns R_1, C_2, D_2, T_3, R_3 and T_4 are obtained by equating the potentials and the fluxes at the 3 common boundaries (e.g. see Mei, 1983, 4.2). The drift force is then simply obtained as

$$F_d = \frac{1}{2} \rho g A^2 [1 + R_1 R_1^* - T_3 T_3^* - R_3 R_3^*] \quad (7)$$



As a numerical application, somewhat arbitrarily (but more or less corresponding to the tests in ECM canal), we take $h_1 = 0.8$ m, $b = 1$ m, $l = 2$ m, and the draft of the barge equal to 0.4 m. The waterdepth on the lee side h_2 is taken successively as equal to h_1 , $h_1/4$, $h_1/16$, $h_1/64$ and then nil (replaced by a wall meaning $R_3 = T_3$). The obtained drift force is shown in the figures above where the right-hand side one is a blow-up of the left-hand side one for wave periods in-between 0 and 2 s. It can be seen that the wall case is asymptotically attained as the waterdepth h_2 goes to zero. A series of peaks, with negative values when h_2 is sufficiently small, appear in the low period range: they are due to sloshing modes in-between the barge and the submarine cliff. What was less expected is that, for wave periods beyond 4 or 5 s, the drift force becomes negative. This is due to some kind of piston mode taking place in-between the barge and the depth transition.



Finally we replace the abrupt transition by a ramp. We take the downwave depth h_2 equal to $h_1/10 = 0.08$ m. The mid-height of the ramp is located at 2 m from the barge. Calculations are made for two slopes, 36 % and 18 %, that is ramps 2 m and 4 m long. Results are shown in the figure above, together with the vertical cliff case, as obtained from the long wave approximation and as obtained by the exact model. In all cases the drift force becomes negative as the wave period increases.

References

- ATHANASSOULIS G.A. & BELIBASSAKIS K.A. 1999 A consistent coupled-mode theory for the propagation of small-amplitude water waves over variable bathymetry regions, *J. Fluid Mech.*, **389**, 275–301.
- BELIBASSAKIS K.A. 2008 A boundary element method for the hydrodynamic analysis of floating bodies in variable bathymetry regions, *Engineering Analysis with Boundary Elements*, **32**, 796–810.
- MEI C.C. 1983 *The Applied Dynamics of Ocean Surface Waves*, Wiley-Interscience.
- MILNE-THOMSON L.M. 1969 *Theoretical Hydrodynamics*, fifth edition.
- REY V., BELZONS M. & GUAZZELLI E. 1992 Propagation of surface gravity waves over a rectangular submerged bar, *J. Fluid Mech.*, **235**, 453–479.