WAVE POWER ABSORBERS AT FLOATING PLATFORM

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1. Introduction

A thin floating platforms can be used as an alternative of such land-based large facilities as airport or off-shore based storage/production units. Bending rigidity of such a floating platform is small, and wave-induced motion of the plate is significantly affected by its elastic deflection. Analysis of floating plate behaviour in waves is based on hydroelasticity, in which the coupled hydrodynamics and structural dynamics problems are solved simultaneously.

In our previous study [1] a goal of the analysis was to predict accurately both the plate deflection and stresses in the plate, as well as to find means of their reduction. The latter is of great importance for safety of the platform and its structure performance. Two approaches for reduction of plate vibrations were suggested. The first approach is based on the concept of vibration absorber well-known in many engineering applications. In the framework of this approach in order to reduce the floating plate vibrations, a rigid plate of smaller length has to be simply connected to the front of the main structure. Within the second approach the floating beam is connected to the sea bottom with a spring, rigidity of which can be adjusted in such a way that the beam deflection due to incident waves is reduced in the inner part of the plate.

A main idea of the present study is to consider a floating elastic plate connected to the sea bottom at its both ends by springs with attached wave energy absorbers, which are modelled as linear dampers. Such connectors are expected to reduce the amplitude of the plate vibrations in wave and, in addition, to generate electric power which may well cover local needs in electric energy. Note that the energy production is not a main function of the connectors. Correspondingly, the wave energy absorbers, which could be installed at the offshore platform, are not necessary the most efficient ones but they must be robust and cheap in terms of their maintenance. In order to estimate the power which could be generated at an offshore platform and optimal reduction of the platform response in waves, we shall perform a parametric analysis with respect to the characteristics of the connectors. The power available from an incident regular wave was estimated by Mei *et al* [2].

We consider two-dimensional problem of hydroelasticity for homogeneous floating plate in waves. The plate is elastically connected to the sea bottom at its both ends. The scheme of the flow and main notations are shown in Fig. 1.

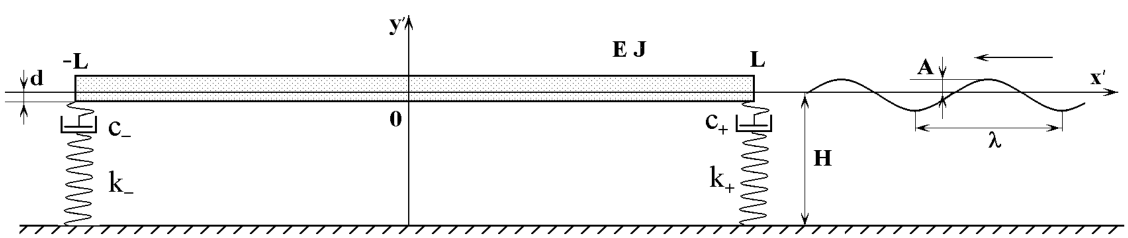

Fig. 1.

2. Formulation of the problem

The plane linear problem of a floating elastic plate in waves is considered. The plate vibration is caused by periodic incident wave of frequency ω and small amplitude A. The incident wave propagates from the right-hand side. The plate vibration is described by the Euler beam equation.The beam bending stiffness EJ and its draft d are given. The beam draft d is assumed much smaller than both the total beam length $2L$ and liquid depth H. Both ends of the beam are connected to the bottom by springs with dampers. The spring rigidity coefficients $K_\pm$ and the damping coefficients $C_\pm$ of the connectors are given or determined from the additional conditions that the power extracted by the dampers from the waves is maximum and the plate deflection or bending stresses in the floating plate are minimum. Here $+/-$ correspond to the right and left ends of the floating plate, respectively.

There are no analytic formula for the extracted power and/or the maximum deflection and stresses in the elastic plate in waves as in the theory of power absorption by a floating rigid body (see Mei *et al* [2]). Therefore we should compute the plate deflection, the bending stresses and the extracted power from the wave with respect to the four parameters K_+, K_-, C_+, C_- and distinguish an optimal set of these parameters. We do not discuss here how this optimal set is defined.

For given elastic characteristics of the plate and the parameters of the connectors $K_\pm$, $C_\pm$, the hydroelastic problem of floating plate is formulated. Non-dimensional variables are used below: L is taken as the length scale, $1/\omega$ as the time scale, the amplitude of the incident wave A as the deflection scale, the product $\rho g A$, where ρ is the liquid density and g is the acceleration due to gravity, as the pressure scale, and the product $A\omega L$ as the scale of the velocity potential. Within the periodic linear wave theory the non-dimensional hydrodynamic pressure $p(x,0,t)$ along the beam, $-1 < x < 1$, and the beam deflection $w(x,t)$ are given as $p(x,0,t) = \Re[e^{it}P(x)]$ and $w(x,t) = \Re[e^{it}W(x)]$, respectively. The new unknown complex-valued functions $P(x)$ and $W(x)$ satisfy the following equations and the boundary conditions [1]:

$$P(x) + \frac{\gamma}{2\pi}\int_{-1}^{1} P(x_0)K(x-x_0)dx_0 = e^{ikx} - W(x), \qquad (1)$$

$$\beta W^{IV} - \alpha W = P(x) \qquad (-1 < x < 1), \qquad (2)$$

$$W''(\pm 1) = 0, \qquad W'''(-1) = -(k_- + ic_-)W(-1), \qquad W'''(+1) = (k_+ + ic_+)W(+1), \qquad (3)$$

where

$$\gamma = L\omega^2/g, \quad \alpha = \gamma d/L, \quad \beta = EJ/(\rho g L^4), \quad k_\pm = K_\pm L^3/EJ, \quad c_\pm = C_\pm L^3\omega/EJ$$

and k is the positive root of the dispersion equation $k\tanh(kH_0) = \gamma$, $H_0 = H/L$. The function $K(z)$ in (1) is given as

$$K(z) = -2\pi i \frac{ke^{-ik|z|}}{H_0(k^2-\gamma^2)+\gamma} + 2\pi\sum_{j=1}^{\infty}\frac{s_j e^{-s_j|z|}}{H_0(s_j^2+\gamma^2)-\gamma},$$

where $s_j = (\pi j - \delta_j)/H_0$ and δ_j is the solution of the equation $\delta_j = \arctan(\gamma H_0/(\pi j - \delta_j))$, $j \geq 1$.

Ones the problem (1)-(3) has been solved for a given set of parameters, we calculate the maximum plate deflection

$$w_{max} = A \max_{|x|<1} |W(x)|,$$

the maximum bending moment in the plate

$$M_{max} = EJA\,L^{-2}\max_{|x|<1}|W''(x)|,$$

and the wave power extracted by each absorber

$$\mathbf{E}_\pm/T = \frac{1}{2}C_\pm A^2\omega^2|W(+1)|^2.$$

3. Method of solution

Problem (1) - (3) can be solved with the help of the normal mode method in the same manner as in [1]. This method reduces the integral equation (1) to infinite system of algebraic equations with respect to the principle coordinates of the pressure $P(x)$. A main idea of this method is to use different basic functions for the pressure distribution along the plate and for the beam deflection. Trigonometric functions are used as basic functions to present the pressure distribution in the form

$$P(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} ac_n \cos\pi nx + \sum_{n=1}^{\infty} as_n \sin\pi nx. \qquad (4)$$

Substitution of expansion (4) into equation (2) leads to the following expansion for the beam deflection

$$W(x) = \frac{1}{2} a_0 wc_0(x) + \sum_{n=1}^{\infty} ac_n wc_n(x) + \sum_{n=1}^{\infty} as_n ws_n(x). \tag{5}$$

The functions $wc_j(x)$ and $ws_j(x)$ satisfy conditions (3) and equation (2) with $P(x)$ being replaced by $\cos(j\pi x)$ and $\sin(j\pi x)$, respectively. The functions $wc_j(x)$ and $ws_j(x)$ are considered here as basic functions for the beam deflection. The integral equation (1) with account for expansions (4) and (5) leads to the infinite system of algebraic equations with respect to the coefficients ac_n and as_n.

$$(\mathbf{I} + \frac{\gamma}{2\pi}\mathbf{S} + \mathbf{A})\,\vec{a} = \vec{e}. \tag{6}$$

Here $\mathbf{I} = \mathrm{diag}(2, 1, 1, ...)$ is diagonal matrix, symmetric matrix $\mathbf{S}$ comes from the integral term in (1), symmetric matrix $\mathbf{A}$ comes from the term $W(x)$, and $\vec{a} = (ac_0/2, ac_1, ac_2, ...ac_n, as_1, as_2, ...as_n)^T$. The elements of the vector $\vec{e}$ are the coefficients in the expansion of $\exp(ikx)$ with respect to the trigonometric functions. All elements of the matrices $\mathbf{S}$ and $\mathbf{A}$ and those of the vector $\vec{e}$ are given by analytical formulae.

4. Numerical results and discussion

Numerical calculations were performed for the conditions of the experiments carried out by Wu *et al* [3] for homogeneous narrow plate in a channel: d =8.36mm, H =1.1m, h =38mm, EJ = 471N·m, L =5m. Convergence of the numerical algorithm was checked by changing the number of terms taken into account in sums (4) and (5). Ninety terms were used to plot the obtained numerical results. The present results for a homogeneous beam, are identical to those obtained by other methods (see [1]).

Calculations with different values of the parameters $c_\pm$, $k_\pm$ showed, that the connectors with $c_- = 0$ and $k_+ = 0$ are most suitable for the goal of wave energy extraction. All results presented below are obtained for such connections only.

Fig. 2 shows the dimensional extracted energy $\mathbf{E}/T$ (in W/m) depending on the period of incident wave T (in s). The results depicted in Fig.2 correspond to the values $c_+ = 600$ and $k_- = 1000$. It can be seen that the value of the absorbed energy for different T is rather different. Calculations for other values of c_+ and k_- give similar curves, but values of local maxima are different and, in general, are smaller.

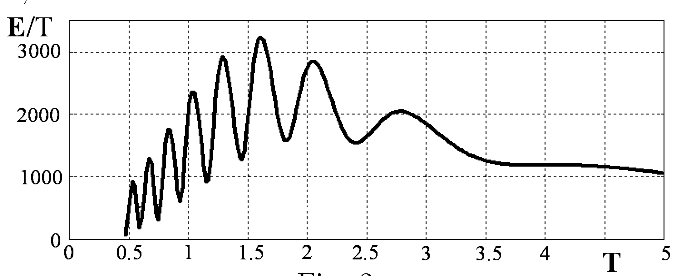

Fig. 2.

According to Fig. 2., the numerical calculations, which are shown below, were performed for $T = 1.6$s where $\mathbf{E}/T$ has maximum for $c_+ = 600$ and $k_- = 1000$.

Figs 3 show results of calculations for different values of spring rigidity k_- at the left end of the beam. Fig. 3 *a* is for $\mathbf{E}/T$ as function of c_+ and Fig. 3 *b* is for maxima of deflections at the central part of the beam. (You can see below, in Figs. 5 and 6, that vibrations of the beam are rather regular along the beam, except vicinities of the beam ends.) For all considered cases curves for the energy have single maxima, approximately at c_+=600 and decrease after that as c_+ increases. Growth of k_- increases the energy maximum, but at the same time it increases the amplitude of beam deflection (see Fig. 3 *b*).

Fig. 4 shows results of calculations for different values of c_+. As before, Fig. 4 *a* is for $\mathbf{E}/T$ and Fig. 4 *b* is for $|W|$ maximum at the center part of the plate as function of k_-. The greater k_-,

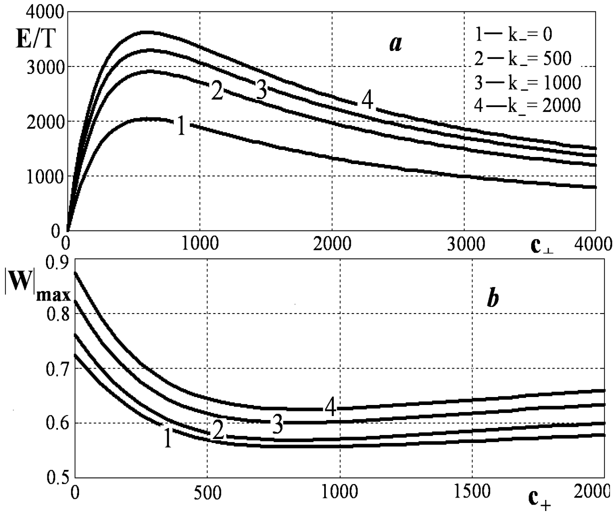

Fig. 3

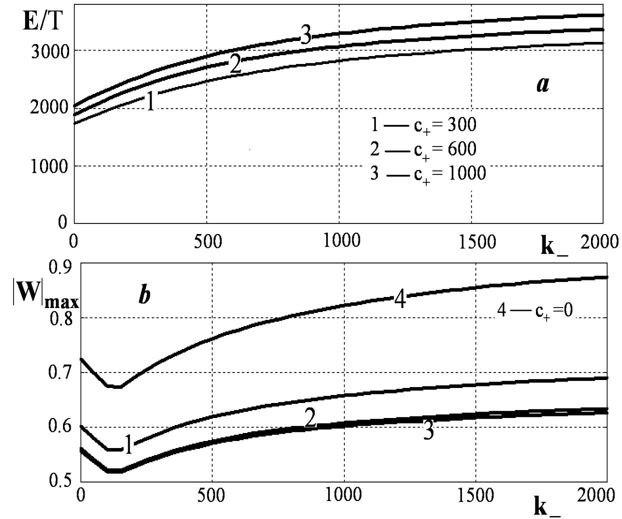

Fig. 4

the bigger both $\mathbf{E}/T$ and W_{max}. For all values of k_- the presence of the absorber decreases the deflections by approximately 20-25%.

The calculated amplitudes of both the beam deflections $|W(x)|$ and the bending moments $|M(x)|$ are shown in Figs. 5 and 6 for $T = 1.6$ s, in Fig. 5 for $c_+ = 600$ and for several values of k_- and in Fig. 6 for $k_- = 1000$ and for several values of c_+. The results for free-free beam are shown for comparison.

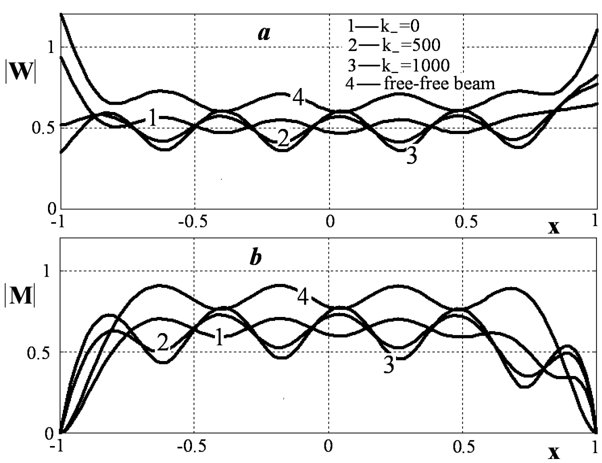

Fig. 5

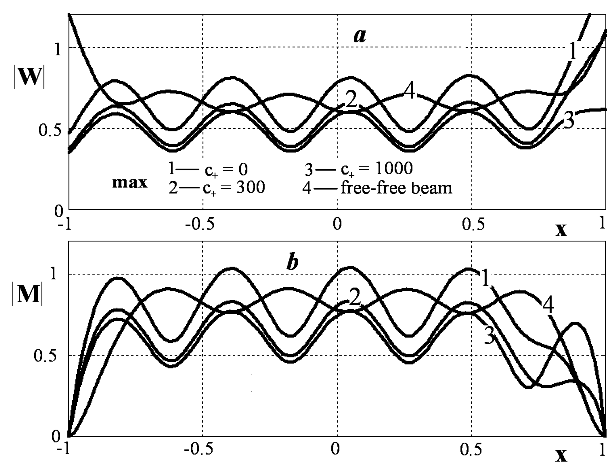

Fig. 6

5. Conclusion

Possibility of surface wave energy absorption by an elastic floating plate was known. At the same time the effect of the beam vibration reduction is well pronounced. Combination of a wave absorber at the front end of the plate and a pure spring connector at the rear end is expected to be perspective and can be utilized at the design stage.

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