Viscous calculations of hydrodynamic forces on marine bodies

by

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1. Motivation

Wave effects on ships and offshore structures enjoy a highly powerful modelling by potential theory. Theoretical analyses and computational methods are developed for use in both academia and industry. The potential theory formulation provides efficiently the dominant forces and behaviour of large volume bodies in waves. Particularly the linear and second order formulations predict the motions that oscillate at the fundamental and second harmonic frequencies. Higher order and fully nonlinear methods are developed for motions of very large amplitude of both the waves and the body responses. In strong seas, a fully nonlinear definition of the incoming wave field is required for a subsequent analysis of the interaction between the waves and fixed or floating bodies. If the geometry is rather slender, wave diffraction and radiation theories appear in the long wave regime and are complemented by slender body theories, use of Morison's equation or related coefficient based load models.

With regards to offshore industrial applications, one have systematically considered the leading order forces on ships and offshore structures that may be considered to be of large volume, where the amplitude of the incoming wave field is typically much smaller than the diameter of the offshore platform leg, for example. Today, a new, important application of marine hydrodynamics is evolving; this is in relation to the growing interest and practice with installing wind turbines in the offshore environment. In many ways, these new structures represent a continuation of the development of theories and models for safety of structures in the offshore environment, however, existing theories and practices are not directly applicable for several reasons. One may of course use the classical long wave theories or apply variants of the coefficient based methods to analyse the loads and motions of the slender structues. However, with a diameter of the supporting pile of the wind turbine typically comparable to the wave amplitude, and in strong seas even less than the wave amplitude, classical theories are less useful for an accurate prediction of the local motion and loads on the structure. This affects the construction and economy of the structure.

The flow at slender geometries penetrating the wave surface may experience local wave breaking at the water line. Strong waves interacting with slender piles may cause effects like ventilation, and splashing of the water may be experienced. In such cases a potential theory formulation is less useful, and one has to look to other alternatives. A formulation that models the incoming wave motion and leading order forces, but also resolves the fluid motion at the very short scales, i.e. very high wavenumbers, is what one is looking for. We are thinking of a viscous modelling of the flow as a candidate that satisfies this global goal. Because the flow is typically in the large Reynoldsnumber range, direct numerical simulations are not realistic. A modelling using a variant of large eddy simulations (LES) represents an alternative, however, where filtered velocity and pressure fields are computed, incorporating the motion at turbulent scales by a LES modelling.

2. Focus of present work

Present calculations evaluate the flow field and the hydrodynamic forces acting on bodies that move or oscillate, including viscous and boundary layer effects. We are currently generalizing a complex method and code developed for general fluid flow problems, where the algoritms avoid numerical dissipation and dispersion (see below). The generalization involves the representation of moving geometries. Although work is intended for free surface flow simulations, using e.g. level-set methods, implementation of the latter is under dvelopment, results have yet to be made where a wave motion is included; the free surface is in present calculations replaced by a rigid lid. Wave-body simulations by LES are increasing, see e.g. Lin and Li (2003) and Liu et al. (2005).

3 Equations for filtered velocity and pressure fields

The velocity and pressure fields are decomposed into filtered and remaining parts, where the filtered velocity field is obtained by $\overline{\mathbf{U}}(\mathbf{x}, t) = \int \mathbf{U}(\mathbf{x}', t)G(\mathbf{x}' - \mathbf{x})d\mathbf{x}'$, G a low pass filter of width Δ, the latter proportional to the grid spacing. Δ is assumed to be somewhat smaller than the scale of the smallest energy containing motion. Various filters used in this kind of modelling, see e.g. Pope (2000), all share the property $\int G(\mathbf{x}' - \mathbf{x})d\mathbf{x}' = 1$. An incompressible constant density flow formulation based on the filtered Navier-Stokes equation is assumed, which reads $\overline{D}\,\overline{U}_i/\overline{D}t = -(1/\rho)\partial\overline{p}/\partial x_i + \nu\nabla^2\overline{U}_i - \partial\tau^r_{ij}/\partial x_j$ where $\overline{D}/\overline{D}t = \partial/\partial t + \overline{\mathbf{U}} \cdot \nabla$. The residual stress tensor is represented using the Smagorinsky model and is related to the filtered shear by $\tau^r_{ij} = -2\nu_r\overline{S}_{ij}$. In this model the eddy viscosity is related to $\overline{S}_{ij}$ by $\nu^r = l_s^2\overline{\mathbf{S}}$ where $\overline{\mathbf{S}} = \left(2\overline{S}_{ij}\overline{S}_{ij}\right)^{1/2}$ and l_s denotes the Smagorinsky lengthscale. l_s is related to the discretization (of the grid) by $l_s = C_s\Delta$ where $\Delta = (\Delta x_1\Delta x_2\Delta x_3)^{1/3}$. Various filters give various estimates of C_s, of about 0.15 (some gives 0.13 and other up to 0.2).

4 Numerical method

The finite volume method uses the integral form of the conservation equations as the starting point and refers to the small volume surrounding each node point on a mesh. The method is conservative, and the solution domain is subdivided into a finite number of contiguous control volumes (CVs) where the conservation equations are integrated. At the centroid of each CV lies a computational node at which the variable values are to be calculated. Interpolation is used to express the variable values at the CV surface in terms of nodal value. For our calculations we employ a modified version of the computer code CDP (see Ham et al., 2006), an unstructured finite volume flow solver to perform LES in complex geometries. Finite volume operators developed for CDP are for a node-based discretization on general polyhedral meshes, where both grid coordinates and the unknowns are collocated at nodes. Node-based operators facilitate the reconstruction of values and gradients along the boundaries of the dual volumes that surrounds each node.

The node-based volumes required for integrating any source term are computed by tessellating each cell into 'sub-tets', each defined by a node, an edge, a face, and a cell. The volume of sub-tets are then added to the volume associated with node P, i.e. $\int \overline{U}_i \simeq \overline{U}_{i,P} V_P \simeq \overline{U}_{i,P} \sum V'_{subtets}$. Unlike CV-based formulation, the option of average for center locations ensures that interpolation based on simple averages of nodal data will be limited and linearly exact.

5 Calculations

Inviscid and viscous computations are carried out for circular cylinder and sphere geometries. The computational domain is 5d by 5d wide and 10d long, where d denotes the cylinder or sphere diameter. Gridsize at the geometry boundary is about 0.025d.

The laminar solution by Wang (1968) represents a reference, providing the leading order contributions to the added mass and damping coefficients on a circular cylinder standing in an oscillatory flow, with the force obtained by $F = \frac{1}{2}\rho d C_d |U| U + \frac{1}{4}\pi \rho d^2 C_m \dot{U}$, where ρ denotes density and $U = U_m \cos 2\pi t/T$ the velocity. Drag coefficient and inertia (added mass plus displaced mass) coefficients are given by, respectively, $C_d = [3\pi^3/2K]\,[(\pi\beta)^{-1/2} + (\pi\beta)^{-1} - \frac{1}{4}(\pi\beta)^{-3/2}]$, $C_m = 2 + 4(\pi\beta)^{-1/2} + (\pi\beta)^{-3/2}$, where $K = U_m T/d$ denotes the Keulegan-Karpenter number, $\beta = d^2/\nu T$ and $Re = \beta\, KC$ the Reynoldsnumber. For the sphere, the inviscid solution for added mass represents another reference. The viscous flow and forces on a sphere have been obtained by Chang and Maxey (1994) for Re up to 50.

A few computational results include:

• Inviscid calculations for cylinder: added mass = 1.004 x displaced mass.

• Viscous calculations for cylinder; Re=400, KC=0.4: added mass = 1.09 x displaced mass; analytical, Wang (1968): 1.07 x displaced mass.

• Inviscid calculations for sphere: added mass = 0.504 x displaced mass.

• Viscous calculations for circular cylinder; Re=400, KC=0.4: steady streaming, see figure 1.

6 Unfinished work

Sets of computations for bodies in oscillatory flow are currently pursued for a range of Reynoldsnumbers and Keulegan Carpenter numbers. Features and limitations of the viscous and LES computations are tested out. Various boundary layer effects are calculated and compared to available published results. More complex boundary layer phenomenas involve e.g. the Honji instability and its effect of on the forces. A parallel activity involves the implementation of a free surface motion. More results will presented at the Workshop.

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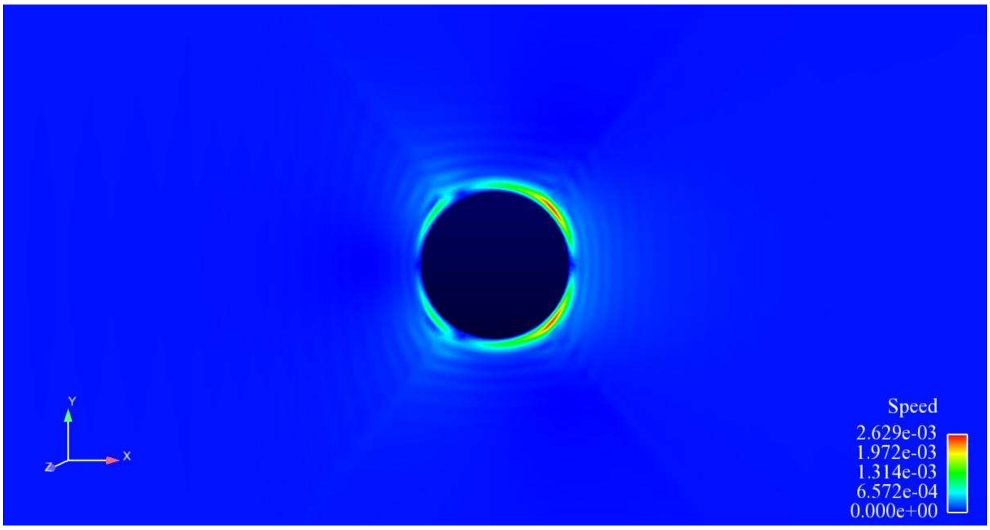

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Figure 1: Steady streaming at circular cylinder in oscillatory motion, after one cycle; Re=400 and KC=0.4.