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# Linear Modelling of Wave Device Arrays And Comparison To Experimental and Second Order Models

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## 1 Introduction

Many types of device have been proposed for generating useful electrical energy from the kinetic and potential energy of ocean waves. One class of device consists of a closely spaced array of floats whose vertical oscillation drives a power take off system (for example Manchester Bobber, Fred Olsen  $FO^3$ , Trident Energy, Wavestar). Typical dimensions are a radius  $a \sim 5$  m, a centre to centre separation  $s \sim 4a$  to operate in wavefields with peak periods in the range  $5 \leq T \leq 12$  and water depth of 25m-50m.

For these systems, the response of each float is dependent on both the excitation force due to the diffracted wave-field and forcing due to waves radiated by the oscillation of the devices. It is widely known that these interactions cause both the response and power output of a float within an array to differ from the same device in isolation. High power interaction factors can be attained in regular waves providing that both the mass and mechanical damping on all floats are specified in terms of wave frequency. To account for hydrodynamic coupling between the floats, optimal tuning requires that the mechanical damping on each float is specified in terms of the velocity and acceleration of all floats in the array. In practice, this is non-trivial. A simpler system in which mechanical damping on each float is based only on its own motion in regular waves has been shown to produce slightly less power, but still greater power than if the devices were in isolation [Justino & Clement, 2003]. A similar approach has also been shown to increase power output in irregular waves [De Backer et al., 2009] when response is modelled by superposition. Mass variation is typically limited to a finite range and, for a buoyant device, a variation of mass would modify draft and hence radiation damping and added mass. It is therefore simpler to modify only the mechanical constraint on each float. A simpler system in which the masses of all the floats are fixed, and the mechanical damping values of each float are selected independently of the motion of the other floats was shown to increase power output of a  $5 \times 1$  array in regular seas [Thomas et al., 2008]. Although these predictions of increased power output are promising they are reliant on the validity of linear theory for modelling shallow-draft float response in regular waves and do not describe irregular wave response. Response and power output of a comparable mechanical system has been studied in irregular waves by superposition [Cruz et al., 2009] but it is unclear whether steady-state regular wave responses can develop. After demonstrating increased power output by application of float-specific mechanical damping, the validity of linear analysis for predicting shallow-draft float response is addressed by i) comparison of predictions of undamped float response to experimental measurements of an array of heaving devices and ii) modification of the forcing spectrum due to 2<sup>nd</sup> order sum- and difference- frequency forces. Hydrodynamic analysis is conducted using WAMIT.

## 2 Interaction Factors due to Variation in Mechanical Damping

According to linear wave theory the net power from an array of  $N$  devices can be calculated using the following equation:

$$P = \frac{1}{8} F^* B^{-1} F - \frac{1}{2} \left( U - \frac{1}{2} B^{-1} F \right)^* B \left( U - \frac{1}{2} B^{-1} F \right) \quad (1)$$

where ‘\*’ represents the conjugate transpose,  $F$  the excitation force vector,  $B$  the full radiation damping matrix, and  $U$  the velocity vector. Modelling an array as a system of linear spring-mass-dampers, the velocity vector is a function of the device mass, stiffness and damping. Maximum power output from an array of equal mass floats is obtained by selecting a different mechanical damping value for

each float. An iterative approach is used by applying the Gauss-Newton Algorithm to Equation (1) to determine a diagonal mechanical damping matrix which results in a specific power value. This is then repeated for each power value within a specified range, and both the maximum power value and the corresponding damping matrix are stored for that frequency. The process is repeated for each frequency to obtain a set of iteratively selected float-specific values of mechanical damping, which yield greatest net power from the array. Individual damping values are sought for a  $5 \times 1$  array of hemispheres in incident waves both perpendicular and parallel to the line of the arrays (beam and head seas respectively). Each float within the array has a mass equal to twice the displaced mass of the fluid, a radius of  $a = 5m$  and is located such that its centre is a distance of  $s = 4a$  from the centres of adjacent floats. The damping values are both selected from an unlimited range, and also from the restricted range of less than twice the maximum radiation damping on an isolated device ( $R < 2 \max(B_0)$ ). The resulting power interaction factors can be seen in figure 1, along with the individually selected damping values. Comparison is drawn to a system in which the mechanical damping on all floats is given by  $R_{opt}$ ; the mechanical damping required to maximise power output from an isolated device.

When compared to the base case, the maximum increase in interaction factor obtained by applying the iteratively selected float specific values is 12.3% in beam seas and 26.5% in head seas, corresponding to interaction factors of  $q = 1.18$  and  $q = 1.2$  respectively. In beam seas these values were achieved by applying high- and low-damping to alternate floats. Much lower damping is applied to the middle and end floats (1, 3 and 5) of each row than the intermediate floats (2 and 4). In head seas the variation of mechanical damping with float number is similar to that of the base case with the mechanical damping distribution reversing over the range of increased interaction factors.

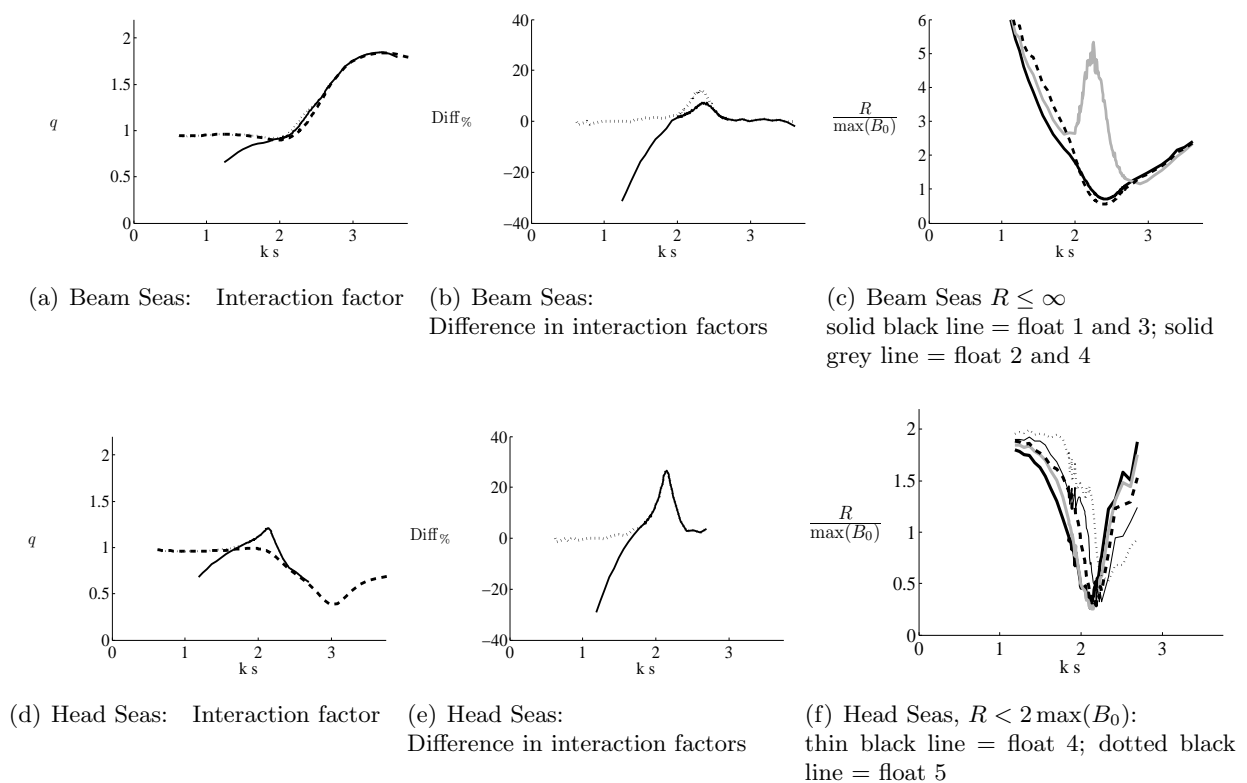


Figure 1: Interaction factors [(a) & (d)] for a  $5 \times 1$  array of hemispheres of radius  $a$ , separation distance  $4a$  in both beam and head seas obtained by applying the base case of  $R = diag(R_{opt})$  (dashed line) and by individually selecting mechanical damping values with no cross-knowledge of the motion of other floats both from an unlimited range of values (dotted line) and from the range  $0 \leq R \leq 2 \max(B_0)$  (solid line); The differences between the  $q$ -factors obtained from the individually selected values and the base case are also shown [(b) & (e)] as are the individually selected damping values [(c) & (f)] (thick solid black line = float 1; solid grey line = float 2; dashed black line = float 3)

### 3 Experimental Comparison of Array Response

To improve understanding of the suitability of linear analysis for modelling the behaviour of an array of shallow draft floats, the free-response of several small arrays has been studied experimentally. The fixed mass array predictions, calculated using linear theory with zero mechanical damping, are compared to experimental measurements in which floats and counterweights are supported from a pulley. The array configuration considered here comprises a  $5 \times 1$  array with geometry as described in Section 2 arranged in both beam and head seas. Peak predicted response amplitude ratios are up to 6 in head seas and 4 in beam seas whilst measured response amplitude ratios are up to 3.5 in both cases (Figure 2 shows head seas case). Measured response amplitudes are qualitatively similar to the predicted response except when predictions exceed four times the incident wave amplitude. The asymmetric response of devices within a head seas array is clearly observed. As expected, the predicted amplitudes of eight times the wave amplitude are not observed experimentally since such large motions would invalidate the small-amplitude assumption. Additional damping, perhaps caused by differences in hydrodynamic damping due to small variations in device spacing or to viscous losses, improves agreement in this region. At low frequencies, measured responses are higher than predicted although this can, in part, be attributed to a reduction of average water plane area as the response amplitude approaches the float radius. A modification to the calculation of hydrodynamic stiffness to account for this reduction results in improved agreement between response predictions and measured response.

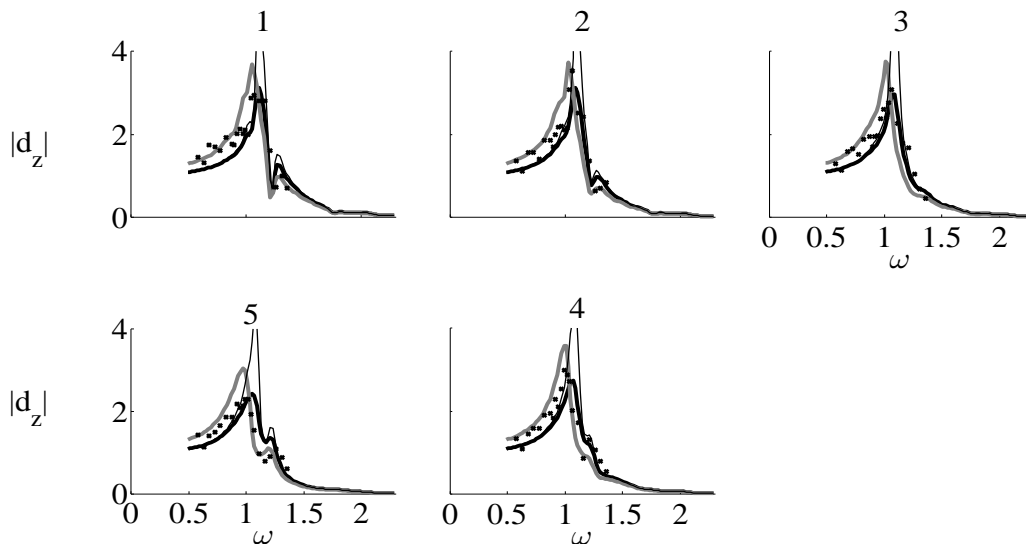


Figure 2: Response Amplitude Ratio for a  $5 \times 1$  array (float 1 at the front) of hemispheres of radius  $a$ , separation distance  $4a$  calculated using linear theory with zero mechanical damping (thin black line), linear theory with a small amount of damping (thick black line), linear theory with a small amount of damping and a modification to the stiffness matrix (thick grey line) and experimentally (dots)

### 4 Irregular Wave Excitation Force

The second order force for a  $2 \times 1$  array of hemispheres is calculated for a range of frequencies. These forces are dimensionalised in Figure 3 according to the first order amplitudes of the constituent waves according to the Bretschneider spectrum with peak wave period equal to 10 seconds. As expected from fixed structure analysis [Malenica et al., 1999] the second order forces are of larger magnitude at the smaller spacing of  $3a$  than  $4a$ , these forces are small relative to the first order components; typically of the order of 10 - 15%.

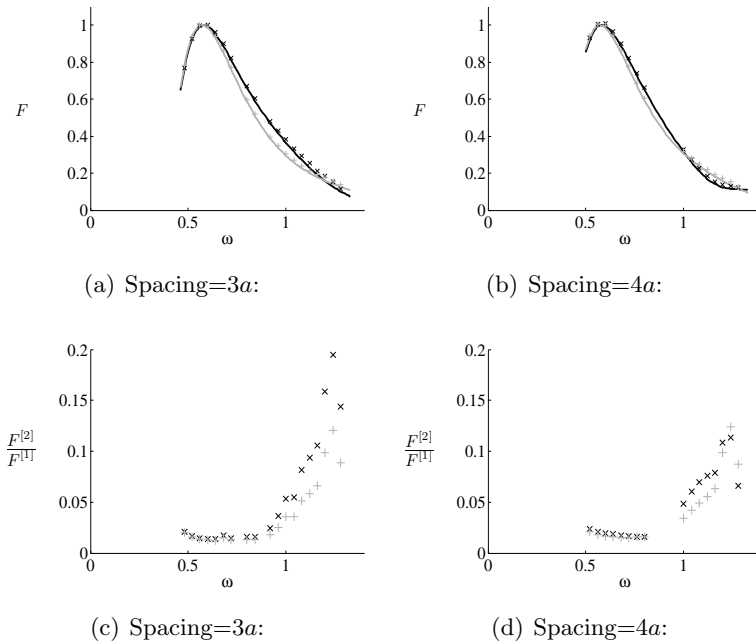


Figure 3: Forces for a  $2 \times 1$  array of hemispheres of radius  $a$  with separation distances of  $3a$  and  $4a$  calculated using amplitudes from the Bretschneider spectrum with significant wave period of 10 seconds: Upper graphs show forces including only linear force components (solid lines) and including both first and second order force components (markers) given relative to the maximum first order force; bottom graphs show second order force components relative to first order force components at each frequency; black represents float 1 (at front) and grey float 2

## 5 Concluding Remarks

It is shown that power interaction factors for a small line-array of equal mass heaving wave devices in regular waves can be increased by employing different values of mechanical damping to each float. This is a mechanically simplistic system which would be straightforward to implement since the damping force on each float is dependent on the motion of only one float. These increased interaction factors occur with float response amplitudes up to 3.5. Comparison to experimental measurements of undamped float response of an array of heaving floats suggest that device response amplitudes are predicted with reasonable accuracy within this range although some modification is required for variation of water plane area at higher motions. A preliminary evaluation of the force spectrum due to irregular waves has been conducted indicating that the magnitude of sum-frequency forces is small relative to first order forces although may coincide with the natural frequency of small heaving devices.

## References

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