Sound Scattering and Noise Control by Free Surface Piercing Cylinders

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Introduction and Objectives:

The problem of sound scattering by vertical structures piercing the free surface is of importance for detection and structural integrity assessment. In this study we look at the scattering of an incoming monochromatic sound wave impinging on a vertical flexible (hydro-elastic) circular cylinder clamped to the bed floor and piercing the shallow water surface. This model problem is relevant for a variety of engineering applications ranging from off-shore ocean structures to a submarine periscope [1]. The vertical uniform elastic cylinder is taken as hollow (constant thickness) and subject to a distribution of internal pressure (serving as our control) which must be optimized.

The problem is of acoustic-structure interaction nature and falls into the ongoing effort of developing active wave control by deforming bodies using direct or indirect means e.g. [2]. Previously, the authors presented a similar approach to reduce the acoustic signature from a floating flexible plate, demonstrating the plausibility of such a method [3, 4]. The current cylinder problem is a step forward looking at a model problem directly applicable for an actual hydroelastic engineering structure. We aim at investigating the effect of the flexibility of the cylinder on the scattered sound in the presence of a free surface and the potential of applying internal pressure patches (continuous or discrete) to the inner side of the cylinder in order to reduce the structure acoustic signature and the level of the scattered sound wave.

Mathematical and Numerical Formulation:

Linear acoustics and elastic shell dynamics are assumed as in Refs. [2, 3, 4]. The stationary wave equation is taken as the governing equation for the sound field. This is justified by the very low Mach number of underwater flows. However, caution must be exercised; shear refraction and scattering due to fluid motion is still possible at very short sound wave lengths. Water flow can also cause the free surface to deform, leading to a steep bow wave in front of the cylinder and a V-like wake behind it [1]. This deformation can affect the sound field near the cylinder, depending on the sound wave length and if the Froude number (based on cylinder's diameter) is high, i.e. at the order of unity or above [5]. In calm water the Froude number is expected to be low and thus analysing the interaction between sound wave and the deforming free-surface is postponed for future studies due its complexity and the high computational cost [1]. Hence, the current study is focused on calm water and low to mid frequency sound waves.

The governing sound field equation in the space-frequency domain is the Helmholtz equation;

$$\left[\nabla^2 + (\omega/c_0)^2\right]p = 0\,. \tag{1}$$

p is the sound pressure, ω is the incoming wave frequency and c_0 is the speed of sound. The free surface is taken as having zero impedance and the rigid bottom has infinite impedance.

$$p = 0, \text{ at } z=h\,, \tag{2a}$$

$$\partial p/\partial z = 0, \text{ at } z=0\,, \tag{2b}$$

where h denotes water depth and the z is the upwards coordinate. The floor may have finite impedance which will affect the modelling of the vertical sound wave variation as will be noted later, but for simplicity we will stick here with the infinite impedance. At the cylinder's surface

$$\partial p/\partial r= \rho\omega^2 u, \text{ at } r = a, . \tag{3}$$

where a is the cylinder's radius, ρ is the fluid density and u is the radial structure deflection .

The cylinder's radial, tangential and vertical defections u, v and w are governed by the following three linear shell equations given in Ref. [6];

$$\left[\frac{\rho_p\left(1-\upsilon^2\right)\omega^2}{E}-\frac{1}{a^2}\right]u-\frac{1}{a^2}\frac{\partial v}{\partial\theta}-\frac{\upsilon}{a}\frac{\partial w}{\partial z}=-\frac{1-\upsilon^2}{Et}q\,, \tag{4}$$

$$\frac{1}{a}\left(\frac{1}{a}\frac{\partial u}{\partial\theta}+\frac{1}{a}\frac{\partial^2 v}{\partial\theta^2}+\upsilon\frac{\partial^2 w}{\partial z\partial\theta}\right)+\frac{(1-\upsilon)}{2}\left(\frac{\partial^2 v}{\partial z^2}+\frac{1}{a}\frac{\partial^2 w}{\partial z\partial\theta}\right)+\frac{\rho_p\left(1-\upsilon^2\right)\omega^2}{E}v=0\,, \tag{5}$$

$$\frac{\partial^2 w}{\partial z^2}+\frac{\upsilon}{a}\left(\frac{\partial w}{\partial z}+\frac{\partial^2 v}{\partial z\partial\theta}\right)+\frac{(1-\upsilon)}{2a}\left(\frac{\partial^2 v}{\partial z\partial\theta}+\frac{1}{a}\frac{\partial^2 w}{\partial\theta^2}\right)+\frac{\rho_p\left(1-\upsilon^2\right)\omega^2}{E}w=0\,, \tag{6}$$

Here the cylinder's wall thickness t is assumed to be much smaller than its radius a, E is Young's modulus, υ is the Poisson ratio. ρ_p is the cylinder's material density and q denotes the distributed load acting on the cylinder in the radial direction, i.e. $q = f - p$, where f representing the internal pressure serves as the control parameter.

The pressure sound field complying with Eq. (1) can be expressed as

$$p(r,\theta,z)=e^{ik_x r\cos\theta}\cos\left(k_{z,n}z\right)+\sum_{ms}A_{ms}\cos\left(k_{z,m}z\right)\cos(s\,\theta)G(\gamma r), \tag{7}$$

where

$$k_{z,m}=\frac{(2m+1)\pi}{2h},\quad \gamma=\sqrt{\left|(\omega/c_0)^2-k_{z,m}^2\right|}\,, \tag{8a}$$

$$G(\gamma r)=\begin{cases}H_s^{(1)}(\gamma r), & \omega/c_0>k_{z,m}\\ K_s(\gamma r), & \omega/c_0<k_{z,m}\end{cases}. \tag{8b}$$

$H^{(1)}_s$ is the first-kind Hankel function and K_s is the second modified Bessel function. Thus, when $\omega/c_0 > k_{z,m}$ the mode is radiative and when $\omega/c_0 < k_{z,m}$ the mode decays exponentially far from the cylinder. The first term on the right hand side of Eq. (7) is the incoming wave and the second term is the scattered wave. Both waves have a cosine variation in the vertical direction to comply with the boundary condition of Eq. (2). If the sea floor has finite impedance, a full Fourier series representation in the vertical direction must be used. Examining the cylinder deflection Eqs. (4) to (6), shows that the radial and vertical deflection *(u* , *w*) are symmetric with respect to the polar angle θ while the tangential deflection v is asymmetric. Thus, we express u and w using a cosine transform and v by a sine transform in the θ direction. These equations are supplemented by edge conditions (clamped at the floor and free edge at the top).

The system of Eqs. (1) to (6) was solved using a Fast Fourier Transform (FFT) in the θ direction and a second-order central finite difference scheme in the vertical direction *z by*, assuming a uniform discretization grid. The scattered sound amplitudes A_{ms} were found by enforcing the boundary condition on the cylinder surface, Eq. (3). The effect of each A_{ms}- mode on u was found by solving coupled Eqs. (4) to (6) using a matrix pivoted LU solver [7]. The result thus obtained was used to generate a matrix equation for A_{ms}. However, the resulting matrix was found to be ill-posed in many occasions and thus a least square operation with respect to $\cos(k_{z,m}z)$ was instead applied. The matrix was then solved using the LU solver [7].

A similar procedure was also applied to calculate the effect of exerting an internal pressure f. To require zero or minimum level of sound scattering, two approaches were considered. The first assume the ability to apply continuous pressure over the inner surface of the cylinder. The aim was to zero all resolved scattered wave amplitudes A_{ms}. The internal pressure was decomposed using FFT in the θ direction and a cosine transform in the z direction. The effect of each Fourier mode on u was found by solving Eqs. (4) to (6). The result was used to build a matrix equation for the various modes of f by complying with Eq. (3) and requiring that A_{ms}=0.

The second approach assumed a more practical scenario of discrete actuators exerting internal pressure on the cylinder wall. The actuators were assumed to be points or lines along the vertical direction with a cosine spatial variation in that direction. Their effects on u were calculated to generate an equation for A_{ms} in accordance with Eq (3). The L2 norm of A_{ms} was then minimised using Powell's optimisation procedure [7] by varying the actuators amplitudes and their vertical locations as if they were points, or of the vertical cosine wave numbers as if they were lines.

Results and Conclusions

An aluminium cylinder of 15 cm radius and 5 mm thickness was considered. The speed of sound was taken as that of fresh water (1500 m/s) and the incoming acoustic pressure amplitude was taken as of 1 Pa (corresponding to sound pressure level of 120dB). The incoming wave was assumed to have a frequency of 5000 Hz (lying in the low to mid frequency sonar range) and the simplest vertical spatial variation, i.e. k_{z0}. The computational grid consisted of (401, 128, 21) points evenly distributed in the (r, θ, z) directions. The contour levels of the sound pressure modulus near the sea floor and a rigid cylinder (u=0) are shown in Figure 1(a). In front of the cylinder a pattern of standing waves caused by perfect reflection from the cylinder is seen while aft of it a V-wake, similar to that seen in water flow, can be observed. The scattered wave also forms side reflection; all having the same vertical distribution as the incoming sound wave.

When the cylinder is allowed to be flexible as depicted in Figure 1(b), some of the sound energy tends to leak away from the lowest vertical mode n=0 towards higher modes, as a result of enforcing the two edge conditions on the cylinder. The resulted acoustic signature is similar in many respects to that of the rigid cylinder but with a more profound V –wake and a noticeable higher level of reflected sound waves.

Calculations for a continuous internal pressure f yielding a perfect zero scattered wave in the level of the grid resolution, were carried out and the results will be shown at the workshop. In this abstract we show the results when using the more practical approach of applying a finite number of controlled actuators to generate f. Numerical simulations showed that using a dozen of point actuators reduced the L2 norm of the scattered wave amplitude only by about 4dB. On the other hand, using the same number of vertical line actuators reduced the L2 norm by more than 10dB (see Figure 1(c)). It is also seen that the V- wake pattern was considerably reduced and that the level of the reflected sound field upstream was also reduced, although some of the reflected pattern remained. As the incoming frequency increases the number of the discrete actuators should increase, in order to obtain a similar performance, thus rendering this approach applicable to the low frequency range of present-day sonar. The free board effect, resulting from extending the cylinder above the water level, will be also discussed at the work shop.

References:

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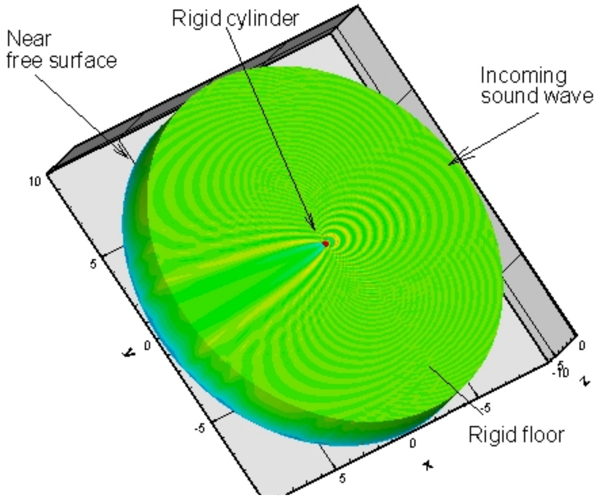
(a)



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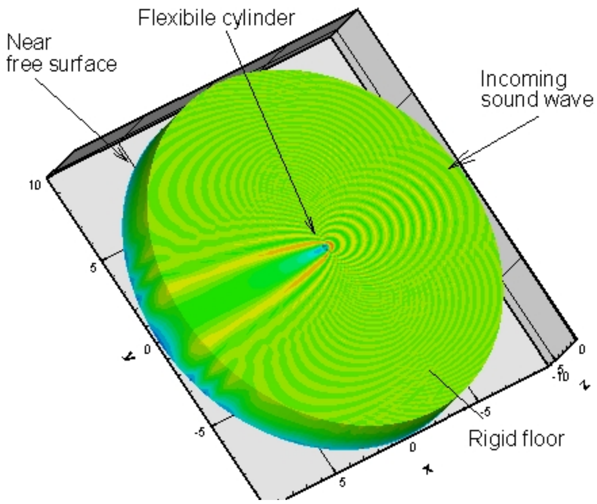

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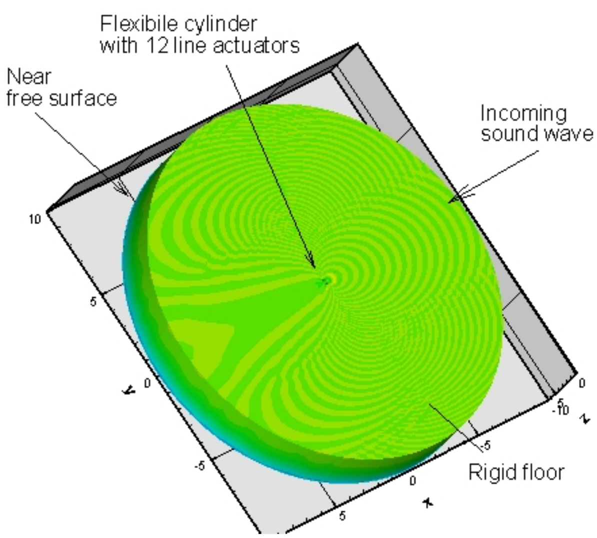

Fig.1: Near bottom sound pressure modulus, ranging from 0.1 to 1.5 Pa for a hollow Aluminium cylinder of 15 cm radius, 5 mm thickness, water depth of 8 m and sound frequency of 5000 Hz.