Tsunamis simulations by using Green-Naghdi theory

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The new wave theory called Green-Naghdi (GN) theory is applied to simulate tsunamis induced by earthquakes and underwater landslides. By satisfying the exact kinematic and dynamic boundary conditions on the free surface and on the seabed, the method developed here by using the GN theory is appropriate to simulate this problem of highly nonlinear type. The good agreement between numerical results and the experimental measurements by Hammack (1997) and Sue (2007) shows the efficiency and accuracy of the GN theory which provides indeed a novel method to simulate earthquake and underwater landslide tsunamis.

1. Introduction

The simulation of fully nonlinear shallow water waves is extremely important for the design of coastal structures. Fundamentally different from the usual perturbation method based on the developments in classical wave theory begun by Stokes and Boussinesq, the GN theory introduces some simplification of the velocity variation in the vertical direction between the fluid sheets but consists of an exact statement of the mass conservation, an approximate statement of the conservation of momentum, and exact statements for the various boundary conditions. Following the development of different polynomial orders for the description of velocity in the vertical direction, the GN theory can be of level I, II or III. Demirbilek and Webster (1992) studied the 2D level II GN theory in shallow water. Webster and Kim (1990) discussed the 2D level III GN theory in deep water while Q. Xu (1993) developed the GN theory in three-dimensional space.

In the present study, the 2D level II GN theory in shallow water is developed and compared with the level III GN theory. The 2D level II and level III GN theories are used to simulate earthquake induced tsunami and underwater landslide tsunami for the first time. The numerical results are compared with measurements of model tests by Hammack (1997) and Sue (2007).

2. Governing equations for tsunamis problems

A fixed Cartesian coordinate system is defined with the Oz-axis oriented vertically upwards and the Ox-axis oriented horizontally. The viscosity of fluid is ignored here. The fluid horizontal and vertical velocity vector is denoted by $u(x,z,t)$ and $w(x,z,t)$ in the position (x,z) at the time t. The free surface and the bottom condition are defined by $z=\beta(x,t)$ and $z=\alpha(x,t)$, respectively. The GN theory in Demirbilek & Webster (1992) introduces the development of velocity vector in the vertical direction by a series of "shape" functions (here polynomials) depending only on z :

$$u(x,z,t)=\sum_{n=0}^{K}u_n(x,t)z^n \tag{1a}$$

$$w(x,z,t)=\sum_{n=0}^{K}w_n(x,t)z^n \tag{1b}$$

The kinematic boundary condition on the seabed $D(z-\alpha)/Dt=0$ becomes

$$\sum_{n=0}^{K}w_n\alpha^n=\frac{\partial\alpha}{\partial t}+\frac{\partial\alpha}{\partial x}\sum_{n=0}^{K}u_n\alpha^n \tag{2}$$

The kinematic boundary condition on the free surface $D(z-\beta)/Dt=0$ is expressed as

$$\sum_{n=0}^{K}w_n\beta^n=\frac{\partial\beta}{\partial t}+\frac{\partial\beta}{\partial x}\sum_{n=0}^{K}u_n\beta^n \tag{3}$$

The conservation of mass is written as

$$u_K = 0 \tag{4a}$$

$$\frac{\partial u_n}{\partial x} + (n+1) w_{n+1} = 0 \quad \text{for} \quad n = 0, 1, \cdots, K-1 \tag{4b}$$

The conservation of momentum is found to be

$$\rho \sum_{m=0}^{K} \left(\frac{\partial u_m}{\partial t} H_{m+n} + \frac{\partial u_m}{\partial x} \sum_{r=0}^{K} u_r H_{m+r+n} + u_m \sum_{r=0}^{K} w_r H_{m+r+n-1} \right) = \frac{\partial P_n}{\partial x} + \hat{p}\, \beta^n \frac{\partial \beta}{\partial x} - \bar{p}\, \alpha^n \frac{\partial \alpha}{\partial x} \tag{5a}$$

$$\rho \sum_{m=0}^{K} \left(\frac{\partial w_m}{\partial t} H_{m+n} + \frac{\partial w_m}{\partial x} \sum_{r=0}^{K} u_r H_{m+r+n} + w_m \sum_{r=0}^{K} w_r H_{m+r+n-1} \right) = P_n - \rho g H_n - \hat{p}\, \beta^n + \bar{p}\, \alpha^n \tag{5b}$$

for $n = 1, 2, \cdot, K$. In (5), we have used :

$$H_n = \frac{1}{n+1}(\beta^{n+1} - \alpha^{n+1}),\; P_n = \int_{\alpha}^{\beta} p\, z^n \, dz,\; P'_n = n \int_{\alpha}^{\beta} p\, z^{n-1} \, dz \tag{6}$$

while $\bar{p}$ is the unknown pressure on the bottom, and $\hat{p}$ is the pressure on the free surface which should be zero if we exclude the surface tension effects. As the value of K increases, the complexity of GN theory increases too. We call the Kth level GN Theory, i.e., the level I, level II and level III GN theories are associated with K =1, 2 and 3, respectively. If the bottom is fixed in time, the expression of bottom becomes $z = \alpha(x)$ and the governing equation here would go back to the equations used by Demirbilek and Webster (1992).

The system of differential equations (2)-(5) is solved by using the finite difference method. We use a unform grid in x direction on which the arbitrary seabed bottom profile is represented by a sequence of straight lines connecting uniformly distributed nodes. In the same way, a unform discretisation in time is adopted as well. The first- and second-order derivatives of velocity field are approximated by the centre difference scheme. The resultant linear system is then of tri-diagonal and solved by Thomas algorithm.

3. Numerical results

First, we consider the case presented in Luth *et al.* (1994) where the sea bottom is fixed as illustrated on the left part of Figure 1. On the right part, the free surface elevation is represented along the model length

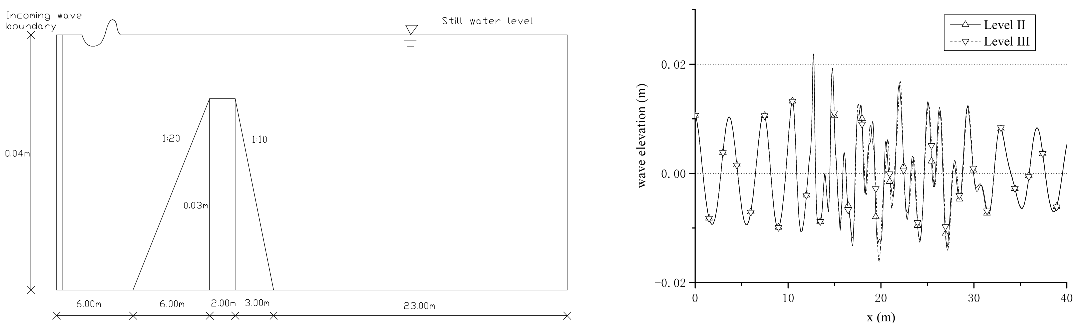

Figure 1: Scheme of Luth *et al.*'s model test (left) and wave elevation (right) at the time t=30.3s

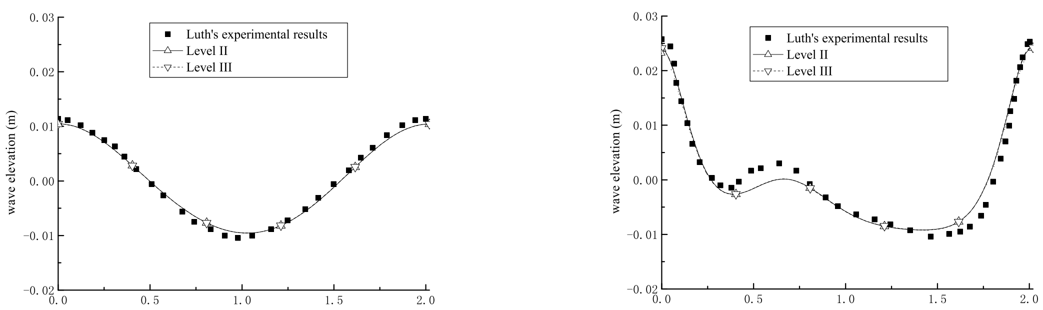

Figure 2: Established wave elevation during one period at x = 2m (left) and at x = 13.5m (right)

at the time $t = 30.3$s. A regular wave of amplitude equal to 0.01m and of period 2.02s is generated on the left boundary where the waterdepth is 0.04m. The measurement of wave elevation during one period was taken when a harmonic wave motion is established. The wave elevations are then computed at $x = 2$m and $x = 13.5$m and represented on Figure 2. The comparison with measurements is very good. Furthermore, the numerical results from level II and level III GN theories are nearly indistinguishable.

Now we consider the free surface elevation due to an earthquake which is assimilated by a part of bottom moves like $z = \alpha(x,t) = -d + \zeta_0(1 - e^{-qt})H(b^2 - x^2)$ with $H(\cdot)$ the Heaviside step function, as shown on the left part of Figure 3 where only the half space ($x \geq 0$) is represented thanks to the symmetry about $x = 0$. The free surface elevation in the quake center region illustrated on the right part presents

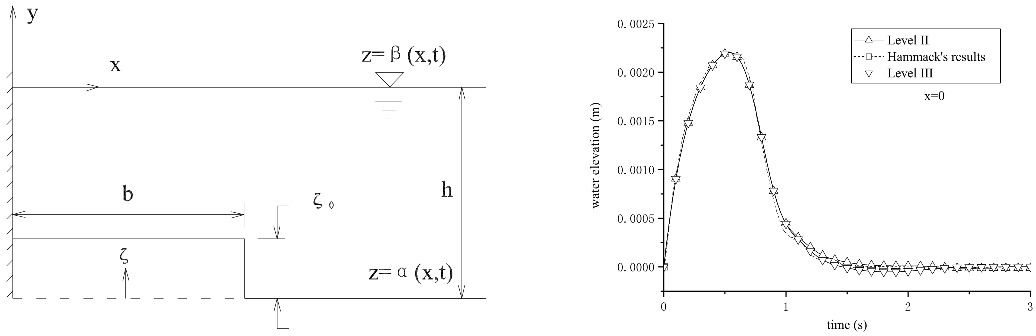

Figure 3: Scheme of Luth *et al.*'s model test (left) and wave elevation (right) at the quake center

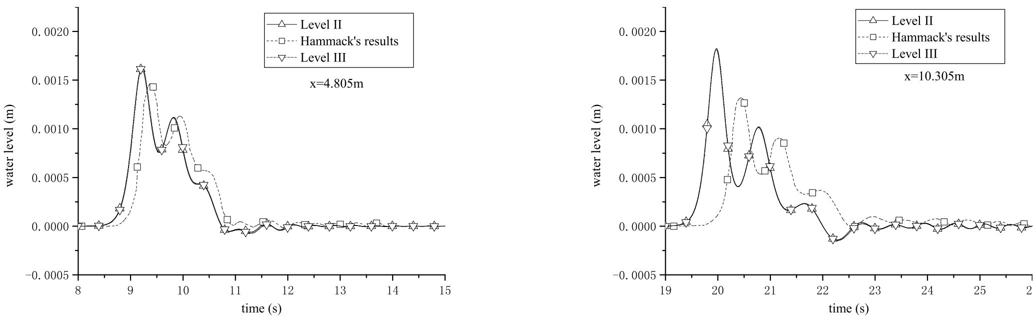

Figure 4: Free surface elevations in the downstream positions $x = 4.805$m (left) and $x = 10.305$m (right)

a maximum initial displacement and rapidly returns to the still-water level. Free surface elevations in the downstream region are presented on Figure 4. After propagation through $b = 4.5$m, this initial wave appears to be separating into approximately three waves, ordered by amplitude and followed by a train of small oscillatory waves. The disintegration of the initial wave into individual waves continues at, where these waves now resemble solitons near their crest. This phenomenon results from the amplitude and frequency dispersion well captured by the GN theory. The major discrepancies between GN theory and experiments appear in the maximum amplitude and speeds of evolving waves. At $x = 10.305$m, the maximum amplitude computed by GN theory is 37% larger than that of measured wave, and the average speed of the computed solitons is approximately 2.3% greater than the measured speed. One explanation of this difference is the presence of viscosity in the model test which attenuate wave amplitudes.

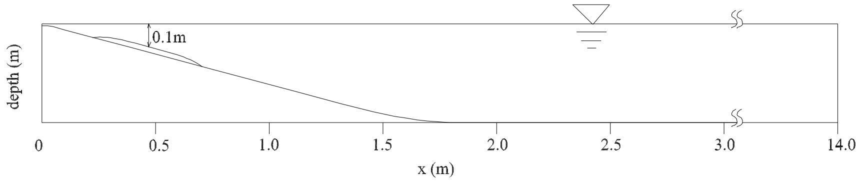

Figure 5: Scheme of landslide model in Sue (2007)

Finally, we consider the underwater landslide tsunami measured in model basin by Sue (2007) as shown on Figure 5. A sliding land of semi-elliptical shape was used in the tests. The basic function for the land shape at the initial position is given by :

$$f(x) = 0.982[1 - (4x)^4]/40$$

for $-0.25 \leq x \leq 0.25$. The free surface elevations along the x direction are illustrated on Figure 6 at

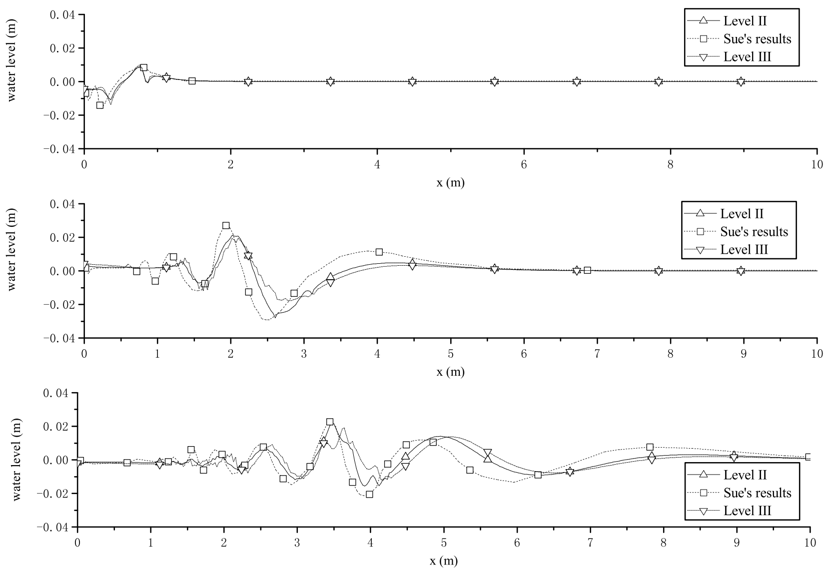

Figure 6: Free surface elevations at $t = 0.6$s (top) $t = 2.6$s (middle) and $t = 4.6$s (bottom)

different time steps. The numerical results by using Level II and level III GN theories are compared with measurements in Sue (2007).

It is shown that the level II GN theory gives already very well results by comparing to measurements. It worth noting that only 12 seconds CPU is needed on a PC with 512Mo RAM and 3.00GHz Processor for each numerical simulation with 500 nodes in space and 1000 time steps.

4. Conclusions

The 2D level II and level III GN theories have been developed to simulate the tsunamis induced by earthquake and underwater landslide tsunami. The 2D GN theory is shown to be very efficient to study full nonlinear wave problems. The good comparison with model measurements shows the accuracy of the method. Furthermore, the level II GN theory gives nearly the same results as those using the level III GN theory.

Acknowledgments

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