Time domain simulation of water entry of twin wedges through free fall motion

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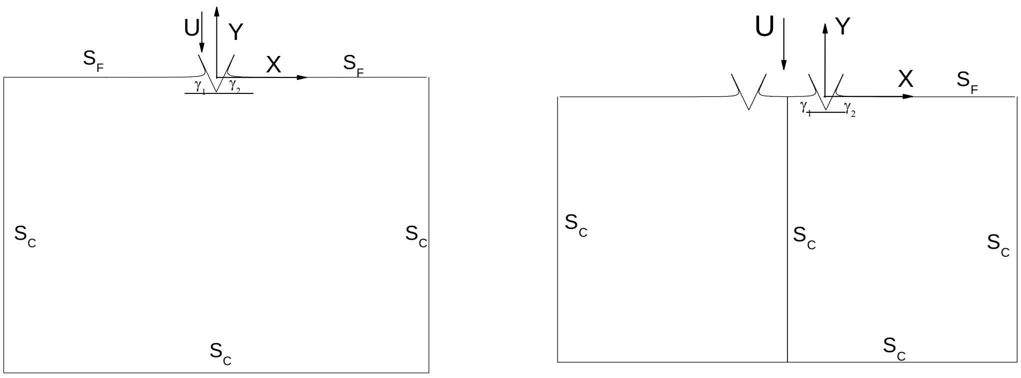
1. Introduction

Slamming is one of the most severe sea load conditions for ships, and it can be critically important for high speed vehicles. Extensive efforts on the slamming problem related to water entry have been made using wedge. Most of the work is, however, for a single wedge (e.g. Zhao & Faltinsen 1993, Wu, Sun & He, 2004, Xu, Duan & Wu 2008). Wu (2006) did consider a case of twin wedges, but the work was limited to the problem of constant entry speed. The present work considers the twin wedges entering the water surface through the free fall motion and therefore is a more general case. The results are evidently relevant to hull forms such as catamaran.

2. Mathematical model



(a) (b)

Fig.1 Sketch of the problem (a) Single wedge, (b) twin wedges

We consider a two dimensional solid wedge or twin-wedges entering calm water at velocity V, as shown in Fig.1. The fluid is assumed to be inviscid and incompressible, and the flow is irrotational. In the Cartesian coordinate system Oxy, the velocity potential ϕ satisfies the Laplace equation

$$\nabla^2\phi = 0 \tag{1}$$

in the fluid domain. On the wedge surface S_0 we have

$$\frac{\partial\phi}{\partial n} = V\cdot\vec{n} \tag{2}$$

where V is defined as positive when the body moves downwards, and $\vec{n} = (n_x, n_y)$ is the normal vector of the body surface pointing out of the fluid domain. The Eulerian dynamic and kinematic boundary conditions on the free surface S_F or $y = \varsigma$ can be written as

$$\phi_t + \frac{1}{2}\nabla\phi\nabla\phi = 0 \tag{3}$$

$$\varsigma_t = \phi_y - \phi_x\varsigma_x \tag{4}$$

where the gravity effect has been ignored in Eq.(3).

In the Lagrangian framework, the free surface boundary conditions can be written as

$$\frac{d\phi}{dt} = \frac{1}{2}\nabla\phi\nabla\phi \tag{5}$$

$$\frac{dx}{dt} = \frac{\partial\phi}{\partial x}, \quad \frac{dy}{dt} = \frac{\partial\phi}{\partial y} \tag{6}$$

The time stepping procedure used here to solve the above problem is similar to that in Wu, Sun & He (2004) and Wu (2006). At the initial stage when body touches the water surface, the flow is obtained through the similarity solution and followed by the time stepping method at the later stages. We define

$$\alpha = x/s, \quad \beta = y/s, \quad \phi = s\varphi(\alpha,\beta,t) \tag{7}$$

The free surface boundary conditions become

$$\frac{d(s\varphi)}{dt} = \frac{1}{2}(\varphi_\alpha^2 + \varphi_\beta^2) \tag{8}$$

$$\frac{d(s\alpha)}{dt} = \varphi_\alpha, \quad \frac{d(s\beta)}{dt} = \varphi_\beta \tag{9}$$

where

$$s = \int_0^t V dt \tag{10}$$

Eqs (7)-(10) show the principle of the stretched system method. This is highly effective for this type of problem, i.e., a body entering the water surface starting with a single contact point. In the method, the sizes of elements and the computational domains vary with the size of the contact zone between the water and the body. For the twin wedges, the flow corresponding to each wedge is virtually independent to the presence of the other wedge at the initial stage. The problem for each wedge can be solved on its own as shown in Fig. 1(a). The interactions between the two wedges become significantly at later stages and they must be included in the solution. This is the so called three step procedure in Wu (2006) and will be followed here.

To solve the boundary value problem at each time step, an integral equation along the boundary of the fluid is established based on Cauchy theorem for the complex velocity potential. The boundary is then divided into small elements along which linear variation of the potential is assumed. The centre line of twin-wedges can be treated as a solid line due to the symmetry of the twin-wedges.

For the free fall motion considered in this paper, it is important to decouple the mutual dependence between the body acceleration and fluid flow. Here we adopt the auxiliary function technique (Wu & Eatock Taylor 2003). In the fluid domain we have

$$\nabla^2\phi_t = 0 \tag{11}$$

The Bernoulli equation gives

$$\phi_t = -\frac{1}{2}\nabla\phi\nabla\phi \tag{12}$$

on the free surface. We also have (Wu 1998)

$$\frac{\partial\phi_t}{\partial n} = -\frac{dV}{dt}n_y + V\frac{\partial\phi_y}{\partial n} \tag{13}$$

on the body surface. To find ϕ_t, we write

$$\phi_t = -\dot{V}\chi_1 + \chi_2 \tag{14}$$

Here the auxiliary functions χ_1 and χ_2 satisfy the Laplace equation in the fluid domain and boundary conditions

$$\frac{\partial\chi_1}{\partial n} = n_y, \quad \frac{\partial\chi_2}{\partial n} = V\frac{\partial\phi_y}{\partial n} \tag{15}$$

on the body surface. On the free surface S_F, their boundary conditions can be written as

$$\chi_1 = 0, \quad \chi_2 = -0.5(\phi_x^2 + \phi_y^2) \tag{16}$$

and

$$\frac{\partial\chi_i}{\partial n} = 0 \tag{17}$$

on the control surface S_C.

With the help of the auxiliary functions, we have

$$F = \rho\int_{s_0}(\phi_t + \frac{1}{2}\nabla\phi\nabla\phi)dS = \rho\int_{S_0}(\dot{V}\chi_1 - \chi_2 - \frac{1}{2}\nabla\phi\nabla\phi)n_y dS \tag{18}$$

From the Newton's law, we then have

$$-(M_b + \rho\int_{S_0}\chi_1 n_y dS)\dot{V} = -\int_{S_0}(\chi_2 + \frac{1}{2}\nabla\phi\nabla\phi)n_y dS \tag{19}$$

This equation means that the acceleration can be found before the pressure distribution as in Wu & Eatock Taylor (2003) and Wu, Sun & He (2004)

3. Numerical results and discussions

The free fall motion of a symmetrical single wedge was studied by Wu, Sun & He (2004), and the results were found to agree well with the experimental data at the initial stage when the effect of gravity was insignificant. However they did not provide any results for pressure distribution. Here we shall first provide some of those missing results and then undertake simulations for twin wedges. Based on the definitions given in Fig.1, Fig.2 gives results for a solid wedge entering calm water with $\gamma_1 = \gamma_2 = 45^0$ at $V_0 = V(t=0) = 5m/s$. Fig.3 gives results for impact of twin wedges with $\gamma_1 = \gamma_2 = 45^0$. Detailed analysis and discussions will be given in the workshop.

4. Conclusions

The present work has solved the problem of water entry of twin wedges in the free fall motion based on velocity potential flow theory. The stretched coordinate system method is used and is combined with the three step procedure for the twin wedges. This has significantly advanced the previous work of Wu, Sun & He (2004). The implication of the newly obtained results will be discussed in the workshop.

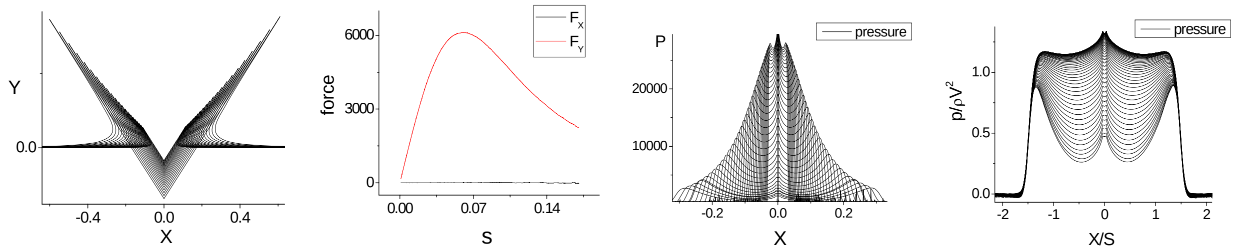


Fig.2 time domain analysis on single wedge

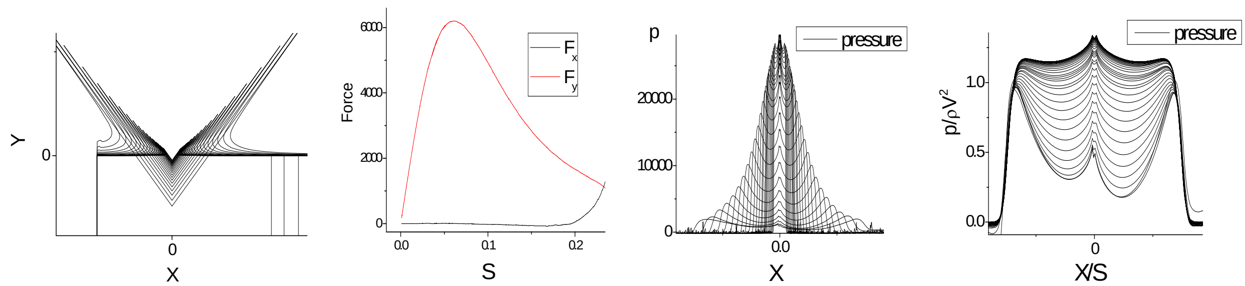


Fig.3 time domain analysis on twin-wedges

Acknowledgement

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References

Wu, G.X. (1998) Hydrodynamic force on a rigid body during impact with liquid, J. Fluids and Structures, 12, 549-559.

Wu, G.X., Eatock Taylor, R., 2003, The coupled finite element and boundary element analysis of nonlinear interactions between waves and bodies. Ocean Engineering, 30, 387-400.

Wu, G.X., 2006. Numerical simulation of water entry of twin wedges. J. of Fluids and Structures 22, 99–108.

Wu, G.X., Sun, H., He, Y.S., 2004. Numerical simulation and experimental study of water entry of a wedge in free fall motion. J. of Fluids and Structures 19, 277–289.

Xu, G.D., Duan, W.Y., Wu, G.X., 2008, Numerical simulation of oblique water entry of an asymmetrical wedge, Ocean Engineering. 35, 1597-1603.

Zhao, R., Faltinsen, O. M. 1993 Water entry of two-dimensional bodies. J. Fluid Mech. 246,593–612.