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Interaction of elastic structure with non-uniformly aerated fluid

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Introduction

The problem of fluid/structure interaction with account for aeration of the fluid is relevant to possible violent sloshing in a tank of LNG carriers accompanied with high slamming loads acting on the tank walls. The overall problem of sloshing impacts is extremely complex. It is known that the impact loads are highly localized both in time and in space. This allows us to model such loads by using simplified models of hydrodynamics [1]. Shape of the wave front just before the impact defines the impact type. Four types of the impact can be distinguished: 'Wagner-type impact', 'Steep wave impact', 'Bagnold-type impact' and 'Aerated fluid impact'. Wagner-type impact can be detected when the wave front is inclined from the wall. The second impact type corresponds to the case where the wave front is almost vertical before the impact and can be modeled as a hydraulic jump. In Bagnold-type impact the wave profile is inclined toward the wall, the wave is almost breaking and its upper part contacts the wall before the wave overturning with trapping an air cavity. The fourth case corresponds to spilling breaker when the wave broke before arriving at the wall and the fluid is mixed with air around the impact region. Steep wave impact was studied in [2] for 2D and [3] for 3D cases. In this report we present the analysis of aerated wave impact.

Two-dimensional problem of steep wave impact onto elastic wall is considered with account for non-uniform aeration of the fluid in the impact region. A fluid of finite depth H is assumed aerated near the wall. The horizontal dimension D of the aerated region is finite. Distribution of the air fraction $\alpha(x, z)$ in this region is given. In the main flow domain outside the aerated region, the fluid is incompressible. The flow is assumed irrotational. In the aerated region, compressibility effects matter and the flow is described by the velocity potential which satisfies the wave equation with variable sound speed. In the main flow region the corresponding velocity potential satisfies Laplace's equation.

The sound speed in the aerated fluid can be as small as $25\, m/sec$. The hydrodynamic pressures induced by the impact are expected to be smaller than those predicted by the model of compressible non-aerated fluid. However, since the dimension of the aerated zone is finite, multiple reflections of the pressure pulses from the interface between aerated and pure fluid and from the wall may lead to certain increase of the pressure magnitude within the aerated region with corresponding increase of magnitude of elastic stresses in the wall.

The problem is solved with the help of the modal decomposition method. The derived initial boundary value problem is reduced to a linear system of ordinary differential equations with respect to generalized coordinates of the wall deflection and the velocity potential in the aerated region. The system has been solved by utilizing its general solution and by the Runge-Kutta 4^{th} order method.

The numerical code was verified with respect to size of the time step of numerical integration and number of modes to be used for accurate predictions of the quantities of practical interest. The code was validated by comparing the results obtained for constant sound speed with those obtained by Steep Wave Impact numerical code.

The effects of non-uniform aeration and thickness of the aerated region on hydrodynamic pressure distribution, elastic deflections of the wall and strains are investigated. It is shown that non-uniform concentration of air in the impact region may increase the stresses in flexible structure compared to the case of non-aerated fluid impact.

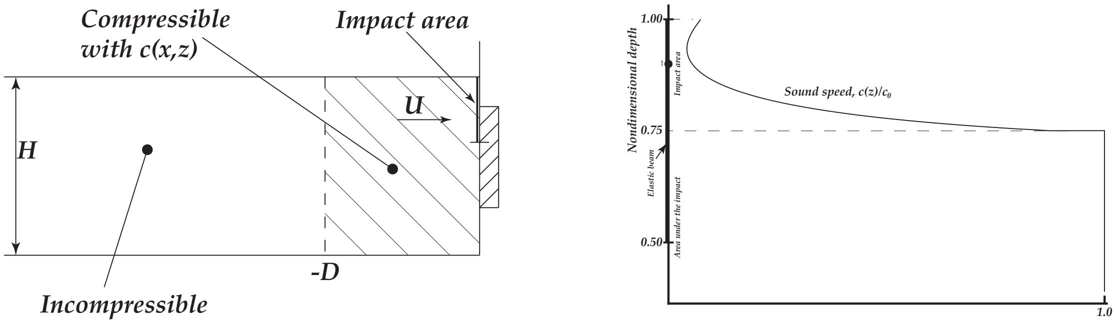

Figure 1. Configuration of aerated fluid impact (left) and sketch of the sound speed dependence on the depth (right).

Mathematical formulation

The fluid of depth H, see fig 1 (left), occupies the domain $x < 0$ and $0 < z < H$, where $z = 0$ corresponds to the flat rigid bottom, and the vertical $z-$axis is directed upward. Before the impact a part of the wall $0 < z < H - H_w$ is in contact with the fluid. A wave with vertical front (hydraulic jump) of height H_w hits the wall in the region $H - H_w < z < H$ with velocity U at the initial time instant $t = 0$. The wall is assumed partly elastic. Elastic part of the wall, $H^- < z < H^+$, is modeled as elastic Euler beam. The fluid is assumed aerated close to the wall, $-D < x < 0$. The aerated fluid is modeled as a fictitious weakly compressible medium with reduced sound speed. In the main flow domain, $x < -D$, the fluid is not aerated and modeled as an incompressible medium. The fluid flow is assumed 2D and potential in the whole flow domain.

The problem is formulated in non-dimensional variables which are chosen in such a way that the impact velocity U, the fluid depth H, the fluid density ρ_f and the maximum of the sound speed in the aerated part of the flow domain,c_{max}, are equal to unity in the non-dimensional variables. During the initial impact stage of short duration the boundary conditions and equations of fluid flow are linearized. The boundary value problem with respect to the velocity potential $\varphi(x, z, t)$ has the form

$$\begin{cases} c^{-2}(x,z)\varphi_{tt} = \Delta\varphi & (0 < z < 1,\ \ x < 0), \\ \varphi_z = 0 & (z = 0,\ x < 0), \\ \varphi = 0 & (z = h,\ x < 0), \\ \varphi_x = w_t(z,t)\chi_1(z) - \chi_0(z) & (0 < z < 1,\ x = 0), \\ \alpha w_{tt} + \beta \left(1 + \gamma_d \dfrac{\partial}{\partial t}\right) w_z^{IV} = -\varphi_t(0,z,t), & (h^- < z < h^+,\ x = 0) \\ \varphi = \varphi_t = w = w_t = 0 & (t = 0), \end{cases} \tag{1}$$

where $w(z, t)$ is the deflection of the elastic part of the wall, where $\chi_1(z) = 1$, on the rigid part of the wall $\chi_1(z) = 0$. Here $\alpha = \frac{m_b}{\rho L}$, $\beta = \frac{EJ}{\rho \bar{c}^2 L^3}$, m_b is the beam mass per unit length, EJ is the bending rigidity of the beam, γ_d is the non-dimensional structural damping. In the impact region, $H_w/H = h_w < z < 1$, we have $\chi_0(z) = 1$, and $\chi_0(z) = 0$, where $0 < z < h_w$. The sound speed $c(x, z)$ is assumed known with $c = +\infty$ in the domain of incompressible flow. On the interface between the domain of aerated fluid, $-d = -D/H < x < 0$, and the domain of fluid without gas bubbles, $x < -d$, the hydrodynamic pressure and the normal to the interface velocity of the flow are continuous

$$\frac{\partial\varphi}{\partial t}(-d-0,t) = \frac{\partial\varphi}{\partial t}(-d+0,t), \qquad \frac{\partial\varphi}{\partial x}(-d-0,t) = \frac{\partial\varphi}{\partial x}(-d+0,t). \tag{2}$$

Note that we do not account for change of the fluid density due to aeration.

The velocity potential $\varphi(x, z, t)$ and the beam deflections are sought in the forms

$$\varphi(x,z,t) = \sum_{k=1}^{\infty} \phi_k(t) u_{n(k)}(x) W_{m(k)}(z), \quad w(z,t) = \sum_{n=1}^{\infty} a_n(t) V_n(z), \tag{3}$$

where the functions $W_m(z)$ and $V_n(z)$ are orthonormal on the interval $0 < z < 1$, and the time-dependent coefficients $\phi_k(t)$ and $a_n(t)$ are to be determined from the body boundary condition and the wave equation.

Substituting (3) into (1) and (2) and after some algebra, the boundary-value problem (1) reduces to the system of ordinary differential equations

$$\begin{cases} \dfrac{\mathrm{d}^2\vec{\phi}}{\mathrm{d}t^2} = \mathbb{S}^{-1}\circ\mathbb{T}\,\dfrac{\mathrm{d}\vec{a}}{\mathrm{d}t} - \mathbb{S}^{-1}\,\vec{v} - \mathbb{S}^{-1}\circ\mathbb{D}^2\vec{\phi} \\ \alpha\dfrac{\mathrm{d}^2\vec{a}}{\mathrm{d}t^2} + \mathbb{K}\vec{a} + \gamma_d\mathbb{K}\dfrac{\mathrm{d}\vec{a}}{\mathrm{d}t} = -\mathbb{T}^*\dfrac{\mathrm{d}\vec{\phi}}{\mathrm{d}t}, \end{cases} \tag{4}$$

where $\vec{\phi} = \{\phi_1, \phi_2, ...\}$, $\vec{a} = \{a_1, a_2, ...\}$, $\vec{v} = \{v_1, v_2, ...\}$, $\mathbb{D} = \mathrm{diag}\{d_1, d_2, ...\}$, $\mathbb{K} = \mathrm{diag}\{\beta\mu_1^4, \beta\mu_2^4, ...\}$, $\mathbb{T} = \{T_{kn}\}$, $\mathbb{S} = \{S_{kl}\}$, d_k and μ_n are eigenvalues which correspond to $u_{n(k)}(x)W_{m(k)}(z)$ and $V_n(z)$, respectively. Here

$$v_n = \int\limits_{1-h_w}^{1} W_n(z)\mathrm{d}z, \quad T_{kn} = \int\limits_{h^-}^{h^+} W_{m(k)}(z)V_n(z)\mathrm{d}z,$$

$$S_{kl} = \int\limits_{-d}^{0}\int\limits_{0}^{1} c^{-2}(x,z)u_{n(k)}(x)u_{n(l)}(x)W_{m(k)}(z)W_{m(l)}(z)\mathrm{d}z\mathrm{d}x.$$

The linear system (4) can be written in the matrix form as

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} = \mathbb{M}\mathbf{X} + \mathbf{C}, \qquad \mathbf{X}(0) = 0 \tag{5}$$

with extended unknown vector $\mathbf{X}(t) = \{\vec{\phi}, \dot{\vec{\phi}}, \vec{a}, \dot{\vec{a}}\}^*$. The matrix $\mathbb{M}$ and vector $\mathbf{C}$ are given as

$$\mathbb{M} = \begin{pmatrix} 0 & \mathbb{I} & 0 & 0 \\ -\mathbb{S}^{-1}\circ\mathbb{D}^2 & 0 & 0 & \mathbb{S}^{-1}\circ\mathbb{T} \\ 0 & 0 & 0 & \mathbb{I} \\ 0 & -\alpha^{-1}\mathbb{T}^* & -\alpha^{-1}\mathbb{K} & -\gamma_d\,\alpha^{-1}\mathbb{K} \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 \\ -\mathbb{S}^{-1}\vec{v} \\ 0 \\ 0 \end{pmatrix}. \tag{6}$$

In (5), the matrix $\mathbb{M}$ and the vector $\mathbf{C}$ are independent of time. This makes it possible to use in computations the exact solution to the initial value problem (5). The matrix $\mathbb{M}$ can be presented as product $\mathbb{M} = \mathbb{R}\circ\mathbb{L}\circ\mathbb{R}^{-1}$, where $\mathbb{R}$ is complex matrix made by eigenvectors and $\mathbb{L}$ is the diagonal matrix formed by the corresponding eigenvalues of matrix $\mathbb{M}$. Then, the solution is

$$\mathbf{X}(t) = \left(\mathbb{R}\circ\mathbb{E}_{\mathbb{L}}(t)\circ\mathbb{R}^{-1} - \mathbb{I}\right)\left(\mathbb{D}^{-2}\vec{v}, \vec{0}, \vec{0}, \vec{0}\right)^*, \qquad \mathbb{E}_{\mathbb{L}}(t) = e^{\mathbb{L}t}. \tag{7}$$

Numerical Results

In order to compute accurately the solution of the initial problem (5), many modes have to be taken into account in both vertical z and horizontal x directions. Computations based on the analytical formula (7) are very time-consuming. The initial problem (5) is solved by the Runge-Kutta 4^{th} order method. The solution (7) is used to check accuracy of the numerical solution.

In the present computations the aerated fluid domain is $H = 2$m deep and $D = 25$cm wide. Elastic steel ($E = 20.7\times10^{10}\mathrm{N/m}^2$, $\rho_b = 7800\mathrm{kg/m}^2$) beam of 2cm thickness corresponds to the part of the vertical wall from $H^- = 1$ m to $H^+ = 2$m. The ends of the beam are simply supported. The structural damping is assumed as $\gamma = 0.001$sec. The wave of height $H_w = 50$cm with vertical front approaches the wall at speed $U = 1$m/s.

The numerical code was validated in terms of convergence of the solution with respect to the time step of integration and numbers of modes in horizontal and vertical directions. Convergence was studied in terms of the hydrodynamic pressure distribution along the wall and in terms of the strain distribution in the beam. The hydrodynamic pressure is much more sensitive to the parameters of calculations than the strains. For example, for accurate prediction of the pressure in the impact region with sound speed in the aerated region being $c = 95.46$m/s, the integration time step should be smaller than 10^{-6}s and number of horizontal modes should be greater than 150. However, calculation of elastic stresses in the beam with the same accuracy requires time step 10^{-5}s and 60 horizontal modes.

To study the effects of non-uniform aeration of the fluid on the elastic response of the beam, we distinguish two values of the sound speed 47.17m/s and 95.46m/s which correspond to "resonance" conditions of the wave-beam interaction in the configuration under consideration. Namely, we consider the case of uniform aeration of

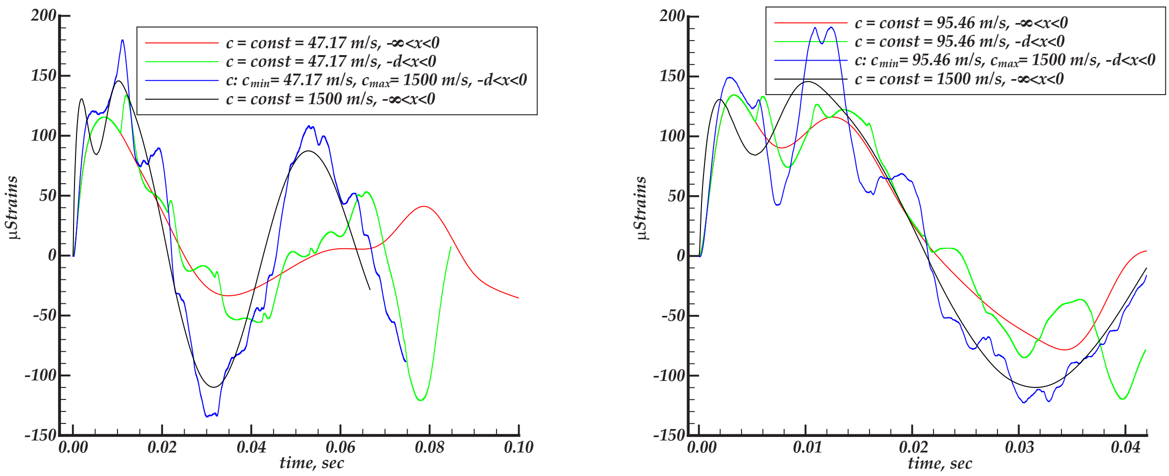

Figure 2. Strain evolution in the impact area at $z = 1.8\,m$ for different air concentrations.

the fluid in the region $0 < z < 1$, $-d < x < 0$ and calculate the corresponding acoustic time scale as $T_{ac} = H/c$. Then we calculate the period $T_1^d = 0.021$s of the first "dry" mode of the beam and the period $T_1^w = 0.0425$s of the first "wet" mode of the beam. Equating the acoustic time scale to these periods of the beam vibrations, we evaluate the sound speeds, 95.46m/s and 47.17m/s, respectively, and correspondingly the air concentrations in the fluid which may provide the most strong interaction conditions. We assume that these conditions are the conditions of strong interaction between the beam and the fluid because the solutions of the corresponding uncoupled problems provide the highest stresses in the beam. The solutions of our coupled problem provide finite stresses.

In fig. 2, evolutions of the elastic strains calculated at the point $z = 1.8\,m$, point 1 in fig. 1 right, are presented. The left figure corresponds to the reference sound speed 47.17m/s and the right figure to the reference sound speed 95.46m/s. In these figures, the red lines corresponds to the strains in the problem with uniformly aerated fluid domain of infinite extend, the black lines to the strains calculated for weakly compressible water without aeration ($c = 1500$m/s). It is seen that uniform aeration reduces slightly the stresses. However, if the thickness of the aerated domain is finite (green lines) the stresses can be higher than for the aerated domain of infinite extend.

The blue lines in fig. 2 represent the strains in the problem with non-uniform aeration in the region of finite extend. Calculations were performed for the sound speed profile shown in fig.1, right. In this problem, the fluid is aerated only in the impact region, $H - H_w < z < H$, $-D < x < 0$, weakly compressible ($c = 1500$m/s) below the impact region $0 < z < H - H_w$, $-D < x < 0$, and incompressible in the main flow domain, $x < -D$, in dimensional variables. In the domain of non-uniform aeration, the air concentration grows with depth starting from the free surface, takes its maximal values (95.46m/s for left figure and 47.17m/s for the right figure) and then decreases again to zero and the lower boundary of the aerated domain. The sound speed as a function of air concentration in the fluid was calculated by using the formula from [4].

The blue lines in the figures clearly indicate that non-uniform aeration increases the stresses even compared with the case of weakly compressible liquid. The obtained result shows that the aeration in a bounded domain near the impact region may increase stresses in flexible structures.

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