# Estimation of wall effects on floating cylinders

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## 1. Introduction

Under the assumptions of classical linearised water wave theory, the time-harmonic two-dimensional motion of a rigid body floating in the surface of a fluid may be characterised by various coefficients which express components of the hydrodynamic forces acting on that body.

For a single body in isolation these hydrodynamic coefficients are relatively simple to calculate. For bodies placed next to a vertical wall, the corresponding calculations are often much more complicated. Moreover, it is well-known that the behaviour of a floating body in waves is radically affected by the proximity of a rigid boundary such as a harbour wall, and this effect is manifested in major changes to the hydrodynamic coefficients. This is due to the near-resonant excitation, close to certain frequencies, of waves trapped between the cylinder and the wall. Examples of the results obtained for bodies oscillating next to walls are given in Wang & Wahab (1971) and Yeung & Seah (2007) where these resonances are identified with large rapid variations in hydrodynamic coefficients.

In this paper we use the well-known wide-spacing approximation (see Martin (2006) for example) to develop approximations to the hydrodynamic coefficients for a body next to a wall solely in terms of the results for the forced motion the same body in the absence of a wall.

Exact results are compared with the wide-spacing approximations for semi-immersed circular cylinders and cylinders of rectangular cross-section and show excellent agreement.

## 2. Formulation

A two-dimensional cylinder is taken to be floating in the surface of a fluid of density $\rho$ and infinite depth. Cartesian coordinates are chosen with the origin in the mean free surface and $y$ pointing vertically downwards.

We consider the time-harmonic small-amplitude forced sway, heave or roll ($j = 1, 2$ or 3 respectively) motion of the cylinder centred at the origin in the presence of a rigid wall at $x = -b$, $y > 0$ on which a Neumann condition is imposed on the potential, $\phi_j^w(x, y)$. (The superscript $w$ identifies quantities associated with the wall). Linearised water wave theory is used, in which potentials $\psi$ (say) satisfy $\nabla^2\psi = 0$ in the fluid, $\psi \to 0$ as $y \to \infty$ and $\partial\psi/\partial y + K\psi = 0$ on $y = 0$ where $K = \omega^2/g$, $\omega$ being the angular frequency and $g$ gravitational acceleration.

The radiation potentials $\phi_j^w$ also satisfy

$$\frac{\partial \phi_j^w}{\partial n} = n_j, \quad (x, y) \in S_B \quad (j = 1, 2, 3) \quad (1)$$

where $S_B$ is the wetted section of the floating body. In the case of sway and heave, $n_j$ are the direction cosines in the $x$ ($j = 1$) and $y$ ($j = 2$) directions of the unit normal directed into the cylinder from the fluid. In the case of roll ($j = 3$) we have $n_3 = xn_2 + (y - c)n_1$ where $(0, c)$ is the point of roll.

As $x \to \infty$,

$$\phi_j^w(x, y) \sim A_j^w \mathrm{e}^{\mathrm{i}Kx - Ky}, \quad (j = 1, 2, 3) \quad (2)$$

in which the far-field radiated wave amplitude $A_j^w$ is to be determined. Other quantities of interest are the added inertia and radiation damping coefficients $a^w_{jk}$ and $b^w_{jk}$ which define the real and imaginary parts of the complex time-independent restoring force matrix $f^w_{jk}$ representing the hydrodynamic force in the component $k$ due to a forcing in mode $j$, defined by

$$f^w_{jk} \equiv -b^w_{jk} + \mathrm{i}\omega a^w_{jk} = \mathrm{i}\rho\omega \int_{S_B} \phi^w_j n_k \mathrm{d}s. \quad (3)$$

It is easy to show that $f^w_{jk} = f^w_{kj}$ and also that

$$b^w_{jk} = \tfrac{1}{2}\rho\omega A^w_j \bar{A}^w_k, \quad (j,k = 1,2,3) \quad (4)$$

which are all real and it follows that $b^w_{jk}b^w_{kj} = b^w_{jj}b^w_{kk}$, $(j,k = 1,2,3)$.

One final quantity of interest is the exciting force on a *fixed* cylinder in direction $j$ (see Mei (1983, p.302)) induced by waves of amplitude $A$ incident from $x = +\infty$ which is

$$f^w_{S,j} = \mathrm{i}\rho\omega \int_{S_B} \phi^w_S n_j ds \quad (5)$$

where $\phi^w_S$ is the scattered potential in the presence of a wall with $\partial\phi^w_S/\partial n = 0$ on $S_B$. Here, it can be shown that

$$f^w_{S,j} = \rho g A A^w_j, \quad (j = 1,2,3). \quad (6)$$

## 3. Wide-spacing approximation

The overall effect of the wall will be equivalent to a radiated wave field travelling away from the cylinder in the absence of the wall, together with an incident wave of unknown amplitude ($D_j$, say) from the left being scattered by the fixed cylinder (assuming the wall is far enough away from the cylinder). Thus

$$\phi^w_j = \phi_j + D_j\phi_S, \qquad (j = 1,2,3), \quad (7)$$

where $\phi_j$ is the radiation potential for a cylinder making sway, heave or roll ($j = 1, 2$ or $3$) motions at the origin but in the absence of the wall and $\phi_S$ is the scattered potential due to a wave incident from $x = -\infty$ on the cylinder held fixed at the origin, again in the absence of the wall.

We have the far-field expressions for each of the potentials in (7) given by

$$\phi_j \sim \begin{cases} (-1)^j A_j \mathrm{e}^{-\mathrm{i}Kx-Ky}, & x \to -\infty \\ A_j \mathrm{e}^{\mathrm{i}Kx-Ky}, & x \to +\infty \end{cases} \quad (8)$$

where $A_j$ are the far-field radiated wave amplitudes (left-right symmetry of the cylinder is assumed for simplicity) and

$$\phi_S \sim \begin{cases} \dfrac{gA}{\omega}(\mathrm{e}^{\mathrm{i}Kx} + R\mathrm{e}^{-\mathrm{i}Kx})\mathrm{e}^{-Ky}, & x \to -\infty \\ \dfrac{gA}{\omega}T\mathrm{e}^{\mathrm{i}Kx-Ky}, & x \to +\infty \end{cases} \quad (9)$$

where $R$ and $T$ are the reflection and transmission coefficients for the fixed cylinder, in the absence of the wall, due to an incident wave of amplitude $A$.

It is assumed that $A_j$, $R$ and $T$ are all known, in addition to $a_{jk}$, $b_{jk}$, the added inertia and radiation damping coefficients in mode $k$ due to forced motion in mode $j$, defined as in (3) but without the $w$ superscript.

It follows from (7) that for large positive $x$

$$\phi^w_j \sim (A_j + (gA/\omega)D_jT)\mathrm{e}^{\mathrm{i}Kx-Ky} \quad (10)$$

and for large negative $x$

$$\phi^w_j \sim \big((-1)^j A_j + (gA/\omega)D_jR\big)\,\mathrm{e}^{-\mathrm{i}Kx-Ky} + (gA/\omega)D_j\mathrm{e}^{\mathrm{i}Kx-Ky}. \quad (11)$$

These asymptotic forms are now assumed to hold near the wall along $x = -b$, $y > 0$ where a Neumann condition is now imposed on the potential $\phi^w_j$. It follows that

$$(-1)^j A_j + (gA/\omega)D_jR = (gA/\omega)D_j\mathrm{e}^{-\mathrm{i}\lambda} \quad (12)$$

with $\lambda = 2Kb$ whence

$$D_j = (\omega/gA)(-1)^j A_j/(\mathrm{e}^{-\mathrm{i}\lambda} - R). \quad (13)$$

Substituting (13) into (10) and comparing with (2) gives

$$A^w_j = \delta_j A_j, \quad (14)$$

where

$$\delta_j = \left(\frac{R - (-1)^j T - \mathrm{e}^{-\mathrm{i}\lambda}}{R - \mathrm{e}^{-\mathrm{i}\lambda}}\right). \tag{15}$$

Note that this implies $\delta_1 = \delta_3$. According to the decomposition made in (7), the restoring force matrix, from (3), is approximated under the wide-spacing approximation by

$$f^w_{jk} \equiv -b^w_{jk} + \mathrm{i}\omega a^w_{jk} = f_{jk} + D_j f_{S,k} \tag{16}$$

where $f_{S,k} = \rho g(-1)^k A A_k$.

Notice that (16) only holds provided $j+k$ is *even* since if $j+k$ is odd, then the symmetry of the cylinder implies that the term $f_{jk}$ is identically zero (for example, a heave motion induces neither sway force nor roll moment on a symmetric cylinder) and (16) is replaced with

$$f^w_{jk} = D_j f_{S,k}. \tag{17}$$

Essentially (16) and (17) define, via (13), the wide-spacing approximations to the added inertia and radiation damping for a cylinder in the presence of a wall in terms of the solution to the wave radiation and scattering by a cylinder in isolation. Additionally (14), via (15), defines the far-field radiated wave amplitudes.

However, we can make further progress by manipulating the equations (16) and (17) that define $a^w_{jk}$, $b^w_{jk}$ using relations such as

$$b_{jk} = \tfrac{1}{2}\rho\omega(1 + (-1)^{j+k}) A_j \bar{A}_k \tag{18}$$

(clearly zero if $j+k$ is odd) and the Newman/Bessho relations (see Mei (1983), p.328)

$$R + (-1)^j T = -A_j/\bar{A}_j = -\mathrm{e}^{2\mathrm{i}\theta_j}, \tag{19}$$

where $\theta_j$ is the phase of the far-field radiated amplitude in the $j$th mode (note $\theta_1 = \theta_3$).

We omit the details here and summarise below the simplified forms of the approximations to $a^w_{jk}$, $b^w_{jk}$.

For $j+k$ odd, we obtain,

$$b^w_{jk} = \frac{2(b_{jj}b_{kk})^{\frac{1}{2}} \cos\mu_j \cos\mu_k}{|\mathrm{e}^{-\mathrm{i}\lambda} - R|^2} \tag{20}$$

and

$$\omega a^w_{jk} = \frac{-(b_{jj}b_{kk})^{\frac{1}{2}} \sin(\mu_j + \mu_k)}{|\mathrm{e}^{-\mathrm{i}\lambda} - R|^2} \tag{21}$$

where

$$\mu_j = \theta_j + Kb. \tag{22}$$

For $j+k$ even,

$$b^w_{jk} = \frac{2b_{jk}\cos^2\mu_l}{|\mathrm{e}^{-\mathrm{i}\lambda} - R|^2} \tag{23}$$

where $l$ is either $k+1$ or $k-1$ provided that number falls in the set $\{1,2,3\}$ whilst

$$a^w_{jk} = a_{jk} - (\beta_k/\omega) b_{jk} \tag{24}$$

where

$$\beta_k = \frac{\frac{1}{2}(-1)^k \sin 2(\theta_l - \theta_2) - \sin 2\mu_k}{|\mathrm{e}^{-\mathrm{i}\lambda} - R|^2} \tag{25}$$

and here $l$ is either 1 or 3.

## 4. Results

We show, in figures 1 and 2, two sets of results for the non-dimensional[1] added inertia and radiation damping coefficients, varying with non-dimensional frequency. In each set of figures, the solid lines correspond to exact calculations including the wall and the points are calculated using the wide-spacing approximation (20), (21), (23), (24).

In figure 1, results are shown for a semi-immersed circular cylinder of radius $a$, whose centre is a distance $b = 2a$ from the wall. In this example, there is no roll component. As expected, the wide-spacing approximation performs better as $Ka$ increases, but still does remarkably well as $Ka \to 0$.

In figure 2, results are shown for a floating rectangular cylinder of width $2a$, draught $d = 2a$ centred a distance $b = 4a$ from the wall. The fluid is now of finite depth $h$

[1] $\mu^w_{jk} = a^w_{jk}/M$, $\nu^w_{jk} = b^w_{jk}/(\omega M)$ for $j,k = 1,2$ where $M$ is the mass of the cylinder, determined by Archimedes' principle. Also, $\mu^w_{33} = a^w_{33}/I$, $\nu^w_{33} = b^w_{33}/(\omega I)$ and $\mu^w_{j3} = a^w_{j3}/\sqrt{MI}$, $\nu^w_{j3} = b^w_{j3}/(\omega\sqrt{MI})$, $j = 1,2$ where $I$ is a moment of inertia about $(0,c)$

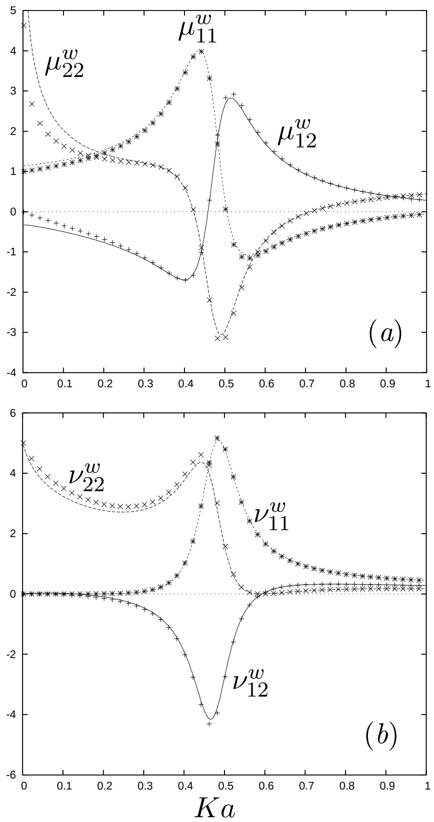


Figure 1: Variation of non-dimensional $(a)$ added mass and $(b)$ radiation damping coefficients for a circular cylinder with $Ka$.

$(= 5d)$, the wavenumber $k$ determined from $K = k \tanh kh$. Again the wide-spacing results, compared to exact calculations are in excellent agreement across the range of frequencies.

## 5. References


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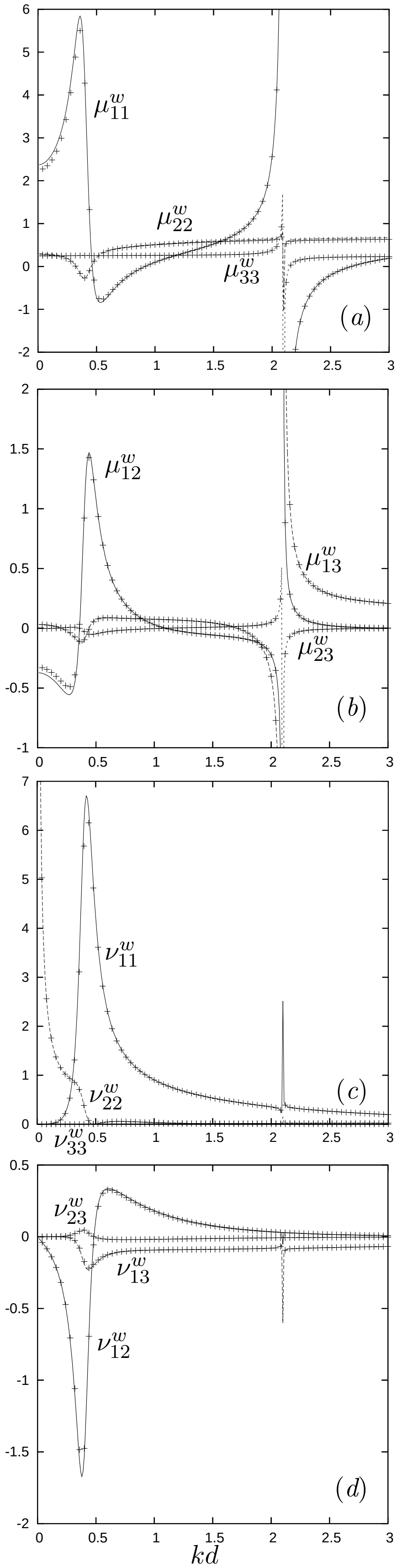


Figure 2: Variation of non-dimensional $(a,b)$ added inertia and $(c,d)$ radiation damping coefficients for a rectangular cylinder with $kd$.