

Estimation of wall effects on floating cylinders

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1. Introduction

Under the assumptions of classical linearised water wave theory, the time-harmonic two-dimensional motion of a rigid body floating in the surface of a fluid may be characterised by various coefficients which express components of the hydrodynamic forces acting on that body.

For a single body in isolation these hydrodynamic coefficients are relatively simple to calculate. For bodies placed next to a vertical wall, the corresponding calculations are often much more complicated. Moreover, it is well-known that the behaviour of a floating body in waves is radically affected by the proximity of a rigid boundary such as a harbour wall, and this effect is manifested in major changes to the hydrodynamic coefficients. This is due to the near-resonant excitation, close to certain frequencies, of waves trapped between the cylinder and the wall. Examples of the results obtained for bodies oscillating next to walls are given in Wang & Wahab (1971) and Yeung & Seah (2007) where these resonances are identified with large rapid variations in hydrodynamic coefficients.

In this paper we use the well-known wide-spacing approximation (see Martin (2006) for example) to develop approximations to the hydrodynamic coefficients for a body next to a wall solely in terms of the results for the forced motion the same body in the absence of a wall.

Exact results are compared with the wide-spacing approximations for semi-immersed circular cylinders and cylinders of rectangular cross-section and show excellent agreement.

2. Formulation

A two-dimensional cylinder is taken to be floating in the surface of a fluid of density ρ and infinite depth. Cartesian coordinates are chosen with the origin in the mean free surface and y pointing vertically downwards.

We consider the time-harmonic small-amplitude forced sway, heave or roll ($j = 1, 2$ or 3 respectively) motion of the cylinder centred at the origin in the presence of a rigid wall at $x = -b$, $y > 0$ on which a Neumann condition is imposed on the potential, $\phi_j^w(x, y)$. (The superscript w identifies quantities associated with the wall). Linearised water wave theory is used, in which potentials ψ (say) satisfy $\nabla^2\psi = 0$ in the fluid, $\psi \rightarrow 0$ as $y \rightarrow \infty$ and $\partial\psi/\partial y + K\psi = 0$ on $y = 0$ where $K = \omega^2/g$, ω being the angular frequency and g gravitational acceleration.

The radiation potentials ϕ_j^w also satisfy

$$\frac{\partial\phi_j^w}{\partial n} = n_j, \quad (x, y) \in S_B \quad (j = 1, 2, 3) \quad (1)$$

where S_B is the wetted section of the floating body. In the case of sway and heave, n_j are the direction cosines in the x ($j = 1$) and y ($j = 2$) directions of the unit normal directed into the cylinder from the fluid. In the case of roll ($j = 3$) we have $n_3 = xn_2 + (y - c)n_1$ where $(0, c)$ is the point of roll.

As $x \rightarrow \infty$,

$$\phi_j^w(x, y) \sim A_j^w e^{iKx - Ky}, \quad (j = 1, 2, 3) \quad (2)$$

in which the far-field radiated wave amplitude A_j^w is to be determined. Other quantities of interest are the added inertia and

radiation damping coefficients a_{jk}^w and b_{jk}^w which define the real and imaginary parts of the complex time-independent restoring force matrix f_{jk}^w representing the hydrodynamic force in the component k due to a forcing in mode j , defined by

$$f_{jk}^w \equiv -b_{jk}^w + i\omega a_{jk}^w = i\rho\omega \int_{S_B} \phi_j^w n_k ds. \quad (3)$$

It is easy to show that $f_{jk}^w = f_{kj}^w$ and also that

$$b_{jk}^w = \frac{1}{2}\rho\omega A_j \bar{A}_k^w, \quad (j, k = 1, 2, 3) \quad (4)$$

which are all real and it follows that $b_{jk}^w b_{kj}^w = b_{jj}^w b_{kk}^w$, ($j, k = 1, 2, 3$).

One final quantity of interest is the exciting force on a *fixed* cylinder in direction j (see Mei (1983, p.302)) induced by waves of amplitude A incident from $x = +\infty$ which is

$$f_{S,j}^w = i\rho\omega \int_{S_B} \phi_S^w n_j ds \quad (5)$$

where ϕ_S^w is the scattered potential in the presence of a wall with $\partial\phi_S^w/\partial n = 0$ on S_B . Here, it can be shown that

$$f_{S,j}^w = \rho g A A_j^w, \quad (j = 1, 2, 3). \quad (6)$$

3. Wide-spacing approximation

The overall effect of the wall will be equivalent to a radiated wave field travelling away from the cylinder in the absence of the wall, together with an incident wave of unknown amplitude (D_j , say) from the left being scattered by the fixed cylinder (assuming the wall is far enough away from the cylinder). Thus

$$\phi_j^w = \phi_j + D_j \phi_S, \quad (j = 1, 2, 3), \quad (7)$$

where ϕ_j is the radiation potential for a cylinder making sway, heave or roll ($j = 1, 2$ or 3) motions at the origin but in the absence of the wall and ϕ_S is the scattered potential due to a wave incident from $x = -\infty$ on the cylinder held fixed at the origin, again in the absence of the wall.

We have the far-field expressions for each of the potentials in (7) given by

$$\phi_j \sim \begin{cases} (-1)^j A_j e^{-iKx-Ky}, & x \rightarrow -\infty \\ A_j e^{iKx-Ky}, & x \rightarrow +\infty \end{cases} \quad (8)$$

where A_j are the far-field radiated wave amplitudes (left-right symmetry of the cylinder is assumed for simplicity) and

$$\phi_S \sim \begin{cases} \frac{gA}{\omega} (e^{iKx} + R e^{-iKx}) e^{-Ky}, & x \rightarrow -\infty \\ \frac{gA}{\omega} T e^{iKx-Ky}, & x \rightarrow +\infty \end{cases} \quad (9)$$

where R and T are the reflection and transmission coefficients for the fixed cylinder, in the absence of the wall, due to an incident wave of amplitude A .

It is assumed that A_j , R and T are all known, in addition to a_{jk} , b_{jk} , the added inertia and radiation damping coefficients in mode k due to forced motion in mode j , defined as in (3) but without the w superscript.

It follows from (7) that for large positive x

$$\phi_j^w \sim (A_j + (gA/\omega) D_j T) e^{iKx-Ky} \quad (10)$$

and for large negative x

$$\phi_j^w \sim ((-1)^j A_j + (gA/\omega) D_j R) e^{-iKx-Ky} + (gA/\omega) D_j e^{iKx-Ky}. \quad (11)$$

These asymptotic forms are now assumed to hold near the wall along $x = -b$, $y > 0$ where a Neumann condition is now imposed on the potential ϕ_j^w . It follows that

$$(-1)^j A_j + (gA/\omega) D_j R = (gA/\omega) D_j e^{-i\lambda} \quad (12)$$

with $\lambda = 2Kb$ whence

$$D_j = (\omega/gA) (-1)^j A_j / (e^{-i\lambda} - R). \quad (13)$$

Substituting (13) into (10) and comparing with (2) gives

$$A_j^w = \delta_j A_j, \quad (14)$$

where

$$\delta_j = \left(\frac{R - (-1)^j T - e^{-i\lambda}}{R - e^{-i\lambda}} \right). \quad (15)$$

Note that this implies $\delta_1 = \delta_3$. According to the decomposition made in (7), the restoring force matrix, from (3), is approximated under the wide-spacing approximation by

$$f_{jk}^w \equiv -b_{jk}^w + i\omega a_{jk}^w = f_{jk} + D_j f_{S,k} \quad (16)$$

where $f_{S,k} = \rho g (-1)^k A A_k$.

Notice that (16) only holds provided $j+k$ is *even* since if $j+k$ is odd, then the symmetry of the cylinder implies that the term f_{jk} is identically zero (for example, a heave motion induces neither sway force nor roll moment on a symmetric cylinder) and (16) is replaced with

$$f_{jk}^w = D_j f_{S,k}. \quad (17)$$

Essentially (16) and (17) define, via (13), the wide-spacing approximations to the added inertia and radiation damping for a cylinder in the presence of a wall in terms of the solution to the wave radiation and scattering by a cylinder in isolation. Additionally (14), via (15), defines the far-field radiated wave amplitudes.

However, we can make further progress by manipulating the equations (16) and (17) that define a_{jk}^w , b_{jk}^w using relations such as

$$b_{jk} = \frac{1}{2} \rho \omega (1 + (-1)^{j+k}) A_j \bar{A}_k \quad (18)$$

(clearly zero if $j+k$ is odd) and the Newman/Bessho relations (see Mei (1983), p.328)

$$R + (-1)^j T = -A_j / \bar{A}_j = -e^{2i\theta_j}, \quad (19)$$

where θ_j is the phase of the far-field radiated amplitude in the j th mode (note $\theta_1 = \theta_3$).

We omit the details here and summarise below the simplified forms of the approximations to a_{jk}^w , b_{jk}^w .

For $j+k$ odd, we obtain,

$$b_{jk}^w = \frac{2(b_{jj}b_{kk})^{\frac{1}{2}} \cos \mu_j \cos \mu_k}{|e^{-i\lambda} - R|^2} \quad (20)$$

and

$$\omega a_{jk}^w = \frac{-(b_{jj}b_{kk})^{\frac{1}{2}} \sin(\mu_j + \mu_k)}{|e^{-i\lambda} - R|^2} \quad (21)$$

where

$$\mu_j = \theta_j + Kb. \quad (22)$$

For $j+k$ even,

$$b_{jk}^w = \frac{2b_{jk} \cos^2 \mu_l}{|e^{-i\lambda} - R|^2} \quad (23)$$

where l is either $k+1$ or $k-1$ provided that number falls in the set $\{1, 2, 3\}$ whilst

$$a_{jk}^w = a_{jk} - (\beta_k / \omega) b_{jk} \quad (24)$$

where

$$\beta_k = \frac{\frac{1}{2}(-1)^k \sin 2(\theta_l - \theta_2) - \sin 2\mu_k}{|e^{-i\lambda} - R|^2} \quad (25)$$

and here l is either 1 or 3.

4. Results

We show, in figures 1 and 2, two sets of results for the non-dimensional¹ added inertia and radiation damping coefficients, varying with non-dimensional frequency. In each set of figures, the solid lines correspond to exact calculations including the wall and the points are calculated using the wide-spacing approximation (20), (21), (23), (24).

In figure 1, results are shown for a semi-immersed circular cylinder of radius a , whose centre is a distance $b = 2a$ from the wall. In this example, there is no roll component. As expected, the wide-spacing approximation performs better as Ka increases, but still does remarkably well as $Ka \rightarrow 0$.

In figure 2, results are shown for a floating rectangular cylinder of width $2a$, draught $d = 2a$ centred a distance $b = 4a$ from the wall. The fluid is now of finite depth h

¹ $\mu_{jk}^w = a_{jk}^w / M$, $\nu_{jk}^w = b_{jk}^w / (\omega M)$ for $j, k = 1, 2$ where M is the mass of the cylinder, determined by Archimedes' principle. Also, $\mu_{33}^w = a_{33}^w / I$, $\nu_{33}^w = b_{33}^w / (\omega I)$ and $\mu_{j3}^w = a_{j3}^w / \sqrt{MI}$, $\nu_{j3}^w = b_{j3}^w / (\omega \sqrt{MI})$, $j = 1, 2$ where I is a moment of inertia about $(0, c)$

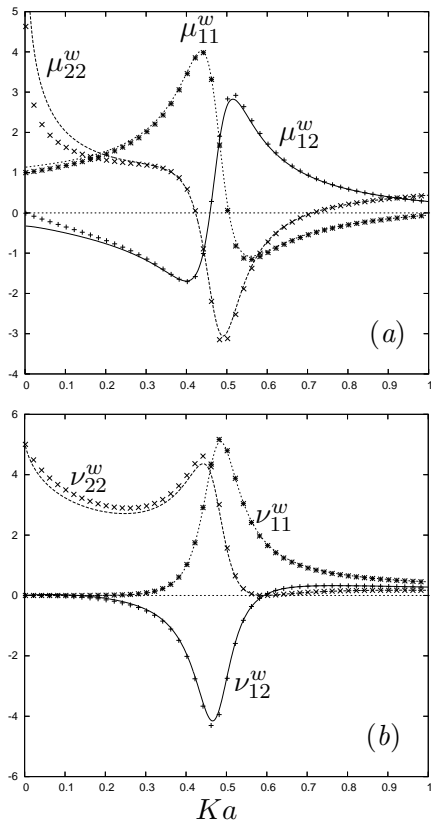


Figure 1: Variation of non-dimensional (a) added mass and (b) radiation damping coefficients for a circular cylinder with Ka .

($= 5d$), the wavenumber k determined from $K = k \tanh kh$. Again the wide-spacing results, compared to exact calculations are in excellent agreement across the range of frequencies.

5. References

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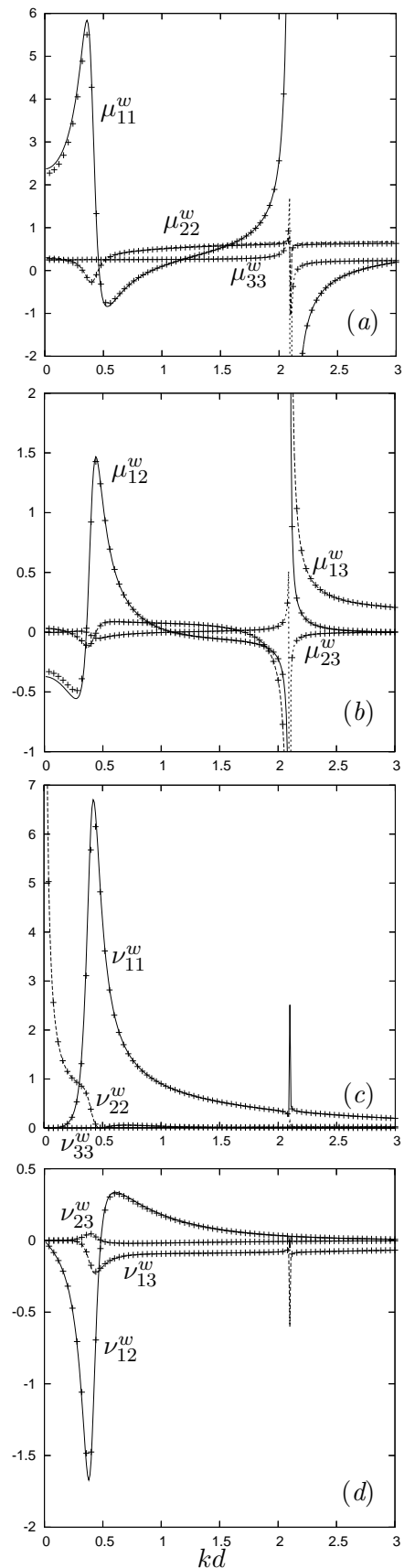


Figure 2: Variation of non-dimensional (a,b) added inertia and (c,d) radiation damping coefficients for a rectangular cylinder with kd .