Bow waves of a family of fine ruled ship hulls with rake and flare

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Introduction

A broad class of ship hulls, including the classical Wigley parabolic hull and the Series 60 ship model, have bows that resemble a wedge. This simple class of bows is characterized by only two parameters: the draft D and the waterline entrance angle 2α, as shown in Fig.1. The bow wave due to such a wedge-shaped bow also depends on two parameters: the draft-based Froude number $F = V_s/\sqrt{gD}$, where V_s stands for the speed of the ship (assumed to steadily advance along a straight path in calm water of infinite depth and lateral extent) and g is the acceleration of gravity, and the waterline entrance angle 2α.

The bow waves generated by the two-parameter family of wedged-shaped ship bows depicted in Fig.1 are considered in [1-3] where several simple relations are given. In particular, expressions that define the height of the bow wave, the distance between the ship stem and the crest of the wave, the rise of water at the stem, and the bow wave profile are given in these three previous studies. The comparisons between the analytical relations and experimental measurements reported in [1-4] show that, in spite of their remarkable simplicity, these relations are sufficiently accurate for practical design applications.

However, the practical usefulness of these relations is restricted by the fact that the two-parameter family of ship bows shown in Fig.1 is inadequate for many applications. Indeed, many ship bows, notably bows of fast ships, have significant rake and flare. Rake and flare must then be taken into account to design a ship bow. The wave created by a ship bow with rake and flare is considered in [5]. There, thin-ship theory is used to extend the relations for the height of the bow wave and the distance between the ship stem and the wave crest previously obtained for wedge-shaped ship bows, and a parametric study of the variations of the bow-wave height and location with respect to the hull speed, draft, rake, and flare is reported. The thin-ship analysis given in [5] is applied further in [6] to systematically investigate the influence of the hull speed, draft, rake, and flare on the bow-wave profile, the rise of water at the hull stem, and the bow-wave length, not considered in [1-5]. The extensive (unpublished) parametric study presented in [6] (recently submitted) and [5] (in press) is briefly summarized here.

Following [5], we consider a family of ruled ship bows defined by four parameters, as shown in Fig.2: the draft D of the ship bow, the entrance angle 2α at the top waterline (at the free surface), the rake angle δ (angle between the ship stem and the vertical) and the hull flare, controlled by $\alpha - \alpha'$. Here, $2\alpha'$ is the entrance angle at the bottom waterline (at the ship draft). The parameter φ defined as

$$-1 \le \varphi \equiv \frac{T-T'}{T+T'} \le 1 \quad \text{with} \quad \left\{ \begin{array}{l} T \equiv \tan\alpha \\ T' \equiv \tan\alpha' \end{array} \right\} \qquad (1)$$

is closely related to the hull flare, and is called flare parameter hereinafter (even though the rake angle δ also affects the hull flare). The special case $\varphi = 0$ corresponds to $\alpha' = \alpha$ and rectangular sections, and the special cases $\varphi = 1$ or -1 correspond to triangular sections with $\alpha' = 0$ or $\alpha = 0$, respectively. The flow (notably the bow wave, of particular interest here) due to

this four-parameter family of ship bows depends on four parameters: $F \equiv V_s/\sqrt{gD}$, α, α' and δ. The two-parameter family of wedge-shaped ship bows depicted in Fig.1 corresponds to the special case $\alpha' = \alpha$ and $\delta = 0$.

The four-parameter family of simple ruled ship bows depicted in Fig.2 is considered for two main reasons: (i) the four parameters F, α, α' and δ are major parameters that have a dominant influence on a ship bow wave, and (ii) the limited number of parameters that define this family of ship bows makes it feasible to perform an extensive parametric study, and to obtain results immediately applicable to ship design. A more general family of ship bows that accounts for the hull curvature would involve a significantly greater number of parameters, for which a systematic parametric study would be problematic. Thus, the four-parameter family of ship bows depicted in Fig.2 is sufficiently general to account for the dominant geometric characteristics of a large class of ship bows, and is sufficiently simple to allow an extensive parametric investigation.

Many alternative methods for evaluating steady free-surface flow about ships have been considered. These methods include semi-analytical theories based on alternative approximations (thin-ship, slender-ship, 2d+t theories), potential-flow (boundary integral equation) methods that rely on the use of a Green function (elementary Rankine source, or Havelock source that satisfies the radiation condition and the Michell linear free-surface boundary condition), and CFD methods that solve the Euler or RANS equations. In principle, any of these alternative methods can be used to evaluate steady flow about the four-parameter family of ship bows considered here. In practice however, most of the existing methods are ill suited for the systematic parametric studies required for our practical goal of obtaining simple analytical relations immediately useful for design. In fact, selection of a calculation method suited for systematic parametric studies or for early design present similar issues, which involve consideration of a tradeoff between competing requirements with respect to accuracy and practicality. Indeed, practical tools that are simple to use and highly efficient, but need not be highly accurate, are required to quickly evaluate the large number of alternative designs that typically need to be considered for concept and preliminary design. On the other hand, detail design, and especially design evaluation, involve many fewer choices and require more accurate computational tools, for which efficiency and ease of use are less important considerations.
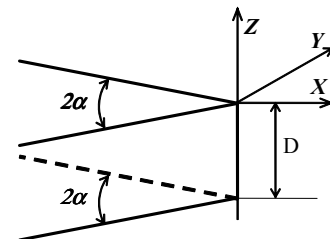

Fig. 1 Two-parameter family of wedge-shaped ship bows defined by the draft D and the waterline entrance angle 2α.
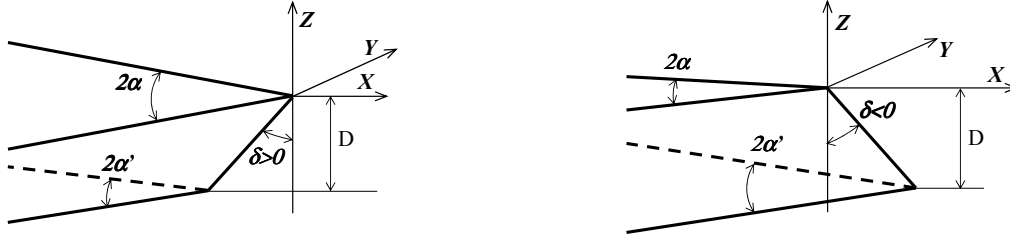
Fig. 2 Four-parameter family of ruled ship bows defined by the draft D, the rake angle δ ($0 < \delta$ on left side, $\delta < 0$ on right side), the top-waterline entrance angle 2α and the bottom-waterline entrance angle $2\alpha'$.
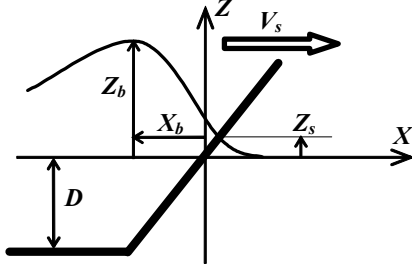

Fig. 3 Definition sketch for the bow wave height Z_b, the distance $-X_b$ between the bow wave crest and the ship stem, and the rise of water Z_s at the ship stem.

Thin-ship theory is used here, as in [5], because this theory is reasonably well suited for the class of fine bows under consideration, and because it significantly simplifies our parametric studies. Specifically, the flow about the four-parameter family of ship bows considered here can be expressed as the product of the factor $(T + T')/2$, which essentially represents an average waterline entrance angle, by a function that depends on three (instead of four) parameters: the (draft-based) Froude number F, the rake angle δ, and the flare parameter φ. In fact, this function of F, δ, φ can be expressed in terms of two functions that only depend on the two parameters F and δ; see [5].

As shown in Fig.1 and Fig.2, the Z axis is vertical and points upward, and the mean free surface is taken as the plane $Z = 0$. Furthermore, the X axis is along the path of the ship and points toward the ship bow, and the intersection of the stem line with the mean free-surface plane $z = 0$ is taken as the origin $X = 0$, as shown in Fig.3. Nondimensional coordinates $\mathbf{x} \equiv \mathbf{X}\, g/V_s^2$ are used hereinafter. As shown in Fig.3,

$$z_b \equiv \frac{Z_b\, g}{V_s^2} \qquad -x_b \equiv \frac{-X_b\, g}{V_s^2} \qquad z_s \equiv \frac{Z_s\, g}{V_s^2} \qquad (2)$$

stand for the (nondimensional) bow-wave height (the elevation of the bow wave crest above the mean free surface), the distance between the bow wave crest and the intersection of the ship stem with the mean free surface, and the height of water (above the mean free surface) at the ship stem, respectively.

Wedge-shaped ship bows without rake and flare

The bow waves generated by the two-parameter family of wedged-shaped ship bows depicted in Fig.1 are considered in [1-3] where the following simple relations

$$z_b \approx \frac{2.2\,T}{1+F} \qquad -x_b \approx \frac{1.1}{1+F} \qquad z_s \approx \frac{2}{\pi}\,\frac{E^s\,T}{1+F^2} \qquad (3)$$

are given. Here, $T \equiv \tan\alpha$ in accordance with (1), F is the draft-based Froude number, the approximation $\cos\alpha \approx 1$ was used because only small values of α are now considered, and

$E^s \equiv E^s(F)$ is the function

$$E^s(F) \approx 1 + \frac{2/3}{1+F^2} + \frac{19/45}{(1+F^2)^2} + \frac{26/105}{(1+F^2)^3} + \frac{601/4725}{(1+F^2)^4}$$
$$+ \frac{1502/31185}{(1+F^2)^5} + 4.16\,(1+F^2)\,e^{-13F-0.26}\,. \qquad (4)$$

These relations are shown in [1-3] to be in fair agreement with experimental measurements for both wedge-shaped ship bows with entrance angle 2α and a rectangular flat plate at a yaw (incidence) angle α.

Ruled ship bows with rake and flare

Expressions (3) for the bow-wave height z_b and location x_b are extended in [5] to the more general four-parameter family of ruled ship bows with rake and flare depicted in Fig.2. The comparisons between experimental measurements and corresponding theoretical predictions given by thin-ship theory reported in that study show that the use of this simple theory to extend the relations given in [1,3] to the more general case $\delta \neq 0$ and $\varphi \neq 0$ is appropriate for small values of α and α', i.e. for a broad class of ships with fine bows.

The thin-ship analysis given in [5] yields the relations

$$z_b \approx \frac{1.1\,\zeta_b}{1+F}\,(T+T') \qquad -x_b \approx \frac{1.1\,\xi_b}{1+F} \qquad (5)$$

where $\zeta_b \equiv \zeta_b(F, \delta, \varphi)$ and $\xi_b \equiv \xi_b(F, \delta, \varphi)$ are functions of the draft-based Froude number F, the rake angle δ, and the flare parameter φ defined by (1). The two functions $\zeta_b(F, \delta, \varphi)$ and $\xi_b(F, \delta, \varphi)$ are depicted in [5] where the values of ζ_b and ξ_b are also listed for six values of F that correspond to $F/(1+F) = 0.3, 0.4, \ldots, 0.8$, nine rake angles $\delta = 60°, 45°, \ldots, -60°$, and nine values of the flare parameter $\varphi = 1, 0.75, \ldots, -1$. The range $0.3 \leq F/(1+F) \leq 0.8$ corresponds to $0.43 \leq F \leq 4$ and — for a ship with typical length/draft ratio $L_s/D \approx 20$ — to length-based Froude numbers in the range $0.1 \leq F_L \leq 0.9$, which encompasses most cases of practical interest.

The relations (5) are further considered in [6]. (5) yield

$$z_b' \equiv \frac{Z_b/D}{T+T'} \approx \frac{1.1\,F^2\zeta_b}{1+F} \qquad x_b' \equiv \frac{-X_b}{D} \approx \frac{1.1\,F^2\xi_b}{1+F}\,. \qquad (6)$$

For a given value of $T + T' \equiv \tan\alpha + \tan\alpha'$, i.e. for a given ship bow, the functions z_b' and x_b' characterize the size of the bow wave with respect to the ship draft D; and the variations of these functions with respect to the Froude number F illustrate the growth of the bow wave with respect to the ship speed V_s. In particular, (6) show that both z_b' and x_b' are $O(F^2)$ as $F \to 0$ and $O(F)$ as $F \to \infty$. Thus, ship bow waves vanish like V_s^2 in the low-speed limit $V_s \to 0$, and grow like V_s in the high-speed limit $V_s \to \infty$, as shown in [1,3]

The functions z_b' and x_b' are depicted in Fig.4 and Fig.5, respectively, for $0 \leq F \leq 4$. The top and bottom rows in these figures show the functions z_b' or x_b' for (top rows) three rake
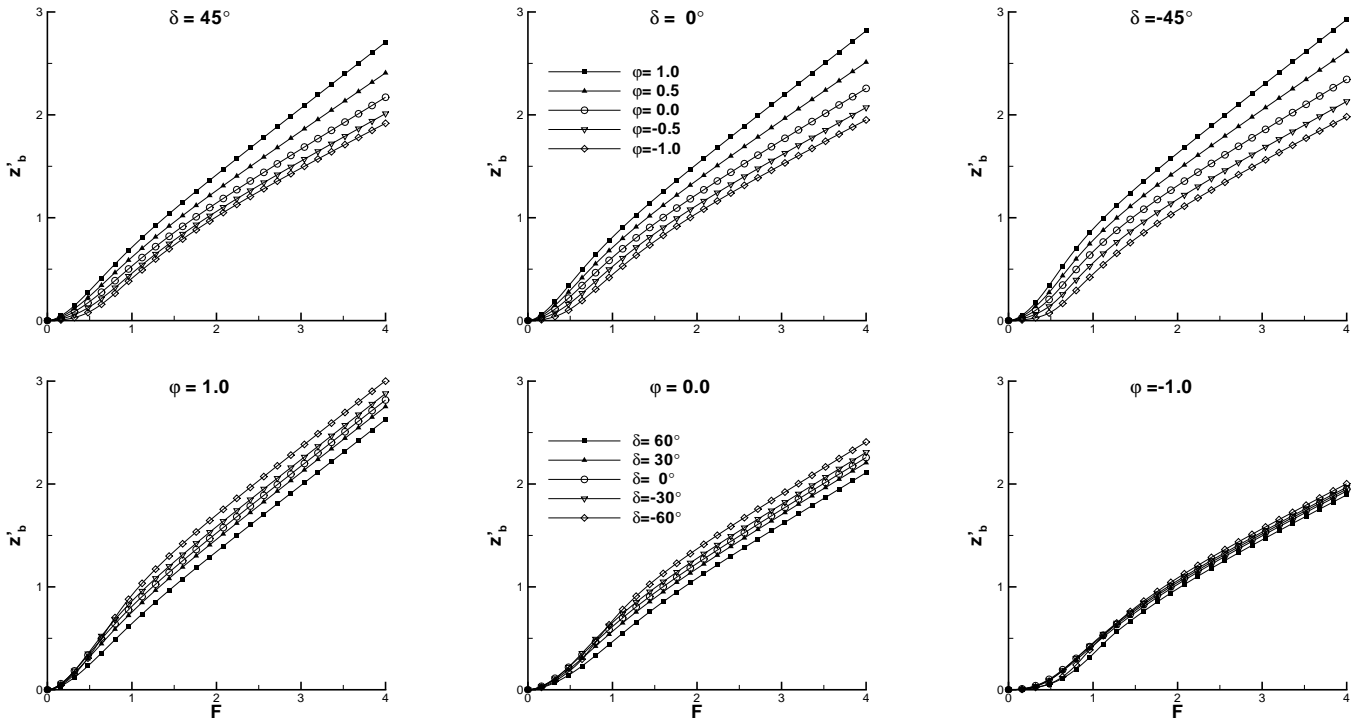
Fig. 4 Variation of the bow-wave height z'_b defined by (6) with respect to the draft-based Froude number F for (top row) three rake angles $\delta = 45°$ (left column), $\delta = 0$ (center) and $\delta = -45°$ (right), and (bottom row) three values of the flare parameter $\varphi = 1$ (left), $\varphi = 0$ (center) and $\varphi = -1$ (right).

Fig. 5 Variation of the bow-wave-crest location x'_b defined by (6) with respect to the draft-based Froude number F for (top row) three rake angles $\delta = 45°$ (left column), $\delta = 0$ (center) and $\delta = -45°$ (right), and (bottom row) three values of the flare parameter $\varphi = 1$ (left), $\varphi = 0$ (center) and $\varphi = -1$ (right).
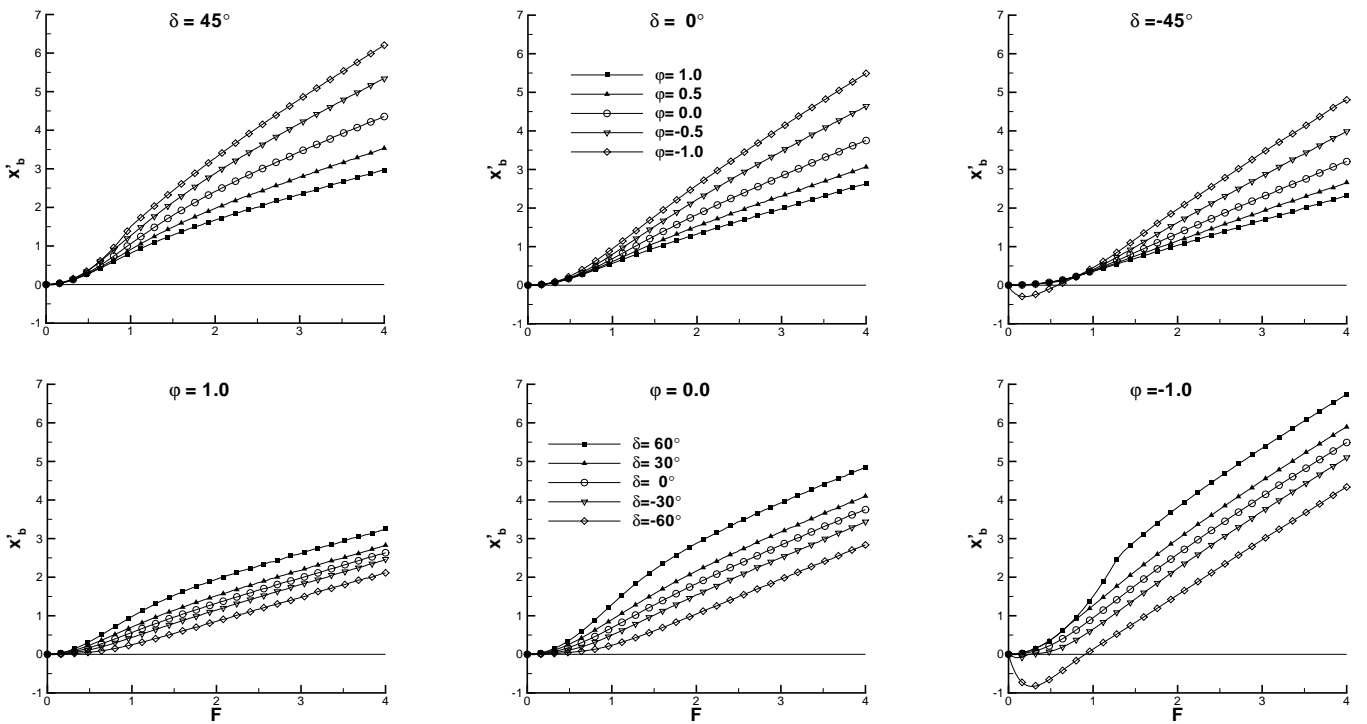
angles $\delta = 0$ and $\delta = \pm45°$, and for (bottom rows) three values $\varphi = 0$ and $\varphi = \pm1$ of the flare parameter φ. The curves in the figures in the top and bottom rows in Fig.4 and Fig.5 correspond to $\varphi = 0, \pm0.5, \pm1$ and to $\delta = 0, \pm30°, \pm60°$, respectively.

Fig.4 and Fig.5 show that the bow-wave height z'_b and the distance x'_b between the wave crest and the ship stem both in-crease monotonically, approximately linearly, with respect to F for $1.5 < F$. Thus — for a given hull — the size of the bow wave, as defined by the wave height z'_b and the bow-wave-crest location x'_b, increases approximately in proportion to the ship speed if F is large enough, specifically for draft-based Froude numbers greater than approximately 1.5. This behav-

ior, illustrated in Fig.4 and Fig.5, is in agreement with (6). For $F < 1.5$, Fig.4 shows that the bow-wave height z'_b increases monotonically with F but at a slower rate, also in agreement with (6). Fig.5 shows that the bow-wave-crest location x'_b also increases monotonically with respect to F for positive rake angles δ. However, for $\delta < 0$ and small values of F, x'_b can be negative, i.e. the wave crest can be ahead of the ship stem.

Fig.4 shows that the rake angle δ has smaller effects on the bow-wave height z'_b than the flare parameter φ. In particular, for $\varphi = -1$ (bottom right corner of Fig.4), the rake angle δ has a negligible influence on z'_b. Fig.4 shows that z'_b increases monotonically as the flare parameter φ increases from -1 to 1. Thus, larger values of the flare parameter φ, which corresponds to hull volume distributed higher (closer to the free surface), yield bigger bow waves as expected. Fig.5 shows that x'_b decreases monotonically as the flare parameter φ increases. Thus, larger values of the flare φ yield bigger bow waves with crests closer to the ship stem, and therefore steeper waves.

Fig.4 shows that the bow-wave height z'_b decreases slightly as the rake angle δ increases. In particular, z'_b is smaller for $0 < \delta$ than for $\delta < 0$. Fig.5 shows that the bow-wave location x'_b increases as the rake angle δ increases. In particular, x'_b is larger for $0 < \delta$ than for $\delta < 0$. Thus, slightly higher bow waves with crests closer to the ship stem, i.e. steeper waves, are obtained for negative rake than for positive rake.

In summary, for a given draft-based Froude number F, Fig.4 and Fig.5 show that bigger and steeper bow waves are obtained as the flare parameter φ increases and the rake angle δ decreases. This result suggests that a ship bow with $\alpha < \alpha'$, i.e. with $\varphi < 0$ (negative flare), and $0 < \delta$ (positive rake angle) may be advantageous.

Bow-wave profiles, rise of water at stem, bow-wave length

Corresponding parametric studies of the variations of the bow-wave profiles with respect to the draft-based Froude number F, the rake angle δ and the flare parameter φ, which characterize the four-parameter family of ship bows depicted in Fig.2, are reported in [6]. In particular, the analysis (based on thin-ship theory) of the rise of water z_s at the stem line $x = z \tan \delta$, reported in [2] for the family of wedge-shaped bows shown in Fig.1, is extended in [6] to the more general family of ship bows shown in Fig.2. This generalization of (3) yields

$$z_s \approx \frac{E^s \left(T + T'\right) \zeta_s}{\pi \left(1 + F^2\right)} \tag{7}$$

where $\zeta_s \equiv \zeta_s(F, \delta, \varphi)$ is a function of the draft-based Froude number F, the rake angle δ and the flare parameter φ. The variations of the function $\zeta_s(F, \delta, \varphi)$ with respect to F, δ and φ are depicted in [6] where ζ_s is also listed for six values of F that correspond to $F/(1+F) = 0.3, 0.4, \ldots, 0.8$, nine rake angles $\delta = 60°, 45°, \ldots, -60°$, and nine values of the flare parameter $\varphi = 1, 0.75, \ldots, -1$. The distance $-x_0$ between the ship stem and the first intersection $x = x_0$ of the bow wave with the mean free surface $z = 0$ is also considered in [6]. These additional results, reported in [6], will be presented at the Workshop.

Conclusion

In summary, the bow waves generated by a four-parameter family of fine nonbulbous ruled ship bows with rake and flare have been considered using thin-ship theory. Specifically, the variations of the bow-wave height and location, the bow-wave profile, the rise of water at the stem, and the bow-wave length with respect to the draft-based Froude number, the entrance angles at the top and bottom waterlines, and the rake angle have been explored via an extensive parametric study. This parametric study has resulted in simple relations, notably (5) and (7), a series of figures, and tables. These relations, figures and tables provide useful insight, and can be used to estimate the influence of main geometrical characteristics of a ship bow. In fact, the estimates provided by the relations, figures and tables given in [5,6] are immediately applicable to ship bow design, notably at early (concept and preliminary) design stages.

For a ship of draft D advancing at speed V_s, the height Z_b of the bow wave, the distance $-X_b$ between the bow-wave crest and the ship stem, the height of water Z_s at the stem, and the distance $-X_0$ (called bow-wave length here) between the stem and the first intersection of the bow wave with the mean free surface behave as

$$\frac{Z_b}{D} \propto V_s^2 \quad \frac{-X_b}{D} \propto V_s^2 \quad \frac{Z_s}{D} \propto V_s^2 \quad \frac{-X_0}{D} = O(1)$$

in the low-speed limit $V_s \to 0$, and as

$$\frac{Z_b}{D} \propto V_s \quad \frac{-X_b}{D} \propto V_s \quad \frac{Z_s}{D} = O(1) \quad \frac{-X_0}{D} \propto V_s^2$$

in the high-speed limit $V_s \to \infty$. These results are interesting and not necessarily a priori obvious.

A notable result of the parametric study reported in [6,5] and summarized above is that it suggests that a bow with positive rake and negative flare appears advantageous. This finding may have implications with respect to bulb design. Specifically, it suggests that a bulb located aft of the ship stem and integrated with the ship hull may be an advantageous alternative to a traditional bulb protruding ahead of a ship stem. The possible advantage of such a bulb design is also suggested by the results of the hull-form optimization reported in [7]. Additional numerical and experimental studies are required to reach definite conclusions with respect to the merit of this alternative to the usual design of bulbs protruding ahead of ship stems.

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