1. Introduction
Sloshing load due to the liquid motion in a partially filled tank is an important load for the structure analysis in design of LNG and FPSO vessels. Significant sloshing load can be observed near the resonant frequencies of the liquid flow motion in the tank. In general, a CFD model is required to simulate the violent non-linear flow in the tank. However, the required computation is very costly and not practical in design and design checking phase, especially considering that the liquid motion in the tank needs to be solved together with the ship motion since they are fully coupled with each other. Instead, the linear velocity potential model is usually applied to analyze this coupled seakeeping problem, at least for the ship motion part. In this study, we first examine the existing velocity potential seakeeping model and check the possible way to improve the accuracy of the ship motion prediction coupled with the liquid flow in tanks, results of the inner tank flow solution can provide the pressure and forces on tank wall as well. To compute the non-linear sloshing loads on tank wall, we assume that the ship motion can be predicted by the linear model where velocity potential theory is applied to both outer flow problem and inner tank flow problem, and the nonlinearity of tank sloshing flow has less effect on the ship motion. We then set up a three-dimensional CFD model of tank with enforced motion to compute the non-linear sloshing flow and the loads on the tank wall. The total pressure force on the tank wall obtained by the linear potential flow model and the non-linear three-dimensional CFD model are compared to the experimental data, and the feasibility of applying the linear potential analysis and the non-linear CFD analysis for the computation of sloshing load on tank wall is discussed.

2. Velocity potential modeling and its refinement
Velocity potential theory can be applied to modeling the flow motion in a partially filled tank if linear assumption is adopted. The velocity potential for the inner tank flow can be constructed with six ‘radiation potentials’ like
\[
\phi = -i \omega e \sum_{j=1}^{6} \xi_j \phi_j ,
\]
where \( \xi_j \) is the motion amplitude of \( j \)-th motion freedom which must be solved from the ship motion equation coupled with the internal tank flow by including the hydrodynamics on the tank boundary in the equation, and tank flow radiation potential, \( \phi_j \), can be solved from the inner tank boundary value problem which is independent from the outer flow problem. Similar to the outer problem radiation potential, \( \phi_j \) satisfies the Laplace equation, the non-penetrable condition on tank wall boundary and a free surface condition on the inner surface between the liquid and air inside the tank.
This non-homogeneous condition is different from that applied for the outer problem. The non-homogeneous term for motions of heave, roll and pitch represents the mass conservation law and must be kept if the tank problem is solved in the ship-fixed frame, where both motion amplitude $\xi_j$ in equation (1) and normal $n_j$ in condition (2) must be given with respective the COG of the vessel. Tank radiation problem can also be expressed in a tank frame with origin at the centre of the liquid surface. In that case, the non-homogeneous term for roll and pitch in free surface condition (2) can be ignored, and only heave potential needs the non-homogeneous term for enforce mass conservation law. Analysis experiences show that the tank heave motion has very small effects on the ship motion prediction of the coupled problem, and the heave potential can therefore either be ignored from the problem or use no-wave analytical solution $\varphi_3 = z_{\tan k}$. The free surface condition without non-homogeneous term expressed in tank frame usually leads to better results for the tank radiation potential, and it has been applied in the computation of present study. We have studied a LNG carrier with two partially filled tanks with 10 meters filling level (37.2% of tank height) by the Lloyd’s Register’s three-dimensional frequency domain panel code FD-Waveload (FDWL). Model is shown in Fig. 1. Ship motion RAOs for sway, roll and yaw coupled with the inner tank flow are shown in Figure 2 to 4.

\[
\frac{\partial \varphi_j}{\partial z} - \frac{\omega^2}{g} \varphi_j = \begin{cases} 
0 & j = 1, 2, 6 \\
n_j & j = 3, 4, 5
\end{cases} \tag{2}
\]

The motion RAO by the linear potential approach agree with the experimental data well, and discrepancies are found for a few frequencies. One is that the yaw motion was under predicted for frequency $\omega < 0.45 \text{ rad/sec}$. We think the reason of this discrepancy may come from the difference between the numerical model and experiment. A mooring system was used in the experiment, while in our numerical calculation this mooring effect has been totally ignored. To
confirm that, a mooring yaw moment was added to the numerical model for checking the possibility of improvement. Another problem is the under prediction of sway motion for \( \omega = 0.7 \text{ rad/sec} \) to \( 0.8 \text{ rad/sec} \) which is close to the natural frequency of the lateral motion of the inner tank flow. This was caused by the over estimation of the tank flow motion near the natural frequency. We found that the amplitude of sway radiation wave was higher than 20 meters for this case, while the liquid depth in only 10 meters. The results may be improved if we can reduce the radiation wave inside the tank around the natural frequency. To do so, an artificial damping term was added on the free surface condition (2). But we need to bear in mind that the artificial damping is needed only for the cases near the flow natural frequency. To identify the natural frequency of the tank flow, we introduced a simplified estimation for a tank with liquid depth of \( h \). The \( n \)-th natural frequency \( \omega_n \) occurs when radiation wave length comes around

\[
\lambda(\omega_n) = \frac{2}{2n-1} l, \quad n = 1,2,... ,
\]

where \( l \) is the tank scale along the flow direction, and radiation wave length \( \lambda(\omega_n) \) can be estimated from

\[
\frac{2\pi}{\lambda} \tanh\left( \frac{2\pi}{\lambda} \frac{h}{\lambda} \right) = \frac{\omega^2}{g}.
\]

The motion RAO of sway and yaw after adding the mooring moment on yaw and artificial damping near natural frequency are shown in Fig. 5. First we can see that the yaw mooring moment has significant effect on the yaw motion for low frequency cases, the prediction after adding that mooring moment agree well with the experimental data, and we can also find that adding that yaw mooring moment has almost no effect on the sway motion results. Comparing to the results shown in Fig. 2 and 4, we can see that sway motion prediction around the natural frequency has been improved significantly by enforce artificial damping on tank free surface, and the yaw motion prediction was improved at those frequencies as well.

3. CFD modeling of the liquid flow in the tank

The results of previous section demonstrated that the coupled ship motion can be obtained on the linear inner tank flow solution. The remaining question is can we use the same approach for the loading computation. To verify this, we constructed a CFD model of tank sloshing flow with the input of coupled ship motion obtained from the linear potential flow analysis. In present study, we selected InterDyMFoam, a three-dimensional CFD module based a RANS with the VOF internal surface capturing algorithm. InterDyMFoam is a special solver developed for the tank sloshing problem using the OpenFOAM CFD libraries. A LNG tank of 47.180 meters long, 39.100 meters wide, 26.860 meters tall and 10 meters of filling level was studied. The viscous effect was not taken into account in present CFD modeling. First we tested the effect of mesh
density to the results. The results of a coarse model of 32,400 meshes \((30 \times 40 \times 27)\) was compared to the results of a fine mesh model of 786,352 meshes \((118 \times 98 \times 68)\). It was found that the discrepancy of the two results was only around 10%, comparing 30 times of computing efforts. We therefore decided to use the coarse model for present inviscid three-dimensional CFD calculation. 13 frequency cases were selected in the range of \(\omega = 0.3 \, \text{rad/sec}\) to \(1.2 \, \text{rad/sec}\), and a very small time step of \(1/10000\) oscillation period was used in a simulation of 20 to 30 oscillation periods for each cases. The mesh motion increased from zero to the full amplitude in two periods. Results of the amplitude of lateral and vertical hydrodynamic forces obtained by the three-dimensional CFD model and potential flow model are plotted together with the experimental observation in Fig. 6. The tank is on the LNG carrier described in previous section in a beam wave condition without forward speed.

![Graph of hydrodynamic forces](image)

**Fig. 6.** Amplitude of the lateral and vertical hydrodynamic forces on the tank wall for the LNG carrier in beam sea \((U=0)\)

### 4. Discussions

From Fig. 6, we see that the vertical hydrodynamic forces obtained by the linear potential flow model agree quite well with the experimental data, while the lateral force along the \(Y\)-direction by the potential flow is under predicted for both low and high frequencies, even though the peak value is close to the experimental observation. On the other hand, the non-linear CFD model predict a much better lateral force, but over predict the vertical force in a large range around the natural frequency of the tank flow. To improve the CFD model, finer mesh model is required and viscosity effects may be needed to be taken into account especially for the strong non-linear cases to dissipate the energy, but this will need a much longer computing time. The linear potential model may be used in practice to provide the loads for the tank design after a carefully scaling on the lateral force.

### References

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