COMPRESSIBLE JET IMPACT ONTO CORRUGATED PLATE

T.I. Khabakhpasheva[1]**, A. A. Korobkin**[1,2]**,**

1 Lavrentyev Institute of Hydrodynamics, pr.Lavrentyeva 15, Novosibirsk, 630090, Russia, e-mail: tana@hydro.nsc.ru

2 School of Mathematics, University of East Anglia, Norwich, UK, e-mail: a.korobkin@uea.ac.uk

Initial stage of a compressible liquid jet impact onto a corrugated elastic plate with account for air trapping and fluid aeration between the corrugations is studied. The jet head is assumed flat and parallel to the plate before the impact. The corrugations are modeled as rigid plates perpendicular to the surface of the main structure. The jet width is greater than the distance between two corrugations. The jet head closes the air cavity between two corrugations and compresses it before the fluid comes in contact with the plate outside the corrugation region. We assume that the air cavity breaks into a cloud of bubbles before the fluid reaches the elastic plate. This implies that in the present model the fluid between the two corrugations is mixed with the air without increase of the local pressure. Within this assumption the impact occurs at the same time instant between the corrugations and outside them but the fluid is aerated between the corrugations and does not contain air bubbles in the main part of the jet. We shall determine elastic deflections of the plate and the bending stresses in the plate due to the jet impact with account for presence of the aerated fluid between the corrugations.

In the present model we assume that (1) the corrugations do not move during the impact stage; (2) the fluid is uniformly mixed with the air between the corrugations; (3) air fraction in the aerated region is given; (4) the distribution of the hydrodynamic pressure in the aerated fluid is described by the acoustic theory with reduced sound speed and the density of the air/fluid mixture. Only symmetric configuration is considered in this study.

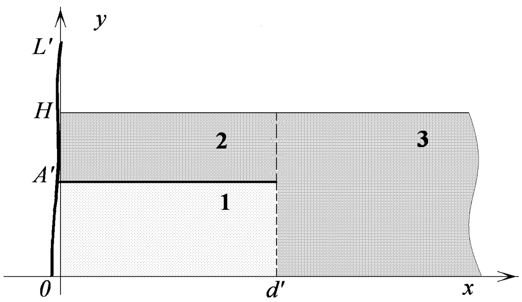

Figure 1.

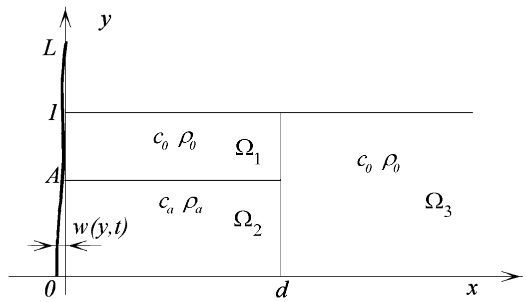

Figure 2.

The hydrodynamic part of the problem is solved by the domain decomposition method. For symmetric configuration three domains are distinguished (see Figure 1). Velocity potentials in each sub-domain are obtained by normal mode method. The matching conditions on the interfaces between these sub-domains and the boundary condition on the elastic plate provide an infinite system of integral and differential equations with respect to unknown time-dependent principal coordinates of the plate deflection and shapes of the interfaces between the sub-domains.

Formulation of the problem

Two-dimensional unsteady problem of compressible jet impact onto a corrugated elastic plate is considered. We assume that maximum bending stresses in the plate occur at the early stage of the impact, when displacements of liquid particles are small and, therefore, equations of motion and the boundary conditions can be linearized around the solution representing the uniform jet flow before the impact.

The plate is modeled as a simply supported Euler beam. The length of the plate is $2L'$ and the plate thickness is h_b. Corrugations are modeled as rigid plates of length d'. The distance between two corrugations is $2A'$. The jet of width $2H$ approaches the corrugated plate from the right (see Figure 1) with the velocity V. The flat head of the jet first touches the edges of the corrugation plates and continue to move towards the main structure. The air trapped between two corrugations is assumed to be well mixed with the fluid by the time instant, $t' = 0$, when the flat jet head touches the plate outside the corrugations. We assume that the mixing process does not increase the pressure between the corrugations (region 1 in Figure 1) before the impact instant $t' = 0$.

The fluid in the jet (regions 2 and 3) is assumed ideal and weakly compressible. Gravity and surface tension effects are not taken into account. The aerated fluid in region 1 is modeled as a fictitious continuum medium with density ρ_a and sound speed c_a being reduced compared with the density ρ_0 and the sound speed c_0 in the main jet regions 2 and 3.

Within the linear theory, which is valid during the initial impact stage of the jet interaction with the plate, the fluid flow is governed by the wave equation, while the response of the plate is governed by the classical linear dynamical plate equation. The coupling between the fluid flow in different regions and the plate deflection is taken into account through the dynamic and kinematic conditions imposed on the wetted part of the plate. Deformations of the jet free surface are neglected. The boundary conditions are linearized and imposed on the initial surface of the jet.

The problem is considered in non-dimensional variables, where the half of jet width H is taken as the length scale, the ratio H/c_0 as the time scale, jet speed V the velocity scale, the "water hammer" pressure $\rho_0 c_0 V$ as the pressure scale, the product VH as the scale of the velocity potential of the flow and HV/c_0 as the scale of plate deflection. Here Oxy is the Cartesian coordinate system with the plate being in the plane $x = 0$ (see Fig. 2) and $w(y, t)$ is non-dimensional deflection of the elastic plate.

In the non-dimensional variables the liquid flow after the impact instant, $t > 0$, is described by the total velocity potential $-x + \varphi(x, y, t)$, where φ is the disturbed potential which satisfies the following equations and boundary conditions

$$\varphi_{tt} = \nabla^2\varphi \quad (\Omega_{2,3}), \qquad \varphi_{tt} = c^2\nabla^2\varphi \quad (\Omega_1) \tag{1}$$

$$\varphi_x = 1 - w_t(y, t), \qquad (x = 0, \quad -1 < y < 1) \tag{2}$$

$$\varphi = 0, \qquad (x > 0, \quad y = \pm 1) \tag{3}$$

$$\varphi_y = 0, \qquad (0 < x < d, \quad y = \pm A) \tag{4}$$

$$\gamma\varphi(d - 0, y, t) = \varphi(d + 0, y, t), \qquad \varphi_x(d \pm 0, y, t) = \eta_t \quad (|y| < A) \tag{5}$$

$$\alpha w_{tt} + \beta w_{yyyy} = \begin{cases} -\varphi_t & (A < |y| < 1) \\ -\gamma\varphi_t & (|y| < A) \end{cases} \tag{6}$$

$$w = w_{yy} = 0 \qquad (y = \pm L) \tag{7}$$

$$\varphi = 0, \ \varphi_t = 0, \ w = w_t = 0 \qquad (t = 0)$$

where

$$\gamma = \frac{\rho_a}{\rho_0}, \qquad c = \frac{c_a}{c_0} \qquad \alpha = \frac{\rho_b h_b}{\rho_0 H}, \qquad \beta = \frac{EJ}{\rho_0 c_0^2 H^3}$$

are non-dimensional parameters of the problem.

The coupled problem (1)-(7) is solved by the normal mode method, which is applied to both the hydrodynamic problem (1)-(5) and the structural problem (6)-(7). This method was successfully used in the problem of homogeneous jet impact onto elastic plate (Korobkin, Khabakhpasheva, Wu 2008). In the present problem the fluid is non-homogeneous but only between the corrugations.

The plate deflection is sought in the form

$$w(y, t) = \sum_{n=1}^{\infty} a_n(t)\psi_n^{(pl)}(y), \tag{8}$$

where the functions $\psi_n^{pl}(y)$ are non-trivial and orthonormal solutions of the eigen-value problem

$$\psi^{pl}_{n\ yyyy} = \lambda_n^4\psi_n^{pl} \quad (-L < y < L), \qquad \psi_n^{(pl)} = \psi^{(pl)}_{n\ \ yy} = 0 \quad (y = \pm L),$$

and λ_n are the corresponding eigenvalues. The principal coordinates $a_n(t)$ satisfy the following system of equations

$$\alpha\ddot{a}_n + \beta\lambda_n^4 a_n = -2\int_A^1 \varphi_t(0, y, t)\ \psi_n^{(pl)}(y)\ \mathrm{d}y - 2\gamma\int_0^A \varphi_t(0, y, t)\ \psi_n^{(pl)}(y)\ \mathrm{d}y. \tag{9}$$

In order to determine the velocity potentials in each sub-domain, we introduce two new unknown functions

$$\varphi_x(d,y,t) = f(y,t) = \sum_{m=1}^{\infty} f_m(t)\psi_m^{(1)}(y) \quad (|y| < A),$$

$$\varphi_x(d,y,t) = g(y,t) = \sum_{m=1}^{\infty} g_m(t)\psi_m^{(2)}(y) \quad (A < |y| < 1),$$

where $\{\psi_m^{(1)}(y)\}_{m=1}^{\infty}$ and $\{\psi_m^{(2)}(y)\}_{m=1}^{\infty}$ are complete systems of orthonormal functions in the regions 1 and 2, respectively.

Equations (9) and the conditions on the interface $x = d$ provide the system of ordinary differential and integral equations:

$$\dot{b}_n = -\beta\lambda_n^4 a_n, \qquad \alpha\dot{a}_n + \sum_{m=1}^{\infty}\int_0^t \dot{a}_m(\tau)S_{nm}(t-\tau)d\tau = b_n(t) + T_n(t) - \tag{10, 11}$$

$$-\sum_{m=1}^{\infty} B_{mn}^{(2)}\int_0^t g_m(\tau)p_m^{(2b)}(t-\tau)d\tau - \gamma\sum_{m=1}^{\infty} B_{mn}^{(1)}\int_0^t f_m(\tau)p_m^{(1b)}(t-\tau)d\tau$$

$$\gamma\int_0^t f_k(\tau)p_k^{(1)}(t-\tau)d\tau + \sum_{n=1}^{\infty}\int_0^t f_n(\tau)P_{kn}^{(13)}(t-\tau)d\tau + \sum_{n=1}^{\infty}\int_0^t g_n(\tau)P_{kn}^{(12)}(t-\tau)d\tau =$$

$$= \gamma e_k^{(1)}\int_0^t p_k^{(1b)}(\tau)d\tau - \gamma\sum_{n=1}^{\infty} B_{kn}^{(1)}\int_0^t \dot{a}_n(\tau)p_k^{(1b)}(t-\tau)d\tau \tag{12}$$

$$\int_0^t g_k(\tau)p_k^{(2)}(t-\tau)d\tau + \sum_{n=1}^{\infty}\int_0^t f_n(\tau)P_{nk}^{(12)}(t-\tau)d\tau + \sum_{n=1}^{\infty}\int_0^t g_n(\tau)P_{kn}^{(23)}(t-\tau)d\tau =$$

$$= e_k^{(2)}\int_0^t p_k^{(2b)}(\tau)d\tau - \sum_{n=1}^{\infty} B_{kn}^{(2)}\int_0^t \dot{a}_n(\tau)p_k^{(2b)}(t-\tau)d\tau. \tag{13}$$

We need to find the solution of this system subject to the initial conditions

$$a(0) = 0,\ \dot{a}(0) = 0,\ b(0) = 0, \quad f_k(t) \equiv 0 \quad g_k(t) \equiv 0 \ \ (t < d). \tag{14}$$

Here $b_n(t)$ are auxiliary functions. The vector- and matrix-functions $T_n(t)$, $S_{nk}(t)$, $P_{nk}^{(13)}(t)$, $P_{nk}^{(12)}(t)$, $P_{nk}^{(23)}(t)$, $p_m^{(1)}(t)$, $p_m^{(b1)}(t)$, $p_m^{(2)}(t)$, $p_m^{(b2)}(t)$ depend only on time and the impact configuration.

This system is truncated and solved numerically. A time stepping method based on the Runge-Kutta scheme is used for the numerical integration of the system. The integrals in (11) are subdivided into two integrals along the intervals $[0, t-\Delta t/2]$ and $[t-\Delta t/2, t]$. The first integral is evaluated by the trapezoidal rule with the integration step equal to $\Delta t/2$. The integral over the interval $[t-\Delta t/2, t]$ is computed by using quadratic approximation of the integrand on the interval $[t-\Delta t, t]$. The integrals in (12-13) are evaluated by the rectangle rule because the functions $p_1^{(1)}(t)$ and $p_1^{(b1)}(t)$ are discontinuous. Numerical tests revealed that more than 10 modes are required to evaluate the functions $f(y,t)$ and $g(y,t)$ with appropriate accuracy. However, accurate values of these functions are not very important if we are concerned with the elastic plate deflections. It was found that 5 modes in the expansions of these functions are needed for convergence of the numerical results in terms of the elastic deflections. Calculations were performed with 5, 10, 15 and 20 modes in these expansions.

Numerical results

Effect of corrugations on elastic vibrations of the plate is investigated. It is show that both the position and length of the corrugations matter. The deflections at the center of the plate (in cm) are shown in Figures 3-6 as functions of the non-dimensional time. Results are obtained for the plate length $2L' = 1\,\mathrm{m}$, the plate thickness $h_p' = 2\,\mathrm{cm}$ and the jet width $2H = 80\,\mathrm{cm}$. The plate is made of steel with $E = 21 \times 10^{10}\,\mathrm{N/m^2}$, $\rho_p = 7875\,\mathrm{kg/m^3}$, $\nu = 0.3$. In the main part of water jet (regions Ω_2, Ω_3),

$c_0 = 1500\,\mathrm{m/s}$, $\rho = 1000\,\mathrm{kg/m^3}$, and in the region Ω_1 of aerated fluid, $c_0 = 500\,\mathrm{m/s}$, $\rho = 900\,\mathrm{kg/m^3}$. Calculations were performed for the initial stage of the impact, duration of which is approximately equal to the quarter of the first (main) period of the plate vibration in the air. It is seen that the maximum values of the plate deflections are achieved at the time interval $25 < t < 35$.

Figures 3 and 4 present time evolutions of the plate deflections at the centre of the plate for different distances between the corrugations. The length of the corrugations is 5 cm for these figures. The fluid between the corrugations is aerated in figure 4 and is not aerated in figure 3. In both cases the deflections increase with increase of the distance between the corrugations. Aeration of the fluid between the corrugations gives rise to high frequency oscillations of the deflections.

Influence of the length of corrugations on the plate deflections is illustrated by Figures 5 and 6, where $A = 0.2$ m. The fluid between the corrugations is aerated in figure 6 and is not aerated in figure 5. In both cases, the longer the corrugations, the higher the plate deflections. It is seen that high frequency vibrations are typical for the problem of aerated fluid between the corrugations. Period of these vibrations increases with the corrugation length d.

Comparing the plate deflections obtained in the problems of corrugated plate impacts, we conclude that the period of plate vibration is longer if the plate is corrugated. Therefore, the shape of corrugations and aeration of the fluid between them are important in hydroelastic response of an elastic plate to liquid impact. High-frequency vibrations of the plate deflection in the case of aerated fluid indicate that the bending stresses are higher than for a non-aerated fluid and that the fatigue of the plate may be expected.

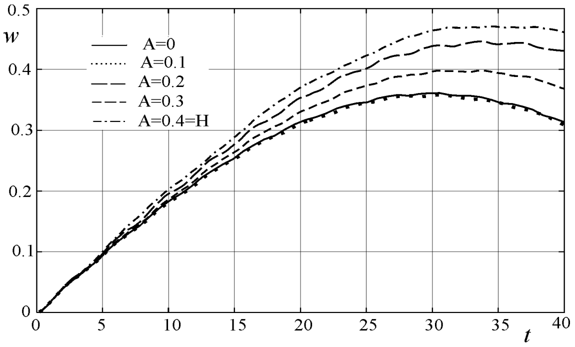

Figure 3.

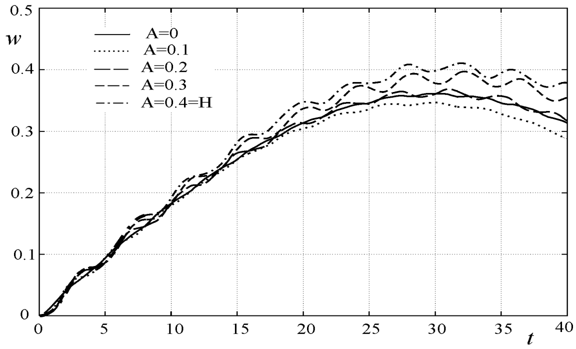

Figure 4.

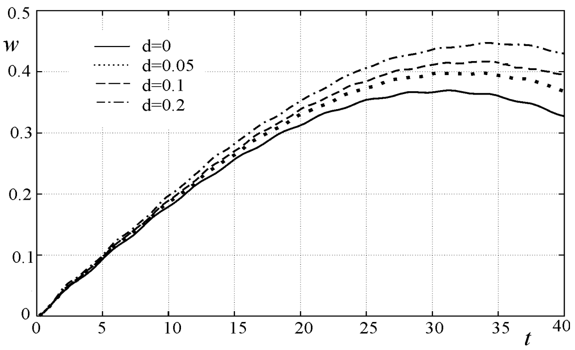

Figure 5.

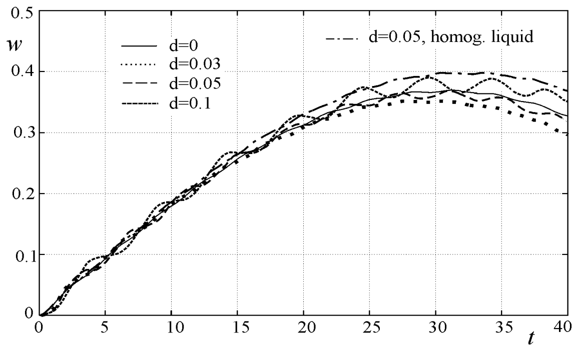

Figure 6.

Acknowledgement. This work was supported by the grant of RFBR 07-08-00145, programm of RAN Department 2.14.2 and grant of President of Russian Federation for the Leading Scientific Schools NS-2260.2008.1.

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