# TANK WALL REFLECTIONS IN TRANSIENT TESTING

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## Introduction

In both numerical and physical wave tanks, tests in short transient wave packets have various advantages. Response amplitude operators may be obtained over a broad frequency range from just one test, and problems associated with finite tank dimensions may be reduced if reflected waves have insufficient time to return to the test zone during the short experiment. Furthermore, tests in focused wave groups (e.g. New Wave) can provide valuable information about nonlinear behaviour of fixed and floating bodies in extreme waves. Radiation tests, in which a body is oscillated in otherwise still water, can also benefit from use of a compact focused group as the input motion time history.

This abstract is concerned with the reflections which may still occur in such transient wave testing, if it proves impractical to test in a large enough physical or numerical tank. The work is motivated by some experiments and nonlinear analysis for a structure with pronounced flare which undergoes transient forced oscillations at the free surface. In an attempt to clarify some comparisons between the experimental and the nonlinear numerical analysis, a linear frequency analysis was conducted for the structure in a finite tank. The linear transient response to the focussed group input was then obtained using an fft approach, with some artificial damping to limit the infinite resonant response peaks and hence to allow solution of the initial value problem using a finite length transform.

## Test configuration

A cone has been tested in vertical oscillations on the centre line of a long tank of width 2.5m. The tip of the cone was 148mm below the mean free surface, and as this was a right circular cone the radius at the waterline was also 148mm. The vertical position $w(t)$ of the cone followed the form of a Gaussian wave packet defined in terms of an amplitude spectrum $W(\omega)$ by

$$w(t) = -A\int_{-\infty}^{\infty} W(\omega)\mathrm{e}^{i\omega t}\,d\omega\,, \tag{1}$$

where

$$W(\omega) = \frac{1}{\sigma_k\sqrt{2\pi}}\exp\left[-\frac{(\omega-\omega_k)^2}{2\sigma_k^{\,2}}\right],\quad \sigma_k = \frac{\omega_k}{2\pi}.$$

With this definition of the packet bandwidth $\sigma_k$, it maintained its shape as the central frequency $\omega_k$ was varied in different tests. In practice the Fourier transform was discretised over a finite range of frequency. Measurements were made of the vertical fluid force on the cone, and the relative vertical motion between the cone and its intersection with the actual free water surface.

A nonlinear hydrodynamic analysis of this problem was undertaken using the boundary element method (BEM) described by Bai & Eatock Taylor (2006). An important difference, however, was that the numerical tank was circular, with the radius of 1.25m matching the half width of the long tank used for the experiments. Figure 1 shows a comparison between experimental and nonlinear numerical results for the transient vertical fluid force and relative elevation on the cone, for two central frequencies $\omega_k = k\pi/3$ rad/s with $k = 3$ and $k = 7$. The time axis is non-dimensionalised by the central period of the group, $T_k=2\pi/\omega_k$. Also shown are the corresponding results predicted by a linear analysis of the cone oscillating vertically in open water (no walls). The nonlinear features of the experimental trace are matched by the BEM analysis over the main central oscillations, and indeed throughout the whole record for the $k$=3 case. Beyond $t/T_k$=1.5 all three traces for force and elevation diverge for $k$=7: the experimental and nonlinear results show evidence of substantial (but different) reflections, while these are absent from the open water linear analysis: in that case the water returns to rest and the force to zero.

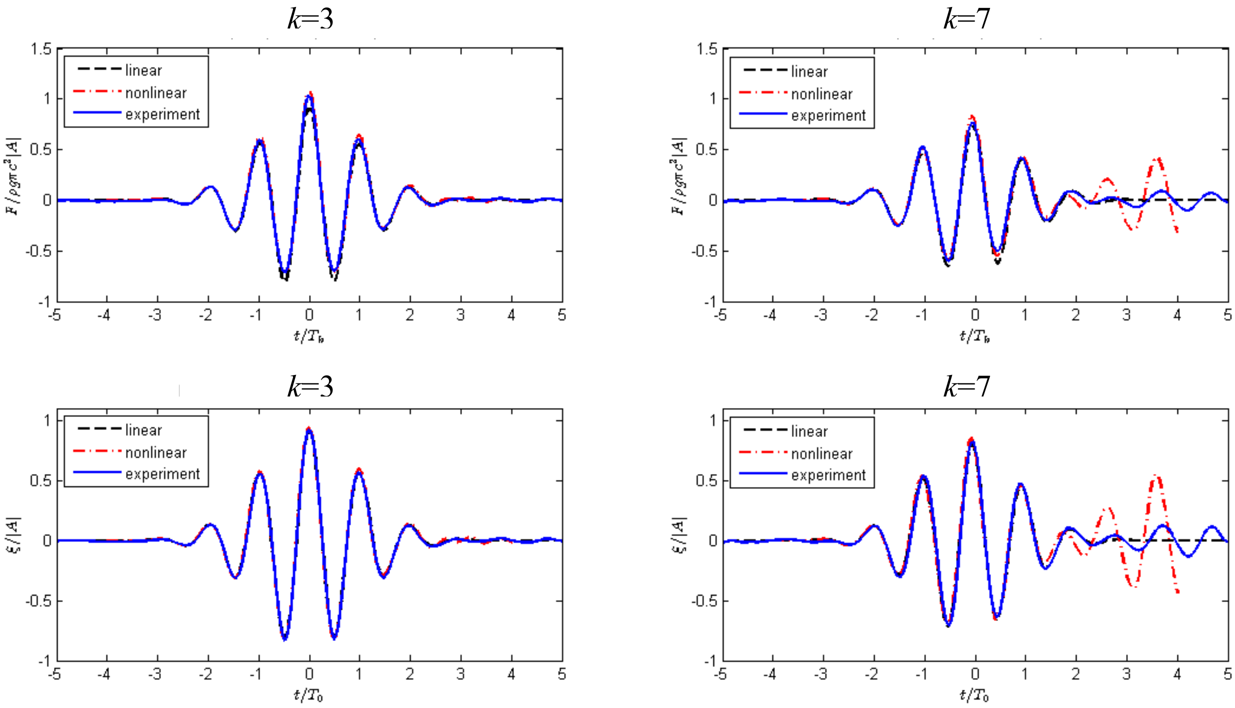


**Figure 1. Force and relative elevation on cone oscillated by two Gaussian motion packets ($k$=3 and $k$=7)**

As part of an investigation of the nonlinear effects, we thought it would be useful to have a linear prediction for the oscillating cone in the same circular tank as used in the nonlinear model. One approach would simply be to linearise the nonlinear code. An alternative, adopted here, was to adapt the linear frequency domain analysis to deal with a closed tank. This provides a degree of independent checking of the nonlinear analysis, and is described next.

**Linear transient analysis**

The starting point was a linear frequency domain wave diffraction-radiation program (AXID) for vertically axisymmetric bodies. This was developed by Zietsman, using a finite element (FE) discretisation of the velocity potential $\phi_1$ in a region $R_1$ close to the body surface $S_B$, coupled with an eigenfunction series representation of the potential $\phi_2$ in the far field region $R_2$, as described for the 2D plane problem by Eatock Taylor & Zietsman (1981). The boundary $J$ between $R_1$ and $R_2$ is taken as a vertical circular cylinder, so that the eigenfunctions may be simply expressed in cylindrical polar coordinates. The finite element equations are then obtained from a variational formulation based on the functional:

$$\Pi(\phi_1,\phi_2) = \tfrac{1}{2}\iint_{R_1}(\nabla\phi_1)^2\,dA - \frac{\omega^2}{g}\int_{S_{F_1}}\phi_1^{\,2}\,dS - \int_{S_B} g(s)\phi_1\,ds + \int_J(\phi_1 - \tfrac{1}{2}\phi_2)\frac{\partial\phi_2}{\partial n}\,ds\,. \qquad (2)$$

Here $S_{F1}$, denotes the free surface in region $R_1$. The FE region is only required very close to the body, and a relatively small number of terms in the far field series (≈ 10) is sufficient to achieve an effectively converged representation of the wave field out to infinity. Quadratic isoparametric elements are used in AXID, and Figure 2a shows a suitable mesh for modelling the cone in the open sea.

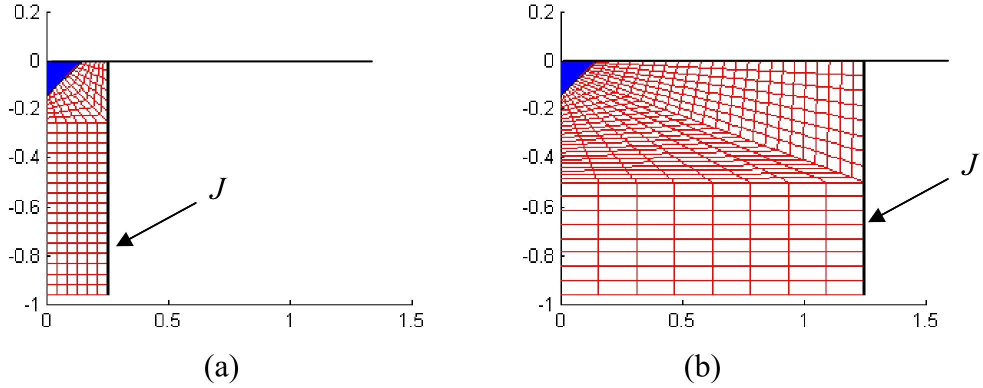


**Figure 2. AXID meshes: a) cone in open sea; b) cone in tank of radius 1.25m**

Combining the resulting added mass and damping terms with the hydrostatic force, one obtains the RAO of vertical force on the cone when oscillated in otherwise still water. In the Gaussian packet defined in (1), the time history of this force is then:

$$f(t) = \int_{-\infty}^{\infty} F(\omega) W(\omega) \mathrm{e}^{i\omega t} d\omega \,. \tag{3}$$

The lines labelled 'linear' in Figure 1 result from the discretised form of (3), with $F(\omega)$ given by AXID for the cone in open water. As usual, care must be taken in the implementation of the FFT to avoid aliasing: one must for example ensure that the frequency spacing is small enough to provide a time history with a sufficient length on either side of the Gaussian packet to prevent corruption from the periodic wrap-around in the finite length transform.

To investigate the effect of reflections in a tank, the functional Π was modified. With $\partial\phi_1/\partial n$=0 one has the case of perfect reflections. It was easy to modify AXID to model this case, by setting the $J$ integral to zero. For a circular tank of radius 1.25m, as modelled in the fully nonlinear BEM analysis, we used the FE mesh shown in Figure 2b (which despite some elements of poor aspect ratio was found to be suitable). Direct use of the resulting RAO, however, would cause problems in obtaining the transient solution with the method described above. The circular wall leads to standing waves, and infinite resonant peaks in the RAO at readily calculated frequencies. This is illustrated by Figure 3, which compares the force spectrum $F(\omega)W(\omega)$ for the open sea and tank cases (normalised to a maximum of 1 in the open sea). In the limit of perfect reflections, an infinite comb of Dirac delta functions is embedded in the RAO $F(\omega)$. An infinite length inverse transform is then required if one wishes to investigate the initial value problem.

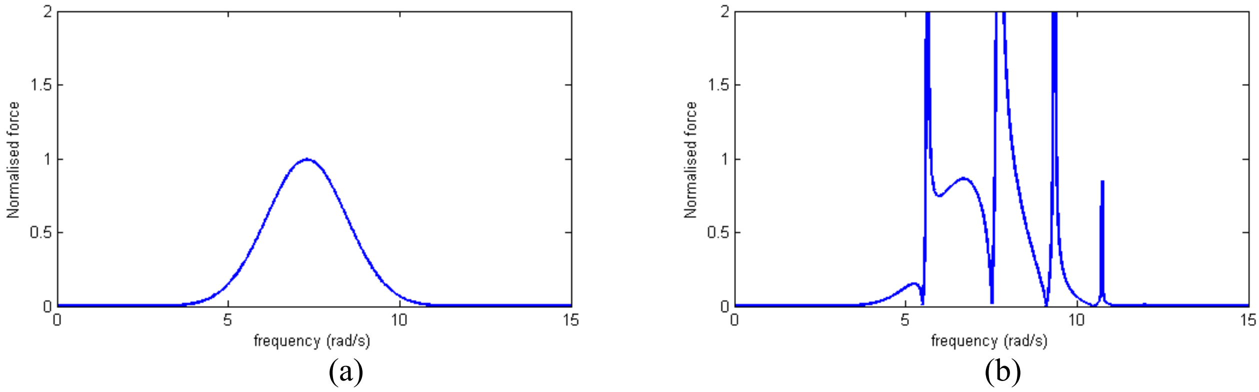


**Figure 3. Force amplitude spectra ($k$=7): a) cone in open sea; b) cone in tank of radius 1.25m**

To overcome this difficulty, a "reflectivity" factor $\alpha$ was introduced as a multiplier of the $J$ integral: $\alpha$=1 corresponds to open sea and $\alpha$=0 to a tank. It was found that by using a small value of $\alpha$, and a correspondingly small frequency increment $\Delta\omega$ in the FFT, it was possible to obtain converged results for the cone during oscillation by the Gaussian packet. Examples of results for forces, computed with different values of $\alpha$ and $\Delta\omega$, are shown in Figure 4. The four subplots may be considered in conjunction with Figure 1, which shows the linear predictions for the open sea ($k$=7) case ($\alpha$=1). It should be noted that in each of the subplots of Figure 4 the line designated "nonlinear" is the same. Examination of the linear results in the range $t/T_k$>1.5 shows the influence of reflections building up as $\alpha$ is reduced, and convergence of the linear predictions as $\alpha \rightarrow 0$. The importance of reducing the frequency increment $\Delta\omega$ as $\alpha$ is reduced can be understood from Figure 4c: in this case $\Delta\omega$ is too large, and non-physical oscillations can be seen in the initial stages of the time history (*i.e.* before the cone has started oscillating).

A significant finding is that the converged linear results for the cone oscillating in the tank agree very well with the nonlinear predictions in the region away from the large primary peak at the focus point. This agreement has also been observed for the Gaussian packets with other central frequencies $\omega_k$. This includes the interesting case of $k$=9, for which the effect of reflections is found to be small.

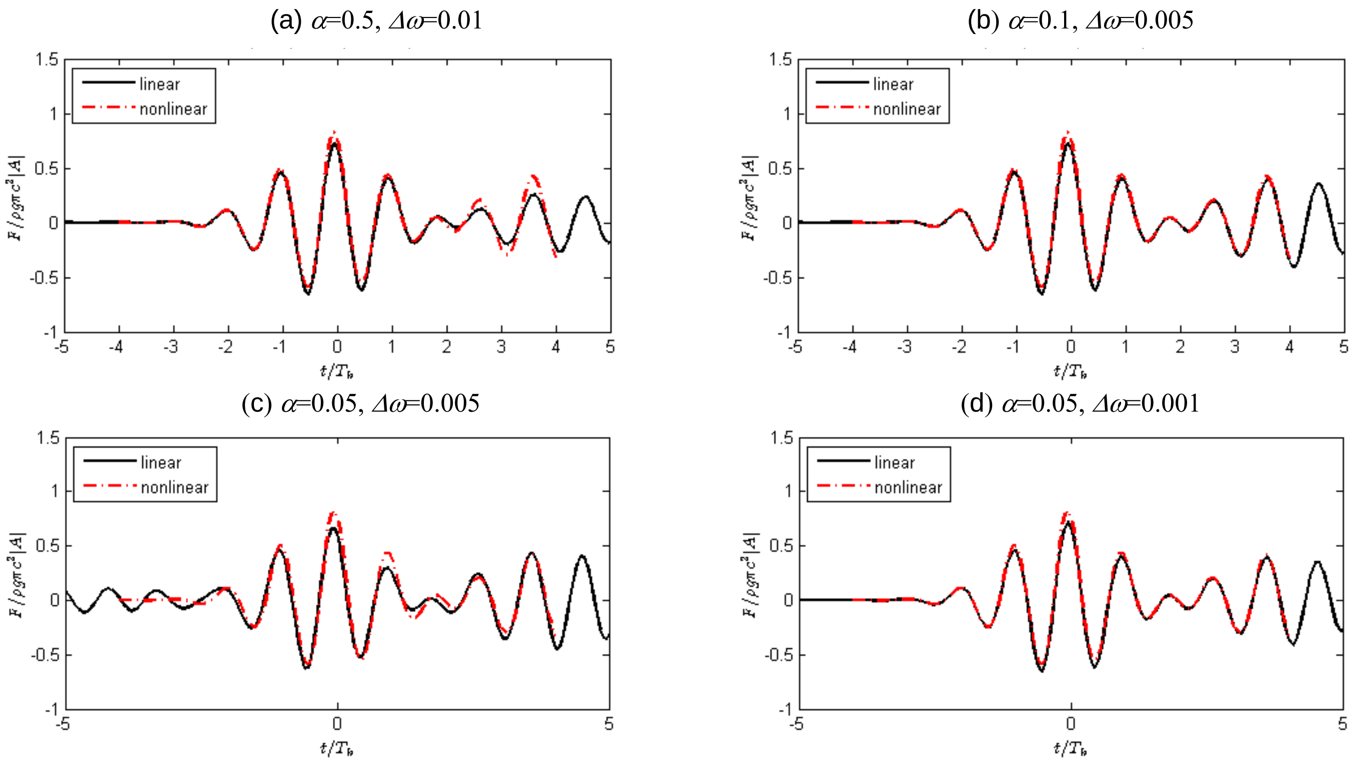


**Figure 4. Force time histories ($k$=7): nonlinear results compared with linear solutions in tanks with different reflectivity**

## References

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