Non-Reflecting Simulation for Fully-nonlinear Irregular Wave Radiation

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1. Introduction

When simulating the nonlinear water wave propagation through an unbounded domain in the time domain, it is necessary to truncate the computational domain into a finite domain in order to reduce computational costs. No-reflecting condition is required for the artificial truncation. The Sommerfeld-Orlanski's condition has been widely used for linear wave radiation simulation, this condition is local in both time and space and depending on the phase velocity of out-going waves and cannot give good results for irregular wave radiation. The global matching or shell function method is very accurate for linear irregular wave radiation but with relatively large computational effect comparing with the local method and can not satisfy the nonlinear condition. Another method in common use is Damping Zone(DZ), which could absorb high frequencies waves efficiently. It is limited by the length of DZ and therefore not very efficient for low frequencies waves. Clément(1996) present coupling piston-like absorbing boundary condition and damping zone method for wide frequency wave radiation. The piston-like condition with feed back signal through wave force on the artificial boundary is just effective for low frequencies waves and is not easy to realize for 3D wave radiation.

In this paper, an Artificial Boundary Condition derived from multi-transmitting formula(MTF) in earthquake engineering is extended to the 2D linear and fully nonlinear water wave radiation simulation. Comparing with Orlanski condition, the high order MTF is efficient for a wide range of phase velocity for low or high frequency waves and kept the easy of operation comparing with piston-like condition. When simulating practical irregular waves, it is proposed to couple MTF and DZ method using MTF method to absorb low frequency waves and using DZ method to absorb high frequency waves. The numerical results obtained by MTF method or the coupling method were found to be excellent for linear and fully nonlinear wave radiation.

2. Mathematical formulation

The 2-D numerical wave tank considered here is illustrated in Fig. 1 which shows wave propagation generated by a piston-type wave maker installed at the left hand. A Cartesian coordinate system is defined, and the x-axis coincides with the undisturbed free surface and the y-axis is positive upwards. The free surface, the artificial boundary, the wave maker and the bottom are denoted as S_F, S_C, S_W and S_B, respectively.

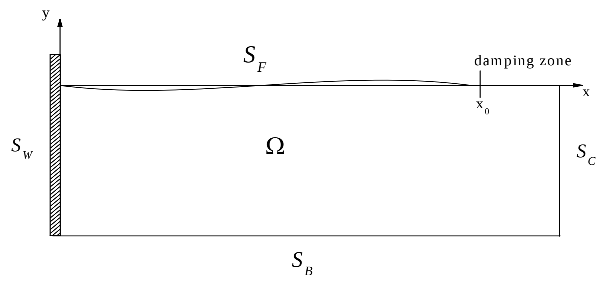

Fig. 1. Sketch of computational domain

We assume the fluid is incompressible and inviscid, and the flow irrotational, the fluid motion can be described by a velocity potential ϕ, which satisfies the Laplace equation within the fluid domain Ω,

The boundary conditions on the instantaneous free surface on S_F can be written in Lagrangian form

$$\frac{Dx}{Dt}=\frac{\partial\phi}{\partial x};\quad \frac{Dy}{Dt}=\frac{\partial\phi}{\partial y};\quad \frac{D\varphi}{Dt}=-gy+\frac{1}{2}\left|\nabla\phi\right|^2 \qquad (1)$$

On the wave maker and bottom surface, boundary conditions can be given as

$$\frac{\partial\phi}{\partial n}=\vec{U}(t) \qquad on\ \ S_w \qquad (2)$$

$$\frac{\partial \phi}{\partial n} = 0 \qquad on\ S_B \qquad (3)$$

Where $U(t)$ is the horizontal velocity of piston motion.

3. Absorbing Boundary Condition

3.1 Multi-Transmitting Formula

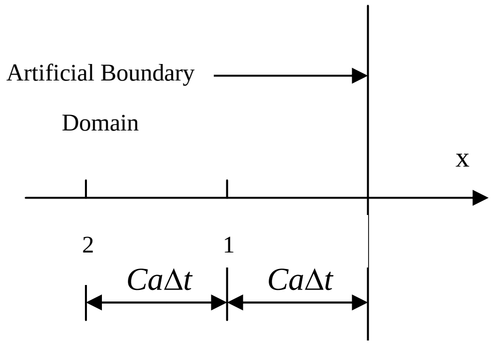

Fig.2. Assemble relation of giving MTF

On the artificial boundary, the Multi-Transmitting Formula (MTF) proposed by Liao(1984) which is applied to the propagation of earthquake wave motion, electromagnetic wave motion is extended to simulate the fully nonlinear water wave radiation. Let x-axis be the normal to the artificial boundary S_C and point to the outer region of the model in Fig. 2. Suppose that the intersection point 0 of the x-axis and the artificial boundary is the radiation boundary point on S_C under consideration. 1,2,3,…,N are the points which are away from point 0 along its normal vector to the inner region. The distance between them is equal to $c_a\Delta t$, where c_a is the artificial velocity. According to the theory of MTF, the velocity potential on S_C may be write

$$\phi_0^{p+1} = \sum_{j=1}^{N} (-1)^{j+1} C_j^N \phi_j^{p+1-j} \qquad (4)$$

where integer p represents the time level, N is the order of the MTF. After numerical investigation, it is found the second order MTF is most better. that is

$$\phi(t+\Delta t, x) = 2\phi(t, x - c_a\Delta t) - \phi(t-\Delta t, x - 2c_a\Delta t) \qquad (5)$$

Because the formula is one dimensional explicit formulation, so it is very easy to realize in the computation.

3.2 Damping Zone

Damping zone is widely used, and is achieved through adding a damping term in the free surface boundary conditions：

$$\frac{Dx}{Dt} = \frac{\partial \phi}{\partial x} \qquad (6)$$

$$\frac{Dy}{Dt} = \frac{\partial \phi}{\partial y} - \nu(x)y \qquad (7)$$

$$\frac{D\phi}{Dt} = -gy + \frac{1}{2}(\nabla\phi)^2 - \nu(x)\phi \qquad (8)$$

where $\nu(x)$ is the damping coefficient. The validity of damping zone is depending on the ratio of damping zone length and wavelength. As the cost for absorbing low frequency waves will increase remarkable, it is only used to absorb high frequency waves. To avoid reflections, the damping coefficient $\nu(x)$ is given as

$$\nu(x) = \begin{cases} 0 & x < x_0 \\ \left(-2\left(\frac{x-x_0}{l}\right)^3 + 3\left(\frac{x-x_0}{l}\right)^2\right)\nu_{max} & x_0 \le x \le x_0 + l \end{cases} \qquad (9)$$

where l is the damping zone length. In the coupling method, we can take $\nu_{max} = 0.2$, $l \ge 2\lambda$ through numerical computation, in which λ is the maximum wavelength that we want to absorb.

4. Numerical method and results

The high-order boundary element method is used, with piecewise cubic polynomial shape functions for geometry and boundary data .A double node technique at the intersection point of different boundary with the continuity of potential is discussed. A standard fourth-order Runge-Kutta scheme is adopted for the integration with respect to time to up-date the wave elevation and the potential on the free surface. When wave nonlinear is obvious, the simulation is over a substantial period of time, the nodes on the free surface may cluster or stretch. In order to avoid over-distortion of elements, nodes on the free surface should be rearranged every several time steps.

4.1 The simulation of monochromatic wave radiation

When simulating monochromatic waves, we use MTF method only. The wave-maker undergoes motion with the following displacement: $S(t)=S_0\sin(\omega t)$. Let $c_a=c_x$. Fig. 3 shows the wave elevation at same point obtained by the linear theory and the linear simulation with excellent coincide, and the results by fully nonlinear simulation with two different tank length L also coincide very good.

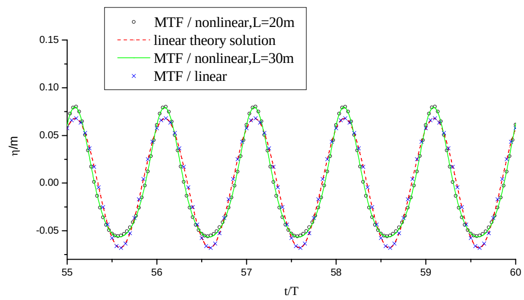

Fig. 3. Wave elevation history at $x=10m$, $S_0=0.1m$, $\eta_0=0.068m$

The numerical results in Fig. 4 show that c_a can be taken within a range around the phase velocity, usually, $0.7c_x<c_a<1.3c_x$. It shows that MTF method is very suitable for narrow spectrum irregular wave radiation.

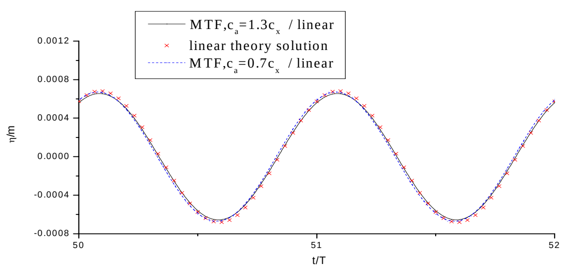

Fig. 4. Wave elevation history at $x=10m$ with different C_a, $S_0=0.001m$

4.2 The simulation of irregular waves radiation

The displacement of the wave maker is taken as: $S(t)=\sum_{i=1}^{N}a_i\sin(\omega_i t+\varepsilon_i)$, wher a_i, ω_i, ε_i are the i-th wave component corresponding wavemaker's amplitude, frequency and phase. ε_i is a random number between $(0,2\pi)$ and N is the total number of wave components. Let

$$\omega_i=\omega_{\min}+\frac{\omega_{\max}-\omega_{\min}}{N-1}(i-1)\ (i=1,2\cdots N),$$

$N=100$. $C_{x_{\max}}$ and $C_{x_{\min}}$ are the physical wave speeds corresponding to $\omega_{\min}$ and $\omega_{\max}$ respectively, if they are in the allowed range(usually $C_{x_{\max}}/C_{x_{\min}}\le 1.3/0.7$), we can use MTF only. Let $\omega_{\min}=0.5$, $\omega_{\max}=5.0$, $a_i=0.002m$ $(i=1,2\cdots N)$, through simple computation, $\omega_i=0.5\sim 5.0$ $(i=1,2\cdots N)$ are known in the allowed frequencies range, and the artificial wave speed is selected as $C_a=0.7C_{x_{\max}}$.

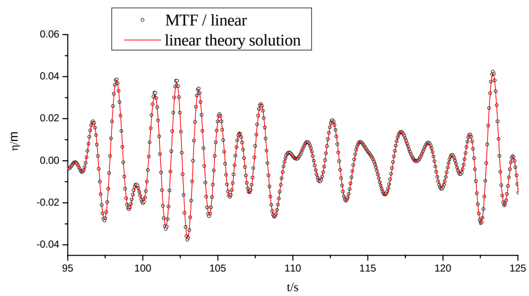

Fig. 5. Wave elevation history at $x=10m$, $L=20m$; linear simulation; $a_i=0.002m$ $(i=1,2\cdots N)$

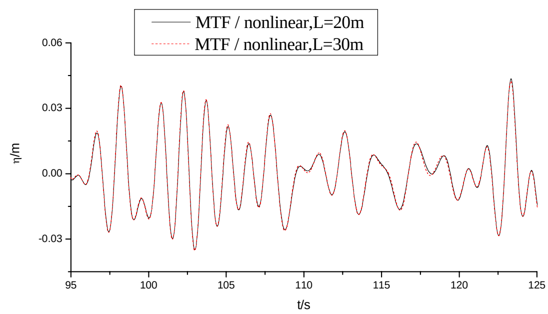

Fig. 6. Wave elevation history at $x=10m$; nonlinear simulation; $a_i=0.002m$ $(i=1,2\cdots N)$

Fig. 5 shows the exact non-reflecting simulation for linear waves. In Fig. 6, the nonlinear results given from the different wave tank length L match very well, which shows there are not reflections at artificial boundary for the fully nonlinear wave radiation.

If there are wave frequencies out of the MTF applying range, we should couple the damping zone to absorb the high frequency waves. Such as, $\omega_{\min} = 0.5$, $\omega_{\max} = 8.0$, $a_i = 0.002m$ $(i = 1, 2 \cdots N)$ and using damping zone to absorb waves with $\omega > 5$,the wavelength corresponding to $\omega = 5$ is $\lambda = 2.43m$, so we can take $l = 5m$,$v_{\max} = 0.2$,and C_a is the same as last sample. Linear simulation in fig. 7 shows that coupling method could absorb all frequencies wave very well and better than MTF only method.

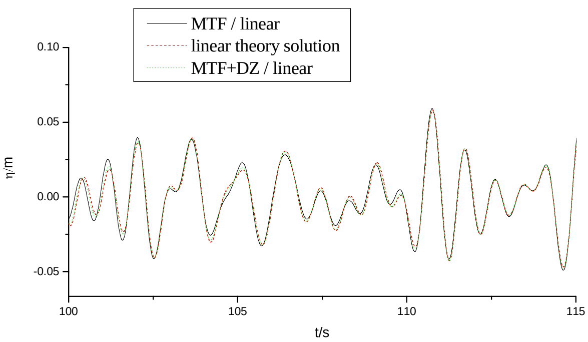

Fig.7 Wave elevation history at $x = 10m$; linear simulation

Fig. 8 shows that coupling method could absorb all frequencies wave very well for nonlinear simulation and the evidence of the nonlinear influence on the results.

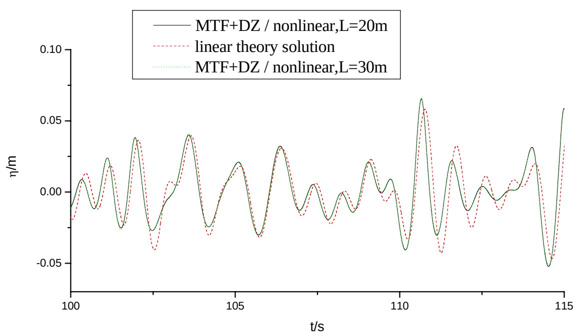

Fig. 8 Wave elevation history at $x = 10m$; nonlinear simulation

5. Conclusion

Nonlinear 2D gravity waves are simulated through the high-order BEM. The mixed Euler-Lagrange(MEL) method is used on the free surface. Attentions are focused on the non-reflecting simulation due to truncated computation domain. It is find that the second order multi-transmitting formula(MTF) may be used to absorb a wide range of frequency irregular waves. The MTF coupled with the Damping Zone (MTF+DZ) method are proposed to realize non-reflecting simulation for fully nonlinear irregular wave radiation.

Acknowledgement

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