Highly simplified Green function for steady flow about a ship

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Introduction

Alternative methods for evaluating steady free-surface flow about ships in deep water have been considered in the literature. These methods include semi-analytical theories based on various approximations (thin-ship, slender-ship, 2d+t theories), potential-flow panel (boundary integral equation) methods that rely on the use of a Green function (elementary Rankine source, or Havelock source that satisfies the radiation condition and the Kelvin-Michell linear free-surface boundary condition), and computational fluid dynamics (CFD) methods that solve the Euler or RANS equations. These alternative calculation methods are reported in a huge body of literature, not reviewed here; a partial list of references may be found in e.g. [1].

Selection of a flow calculation method involves considering a tradeoff between competing requirements with respect to accuracy and practicality. Indeed, practical tools that are simple to use and highly efficient, but need not be highly accurate, are required to quickly evaluate the large number of alternative designs that typically need to be considered for early design stages (concept and preliminary design) and for hydrodynamic optimization. On the other hand, detail design and design evaluation involve many fewer choices and require more accurate computational tools, for which efficiency and ease of use are less important.

Thus, highly-efficient (in terms of user input time and CPU) and robust approximate methods remain important for many practical applications, notably for early design stages and for hull-form optimization; e.g. [2,3]. A classical type of approximate methods are potential-flow panel methods based on a Green function that satisfies the radiation condition and the Kelvin-Michell linear boundary condition at the free surface. A major recommendation of this approach is that flow in an unbounded 3D flow domain is formulated over a finite 2D boundary surface. However, this simplification comes at the price of a relatively complicated Green function. But how complicated does this "free-surface Green function" need to be?

The "steady free-surface flow Green function" can be expressed as the sum of three components: (i) a wave component given by a single Fourier integral with continuous integrand, (ii) the fundamental free-space Green function $-1/r$ where r is the distance between the source point $\mathbf{x}$ and the flow-field point $\widetilde{\mathbf{x}}$ in the Green function $G(\mathbf{x};\widetilde{\mathbf{x}})$, and (iii) a local-flow component that is given by a double Fourier integral with singular integrand. This singular double Fourier integral can be transformed into a single integral, with integrand expressed in terms of the exponential integral with complex argument. Three alternative single-integral representations of the local-flow component are given in [4], and nearfield and farfield asymptotic approximations are given in [5-8]. Approximations based on polynomial expansions [8] or table interpolations [9-11] in complementary separate regions of the flow domain have also been given. Simple analytical approximations — valid within the entire flow region — to the local-flow component have recently been given for two special cases that correspond to thin-ship theory [12] and to flow about air-cushion-vehicles and planing boats [13], for which we have either $y-\widetilde{y}=0$ or $z+\widetilde{z}=0$, respectively.

Extension of the simple expressions given in [12,13] to the general case of arbirary location of the source point $\mathbf{x}$ and the flow-field point $\widetilde{\mathbf{x}}$ is considered here. Thus, steady potential flow about a ship, of length L_s, that advances in calm water (of effectively infinite depth and lateral extent) with constant speed V_s along a straight path is considered. The X axis is taken along the path of the ship and points toward the ship bow. The Z axis is vertical and points upward, and the mean free surface is taken as the plane $Z=0$. The flow is observed from a moving system of coordinates attached to the ship and thus appears steady. Nondimensional coordinates $\mathbf{x}\equiv(x,y,z)\equiv(X,Y,Z)/L_{ref}$ are defined in terms of a characteristic reference length L_{ref}, e.g. the ship length L_s. The Froude number F is defined as

$$F=V_s/\sqrt{gL_{ref}} \qquad (1)$$

where g stands for the acceleration of gravity.

Simplified Green function for steady flow at the centerplane of a thin ship

Within the theoretical framework of Michell's classical thin-ship approximation, the flow due to a ship is represented in terms of a distribution of sources at the ship centerplane $y = 0$. Furthermore, the free-surface elevation along the ship waterline and the pressure at the ship hull can be evaluated at the ship centerplane $\widetilde{y} = 0$. Thus, the Green function of thin-ship theory corresponds to the special case $y - \widetilde{y} = 0$ of the Green function $G(\mathbf{x};\widetilde{\mathbf{x}})$, i.e. $G \equiv G(x,z;\widetilde{x},\widetilde{z}) \equiv G(x-\widetilde{x}, z+\widetilde{z}, z-\widetilde{z})$. A simple approximation to the Green function of thin-ship theory is given in [12]. This simplified Green function is

$$4\pi G \approx H(x-\widetilde{x})\frac{8}{F^2}\,\Im\mathrm{m}\int_0^{t_\infty} dt\,\Lambda\, e^{(1+t^2)(z+\widetilde{z})/F^2 - i\sqrt{1+t^2}(x-\widetilde{x})/F^2} - \frac{1}{r}$$
$$+\frac{1}{r_1} - \frac{2}{F^2+r_1} - \frac{2F^2\psi}{(F^2+r_1)^2}\left(1+\frac{2.3\,\psi\, r_1}{F^2+r_1}\right) \text{ with } \left\{\begin{array}{l} r \equiv \sqrt{(x-\widetilde{x})^2+(z-\widetilde{z})^2} \\ r_1 \equiv \sqrt{(x-\widetilde{x})^2+(z+\widetilde{z})^2} \\ \psi \equiv -(z+\widetilde{z})/(r_1+|x-\widetilde{x}|) \end{array}\right\} \tag{2}$$

Here, F is the Froude number (1) and the function Λ in the wave component filters unrealistic and irrelevant short waves for $t_\infty \le t < \infty$.

Simplified Green function for steady flow due to a free-surface pressure patch

Within the context of linearized potential-flow theory, flows about air-cushion vehicles, planing boats and some types of hybrid ships can be represented in terms of a pressure distribution at the mean free-surface plane $z = 0$. Furthermore, the free-surface elevation can be evaluated at the plane $\widetilde{z} = 0$. Thus, the Green function for steady flow due to a free-surface pressure patch corresponds to the special case $z+\widetilde{z} = 0$ of the Green function $G(\mathbf{x};\widetilde{\mathbf{x}})$, i.e. $G \equiv G(x,y;\widetilde{x},\widetilde{y}) \equiv G(x-\widetilde{x}, y-\widetilde{y})$. A simple approximation to the Green function for steady flow due to a free-surface pressure patch is given in [13]. This simplified Green function is

$$2\pi G \approx H(x-\widetilde{x})\frac{2}{F^2}\,\Im\mathrm{m}\int_{-t_\infty}^{t_\infty} dt\,\Lambda\, e^{-i\sqrt{1+t^2}[x-\widetilde{x}+t(y-\widetilde{y})]/F^2}$$
$$-\frac{1}{F^2+r} + F^2\frac{F^4+2F^2r+5r^2}{(F^2+r)^5}\left(|x-\widetilde{x}| - \frac{4}{5}r\right) \text{ with } r = \sqrt{(x-\widetilde{x})^2+(y-\widetilde{y})^2} \tag{3}$$

As in (2), F is the Froude number (1) and the function Λ in the wave component filters unrealistic and irrelevant short waves for $t_\infty < |t|$.

Simplified Green function for steady flow about an arbitray ship hull

[12] and [13] show that flow calculations based on the highly simplified Green functions (2) or (3) and corresponding more accurate Green functions are indistinguishable. The simple Green functions (2) and (3) can then be used in practice. The Green function $G(\mathbf{x};\widetilde{\mathbf{x}})$ is now considered for the general case $-\infty < x-\widetilde{x} < \infty$, $-\infty < y-\widetilde{y} < \infty$, $-\infty < z+\widetilde{z} \le 0$.

Expressions (7) in [4] and (1) and (3) in [7], or expressions (13)–(16) and (18a) in [11] yield

$$4\pi G = H(x-\widetilde{x})\frac{4}{F^2}\,\Im\mathrm{m}\int_{-t_\infty}^{t_\infty} dt\,\Lambda\, e^{(1+t^2)(z+\widetilde{z})/F^2 - i\sqrt{1+t^2}[x-\widetilde{x}+t(y-\widetilde{y})]/F^2} - \frac{1}{r} + G^L \tag{4}$$

$$\text{with} \qquad G^L = \frac{1}{r_1} - \frac{2}{F^2+r_1}\left(1+\frac{F^2\psi}{F^2+r_1}\right) - \frac{2L'}{F^2} \tag{5}$$

r in (4) and r_1, ψ and L' in (5) are defined as

$$\left\{\begin{array}{l} r \equiv \sqrt{(x-\widetilde{x})^2+(y-\widetilde{y})^2+(z-\widetilde{z})^2} \\ r_1 \equiv \sqrt{(x-\widetilde{x})^2+(y-\widetilde{y})^2+(z+\widetilde{z})^2} \end{array}\right\} \qquad \psi \equiv \frac{-(z+\widetilde{z})}{r_1+|x-\widetilde{x}|} \tag{6}$$

$$L' \equiv \frac{d}{1+d}\left(1+\frac{2+d}{1+d}\,\psi\right) - \frac{1}{\pi}\int_{-1}^{1} dt\,\Im\mathrm{m}\,[\,e^A E_1(A) + \ln(A) + \gamma\,] \tag{7a}$$

$$\text{with} \qquad A \equiv (b\,t - c\sqrt{1-t^2} + i\,a)\sqrt{1-t^2} \tag{7b}$$

$$a \equiv |x-\widetilde{x}|/F^2 \qquad b \equiv |y-\widetilde{y}|/F^2 \qquad c \equiv -(z+\widetilde{z})/F^2 \qquad d \equiv \sqrt{a^2+b^2+c^2} \equiv r_1/F^2 \tag{7c}$$

Expressions (7c) show that the coordinates a, b, c and the related distance d are positive real numbers.

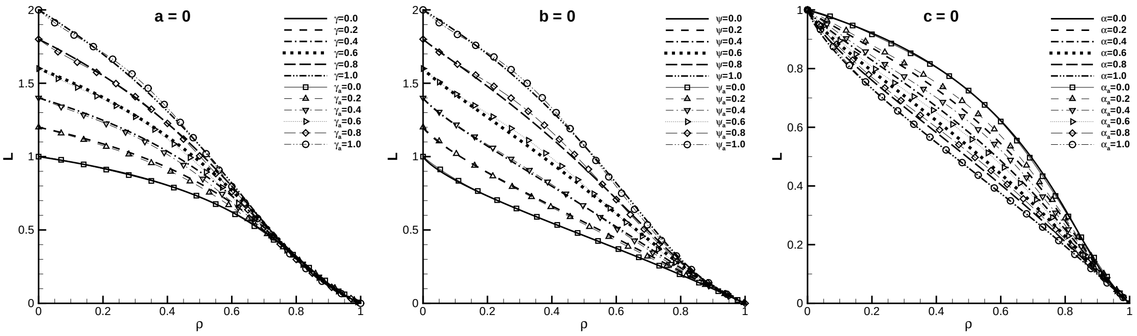

Figure 1: Function $L \equiv [1+\psi/(1+d)]/(1+d)+L'$ with L' evaluated using the (exact) integral representation (7) or the simple approximation (11) in the three particular cases $a=0$ (left), $b=0$ (center) and $c=0$ (right), i.e. for $x-\widetilde{x}=0$, $y-\widetilde{y}=0$ and $z+\widetilde{z}=0$.

The integral representation (7), where $E_1(\cdot)$ is the usual exponential integral and $\gamma = 0.577\ldots$ is Euler's constant, is well suited for numerical evaluation in the nearfield, i.e. for small and moderate values of d , but is not suitable for large values of d, i.e. in the farfield. Expressions (12)–(15) in [9] or (15) and (20a) in [11] yield the farfield approximation

$$L' \sim \frac{1}{d^2}\left((2+c/d)\frac{(1+c/d)\,c/d+b^2/d^2}{(1+c/d)^2}-\frac{c/d}{1+a/d}\right) \text{ as } d\to\infty \tag{8a}$$

This approximation shows that $L'=O(1/d^3)$ as $d\to\infty$ with $b=0$ and $c=0$, i.e. in the farfield along the x axis. In fact, (16) in [12] yields

$$L' \sim -1/d^3 \text{ as } d\to\infty \text{ with } b=0=c \tag{8b}$$

Expression (5) in [7] yields the nearfield approximation

$$L' \sim d\left[1+\frac{a}{2d}\left(\ln\frac{d+a}{4}+\gamma-\frac{1}{2}+\frac{1}{6}\,\frac{c^2-b^2}{(d+a)^2}\right)-\frac{b}{d}\,\frac{b/3}{d+a}-\frac{2c}{3d}\left(1-\frac{3d-c}{d+a}\right)\right] \text{ as } d\to 0 \tag{8c}$$

Let d , a, b, c be expressed as

$$d=\rho/(1-\rho) \qquad a=\alpha\, d \qquad b=\sqrt{1-\gamma^2}\,\sqrt{1-\alpha^2}\,d \qquad c=\gamma\sqrt{1-\alpha^2}\,d \tag{9a}$$

where $0\le\rho\le 1$, $0\le\alpha\le 1$ and $0\le\gamma\le 1$ are defined as

$$\rho\equiv d/(1+d) \qquad \alpha\equiv a/d \qquad \gamma\equiv c/\sqrt{b^2+c^2} \tag{9b}$$

The local-flow component $g^L\equiv F^2G^L$ in (4) can be expressed as

$$g^L\equiv 1/d-2L \quad \text{with} \quad L\equiv[1+\psi/(1+d)]/(1+d)+L' \tag{10}$$

where the identity $r_1=F^2d$ was used and L' is a function of the three variables ρ , α , γ defined by (9b). Expressions (10) and (8) yield $g^L\sim 1/d$ as $d\to 0$, and $g^L\sim -1/d$ as $d\to\infty$. Furthermore, we have $L'/g^L=O(d^2)$ as $d\to 0$, and $L'/g^L=O(1/d^2)$ as $d\to\infty$ with $b=0$ and $c=0$ (along the x axis). Thus, we have $L'\ll g^L$ in both the nearfield and the farfield.

The 'exact' and 'approximate' functions L defined by (10) with L' evaluated using the exact integral representation (7) or the simple approximation

$$L'\approx\{[4-2\rho+24\rho^2+(1+37\rho-62\rho^2)\gamma](1-\alpha)-(4-5\rho+6\rho^2)(1-\rho)\,\alpha\}\,\rho\,(1-\rho)^2/5 \tag{11}$$

are depicted in Fig.1 for $0\le\rho\le 1$ in the three particular cases $a=0$, $b=0$ and $c=0$, i.e. for $x-\widetilde{x}=0$, $y-\widetilde{y}=0$ and $z+\widetilde{z}=0$. The figures for $b=0$ and $c=0$ (in the center and on the right of Fig.1) correspond to Fig.7 in [12] and Fig.7 in [13], respectively, and to the Green functions of thin-ship theory and the theory of flow due to a free-surface pressure patch considered in these two previous studies. The center and right figures in Fig.1 and Figs.7 in [12] and [13] show that the approximation (11) for the general case $a \neq 0$, $b \neq 0$, $c \neq 0$ is no less accurate than the approximations given in [12] and [13] for the special cases $b = 0$ or $c = 0$, respectively.

Conclusion

Expressions (5), (11) and the relation $r_1 = F^2 d$ finally show that the local-flow component G^L in (4) can be approximated as

$$G^L \approx \frac{1}{r_1} - \frac{2}{F^2 + r_1}\left(1 + \frac{F^2 \psi}{F^2 + r_1}\right) - \frac{2}{5}\,\frac{F^2 r_1 P}{(F^2 + r_1)^5} \tag{12a}$$

where r_1 and ψ are given by (6) and P is defined as

$$P \equiv [4F^4 + 6F^2 r_1 + 26 r_1^2 + (F^4 + 39F^2 r_1 - 24 r_1^2)\gamma](1-\alpha) - F^2 \frac{4F^4 + 3F^2 r_1 + 5r_1^2}{F^2 + r_1}\,\alpha \tag{12b}$$

$$\text{with} \qquad 0 \leq \alpha = |x - \widetilde{x}|/r_1 \leq 1 \qquad 0 \leq \gamma \equiv -(z + \widetilde{z})/\sqrt{(y - \widetilde{y})^2 + (z + \widetilde{z})^2} \leq 1 \tag{12c}$$

The Green function G defined by (4) with (12) and (6) is considerably simpler than the alternative expressions given in the literature. In particular, the singular double Fourier integral that defines the local-flow component in the classical representations given in the literature is approximated by the simple expression (12) within the entire flow domain $0 \leq r_1 \leq \infty$. Although the local-flow component (12) is not highly accurate, as shown in Fig.1, the calculations reported in [12,13] for the related approximations (2) and (3) show that the general approximation (12) can be expected to be sufficient for practical applications. This result stems from two main properties: (i) the contribution of the wave component in (4) typically is more important than that of the local-flow component, and (ii) the approximate local-flow component (12) is asymptotically correct in both the nearfield and the farfield. Furthermore, the approximation (12) is most accurate for $x - \widetilde{x} = 0$, as can be seen from the left figure in Fig.1, and in the farfield limit $r_1 \to \infty$ with $a = 1$, i.e. along the x axis; a useful property for practical applications to typical slender ships.

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