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Multi-Hull Interference Wave-Resistance in Finite-Depth Waters

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1 Introduction

To address the need for fast transportation in nearshore, shallow-water areas, ship designers should recognize that the performance of multi-hull ships are different from their counterparts in deep water. The effects of interaction between ship-generated waves, hence the resulting resistance of such a hull system need to be considered. Fast-speed operation in fact can generate sufficiently long waves that can "feel" the finite-depth bottom.

This paper provides a bried description of a formulation for the interference wave resistance between two or more hulls in finite-depth waters, using a distribution of Havelock sources over the hulls. This result will lead to a rapid design-based computation method of the resistance of a multi-hull vessel, as a function of geometric or configuration parameters, as well as the two Froude numbers, the length-based F_n, and depth-based F_h.

Havelock [4] and Lunde [7] were among the first to examine the analytical formulation of water depth to ship resistance. Numerical computations by Kirsch [5] further explored the effects of the depth-based Froude number on the wave resistance of a single hull. A different framework was used by Yeung et al. [10] to examine the issue of total resistance of a multi-hull system. Assuming infinite depth, they consider the total hull resistance to be made up of the sum of the resistance of the individual hulls and the mutual interference effects due to each pair of inteacting hulls. They obtained a multi-hull interference-resistance formula, which allows them to rapidly evaluate the effects of configuration parameters. Their method is generalized for finite depth here and then applied to some sample designs in this paper. As to be expected, the depth-based Froude number $F_h = U/\sqrt{gh}$ plays a critical role.

It is of note that at the critical Froude number of F_h 1, a purely linear solution would not be adequate and in fact there might not be a steady-state solution (Graff et al., 1964) and a nonlinear formulation such as (Ertekin et al., 1984; Chen, 1999) would be needed to handle large and unsteady waves. However, if the design conditions are away from this critical regime, the use of linear theory can be reasonably justified, at least as a first search for the possible optimal configuration of the multiple parameters of hull systems in shallow water.

2 Interference Resistance - Analytical Theory

Michell's single-hull theory

Multi-hull systems often lead to smaller beam-to-length ratios than those of typical monohulls. This enables the individual hulls be treated more favorably with a thin-body approximation, at least as a 'practical approximation for or design evaluation. With this assumption, the free-surface boundary condition could be linearized, then the three-dimensional boundary-value problem for the velocity potential of the "i-th hull" alone in a laterally unbounded fluid can be expressed in terms of a Green function G given in Wehausen & Laitone [9]:

$$\frac{\phi_i(x,y,z)}{U/(2\pi)} = -\iint_{S_i} f_{i\xi}(\xi_i,\zeta_i) G(x,y,z;\xi_i,0,\zeta_i)\, d\xi_i d\zeta_i, \quad (1)$$

where S_i is the surface of hull i, defined by its half-breadth function $y = f_i(x,z)$, and moving with speed U. The boundary conditions satisfied by G include the free-surface condition at $y = 0$ and the bottom condition at $y = h$:

$$\frac{g}{U^2}\phi_z(x,y,0) + \phi_{xx}(x,y,0) = 0 \quad (2)$$

$$\phi_z(x,y,-h) = 0 \quad (3)$$

In addition G satisfies the radiation condition of vanishing disturbance upstream of the source. The total potential associated with any number of hulls can be obtained by a superposition of the potential for each individual hull if the dipole distribution is weak.

The wave resistance of a multi-hull system, in the absence of any mutual interferences, will be given by a simple sum of contributions of sources over each member hull of the system as in (1) This would be the same as assuming each of the hull is infinitely far from the others. In this case, the double integral over the hull surface will involve only the far-field waves from G, which yields the formula below, similar to Kostyukov [6]. It is proportional to the modulus square of the complex wave-amplitude function $A_i(\theta)$ that involves the hull slope f_{ix}:

$$R_{w,i} = \rho g \pi \times$$

$$\int_{\theta_0}^{\frac{\pi}{2}} \cos\theta \, \frac{1 - \nu H \sec^2\theta \, sech^2(k_\theta H)}{k_\theta} \, \left|A_i(\theta)\right|^2 d\theta. \quad (4)$$

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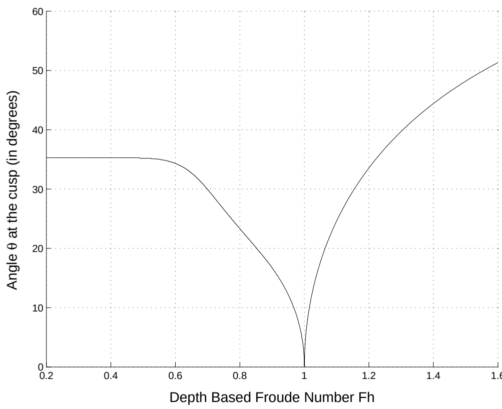
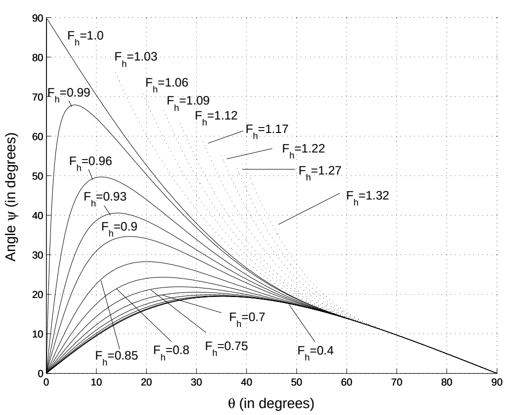
Here k_θ is the wavenumber of the far-field waves making an angle θ with respect to the x axis. k_θ is a pole of the Green function, which, for each wave angle θ, must satisfy the equation: $k_\theta h - \sec^2\theta \tanh(k_\theta h)/F_h^2 = 0$, with given depth h and F_h. The resistance is seen as a weighted integration over the wave angle. Numerical results may be compared to wave resistance measurements by Kirsch (1966), whose results had been confirmed except for some inaccuracies at large, super-critical F_h [1].



(a) Wave angle θ at the cusp line, θ_{cusp}.



(b) Divergent and transverse wave angles θ at for given ray angle ψ from the source

Figure 1: *Characteristic wave angles in the Kelvin-wave system in finite depth*

The computation of the above integral is guided by the properties of the ship-wave pattern at finite F_h. The equation for k_θ defines the restriction on the interval of integration to $\theta > \theta_0$ where $\theta_0 = \arccos 1/F_h$ if $F_h \geq 1$, or $= 0$ otherwise. Physically, this corresponds to the disappearance of the transverse waves in the super-critical regime ($F_h > 1$). The transition from tranverse waves to divergent waves occurs at a constant wave angle at given length-based Froude number and depth.

For a given F_h, stationary-phase analysis [7] will lead to a Kelvin-wave angle ψ_{max}, which encloses the wave pattern, as the solution of:

$$\psi_{max} = \arctan\big(\mathcal{F}(\theta_\psi, F_h, h)\big), \tag{5}$$

where $\theta_\psi = \theta_{cusp}$ for $F_h < 1$ and $\theta_\psi = \theta_0$ otherwise. $\mathcal{F}$ is a continuous function of three variables. The solution of the θ_ψ at the Kelvin wedge, where the transverse and divergent waves meet, is shown in Fig. 1a. This angle separates the contribution of the two wave systems in Eqn. (4). The variations of the wave angle θ for a given angular ray ψ downstream of the source can be obtained in Fig. 1b as the intercepts of each F_h curve. The maximum value of ψ for each Froude-number curve defines the Kelvin angle ψ_{max} of that particular depth Froude number.

Modeling of the interference resistance

For N hulls, the total resistance is a sum of the resistance on each hull, plus a sum of the resistances generated by all other sources in the hull system :

$$R_{wT} = \sum_{i=1}^{N} \left\{ R_{w,i} + \sum_{j,j\neq i} R_{w,j\to i} \right\} \tag{6}$$

where $R_{w,j\to i}$ indicates the force due to j-th hull acting on the i-th. where $j \to i$ indicates the force due to j acting on i. Using a method similar to the one used for the infinite-depth case [10], we can prove that the resistance of hulls i and j on each other is an integral over the wave angle:

$$R_{w,j\leftrightarrow i} = R_{w,j\to i} + R_{w,j\leftarrow i} = 2\pi\rho g \times$$

$$\int_{\theta_0}^{\frac{\pi}{2}} \mathcal{R}e\Big\{\mathcal{H}(\theta, F_h, sp_{i,j}, st_{i,j})\ \mathcal{G}\big(\overline{A_i}(\theta), \overline{A_j}(\theta)\big)\Big\} d\theta \tag{7}$$

where ρg is the specific weight of water, sp and st are the separation and stagger that define the position of hull j relative to hull i, as illustrated in Fig. 2. $\overline{\mathcal{G}}$ and $\overline{\mathcal{H}}$ are relatively simple and continuous functions of θ, the configuration parameters, and the Froude number F_h, and of the wave amplitude generated by the i-th hull, $\overline{A_i}$. The computations of (7) can be done in parallel with (4) and very efficiently.

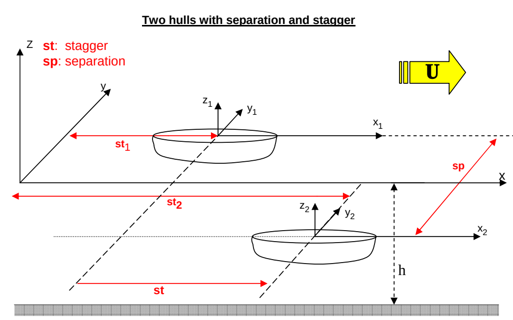
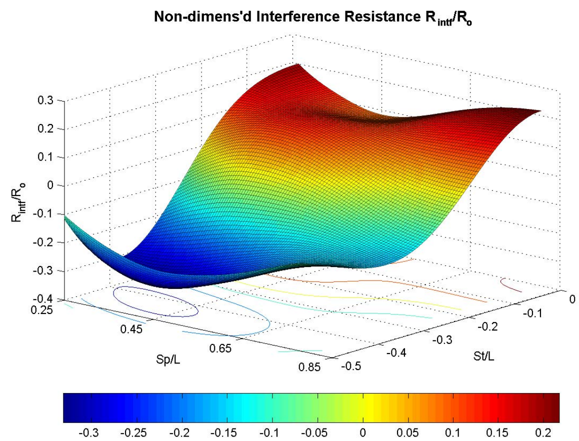


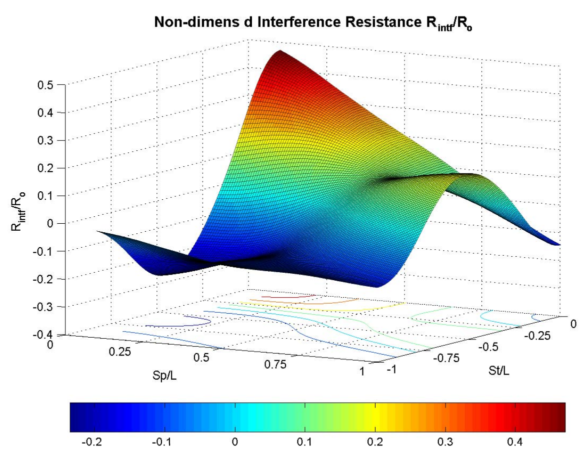
Figure 2: *Frames of reference for two hulls with Separation (sp) and Stagger (st).*

3 Illustrative Results for a Pair of Hulls

The formula (7) was used in conjunction with (4) to compute the intereference and the total resistance of a di-hull ship having two Series 60 hulls, with a block coefficient of 0.6, and dimensions of length $L = 123.96m$, beam $B = 16.5m$, and draft $T = 6.5m$. Computations can be efficiently performed at a fixed length-based Froude number $F_n = U/\sqrt{gL} = 0.33$, for a range of depth parameters, from deep water to shallow water to study the potential advantages of a multi-hull configuration over monohulls. Fig. 3 shows the *interference* resistance of the unconventional (st $\neq 0$) catamarans (sometimes refered to as the Weinblum configuration.) It is non-dimensionalized by R_0, the value of the resistance of a monohull of identical displacement at the same speed and depth. Whenever the interference resistance is half or less the value of R_0, the catamaran in this configuration has a smaller wave resistance than the monohull. A -0.25 contour value would mean a 75% reduction over the monohull.



(a) $F_h = 0.8$: subcritical speed



(b) $F_h = 1.3$: supercritical speed

Figure 3: *Non-dimensionalized interference resistance of a catamaran with two Series-60 demi-hulls as a function of st/L and sp/L, with L being the hull length.*

A systematic search for the minimum values of the non-dimensionalized interference resistance in the surface plots mentioned above leads to Fig. 4. It is a mapping of the optimal combinations of configuration parameters, namely, separation and stagger. It illustrates how the depth Froude number modifies the effects of hull interference.

4 A Configuration Problem of a Trimaran

A procedure named "MULTIRESH" was developed to obtain the total wave resistance for systems with one to five hulls in finite-depth water, parallel to the " MultiRes" code mentioned in [11] for infinite-depth water. Multi-hull optimization at a specified water-depth is illustrated below by examining the design possibilities of a trimaran. Three Series 60 hulls of $C_B = 0.60$ are combined to form a trimaran system with a center (normally the main) hull and two identical outriggers so that the total displacement is constant. For reference and comparison, a monohull of $\forall_o = 1,408m^3$ ($L = 70m$, $B = 9.2m$ and $T = 3.7m$) is introduced as the baseline hull. As in [10], each hull is geometrically similar. Thus, a single parameter, $\Lambda \equiv \forall_1/\forall_o$ characterizes the main-hull volume and relative volumetric fraction of the system. Thus, $\Lambda = 0$ corresponds to a catamaran and $\Lambda = 1$ a monohull; in between, a trimaran.

Consider $U = 15kts$, at a specified F_h, very fast computations using (4) & (7) can be carried out for $st = (-50, 50)m$, the outriggers' stagger relative to the center hull, and a separation between the centerplanes of the outriggers ($sp < 100m$). This is a problem involving four parameters: F_h, Λ, sp, and st. Figure 5 shows a summary plot of this result for three values of F_h: 0.5, 1.0, 1.2, in which we show how the minimum R_{intf}/R_o ratio behaves with volumetric fraction Λ. Points on a constant-F_h curve represents a combination of st and sp that would give the lowest interference resistance. These summary curves pinpoint the best volumetric fraction for a given F_h, and in the case of $F_h = 0.5$, there are actually two minima. To obtain the total R_{wT}, the individual resistance of each hull R_{wi}/R_o needs to be added. However, it turned out that this addition doesn't change the location of the minima.

To obtain a closer look of the sp and st values that may lead to these maximum negative interference, we show in Fig. 6, the contour plots of R_{intf}/R_o in the sp–st plane for the two Λ values corresponding to $F_h = 0.5$. The pattern is quite complex. Similar plots are shown for the $F_h = 1.0$, and 1.2 cases in Fig. 7. The super-critical solution has three competing regions of sp–st combinations of negative interference. In the Workshop, the meaning and implications of these results and those of tetra-hulls are further clarified.

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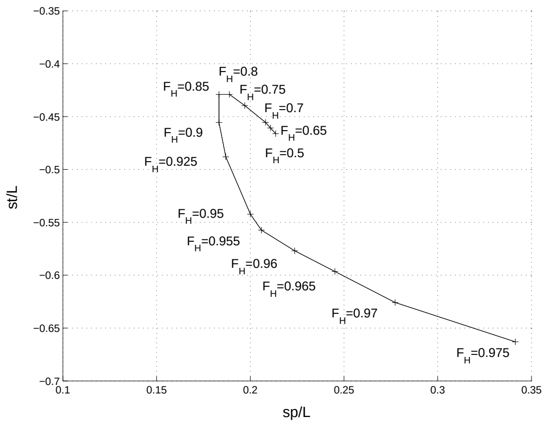


Figure 4: *Non-dimensionalized $sp-st$ value as functions of volumetric fraction Λ of a trimaran, for F_h = 0.5 (solid line), 1.0 (dashed line), and 1.2 (dash-dot-dash line).*

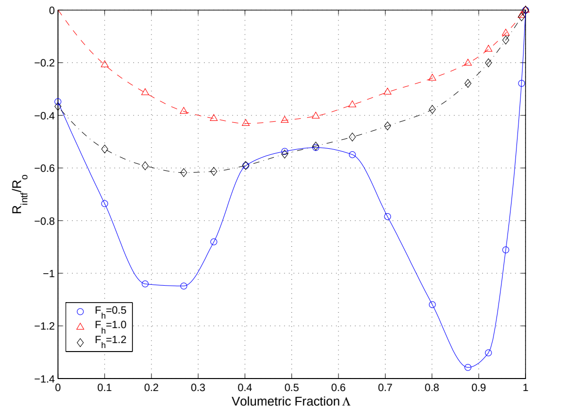
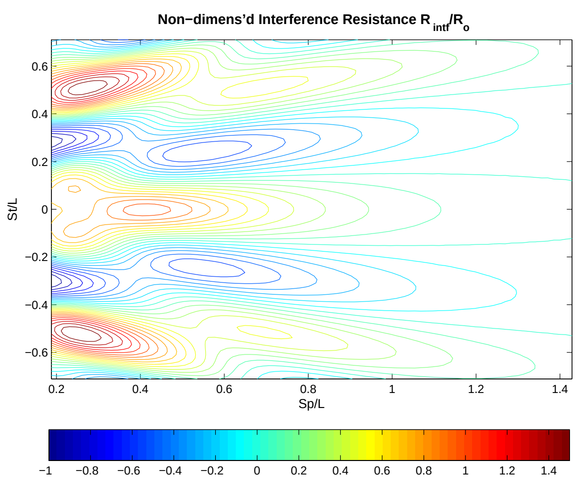
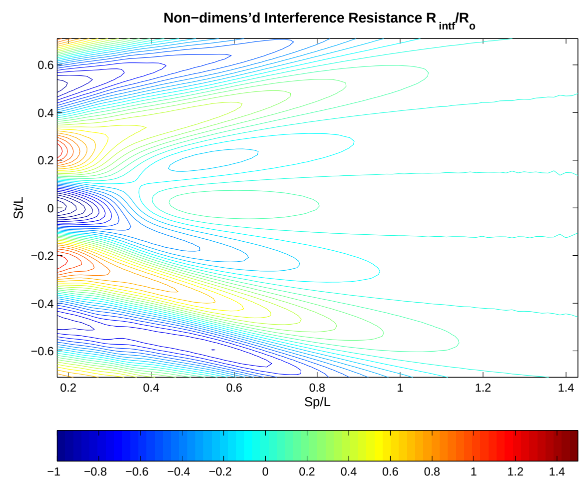


Figure 5: R_{intf}/R_o *at the optimal $sp-st$ value as functions of volumetric fraction Λ of a trimaran, for F_h = 0.5 (solid line), 1.0 (dashed line), and 1.2 (dash-dot line).*

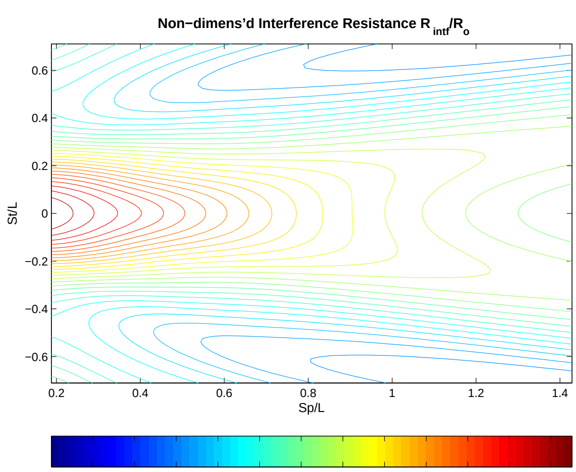


(a) $\Lambda = 0.92$

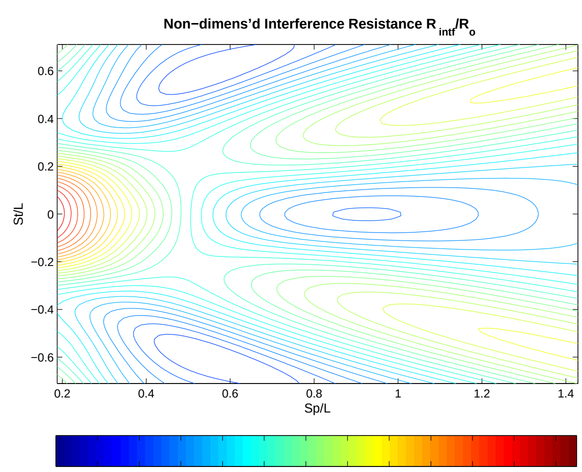


(b) $\Lambda = 0.26$

Figure 6: R_{intf}/R_o *contours of a trimaran with three Series 60 hulls, geo-similarly scaled by volumetric fraction Λ, as functions of st/L and sp/L at $F_h = 0.5$. L of the same-displacement monohull is $70m$.*



(a) $F_h = 1$, $\Lambda = 0.4$



(b) $F_h = 1.2$, $\Lambda = 0.26$

Figure 7: R_{intf}/R_o *contours of a trimaran with three Series 60 hulls, geo-similarly scaled by volumetric fraction Λ, as functions of st/L and sp/L at $F_h = 1.0, 1.2$. L of the same-displacement monohull is $70m$.*