

# NUMERICAL SIMULATION OF THE DAM BREAK PROBLEM BY GENERAL NATURAL ELEMENT METHOD

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This work is devoted to the decision of problems of mechanics of continuous environments with free surface by Natural Element Method. The brief description of a method is given, the scheme of movement on time is considered, results of numerical calculations of test problems, and also the numerical decision of a problem on destruction of a dam are resulted at presence of a layer of a liquid on the basis of.

## Introduction

For modeling the unsteady physical phenomena described by the various equations with partial derivatives, is created the whole class of mesh's methods for which on each time step is required the knowledge of the information of intercentral connectivity. The most popular representatives of this group – finite element method, volumes of fluid, boundary element method. These methods possess one common fault: during calculation the grid on which the decision is under construction, keeps the central connectivity, that, at the big deformations of area can result to it degeneration.

With growth of computers productivity development have received meshless methods which approximate the equations with partial derivatives, being based only on a set of points, without knowledge of the additional information of a grid structure. In such methods the relation of the neighbourhood of particles is not fixed, that is particles, former by neighbours during the initial moment of time, can miss in due course from each other. The characteristic representative of this group of methods is the Smoothed Particle Hydrodynamics method (SPH). This method allows precisely to reproduce kinematics of stream, however obtaining of the dynamic characteristics which necessary for calculation of hydrodynamical loadings, is a labour-intensive problem.

These circumstances have forced researchers to search for the new methods combining ideas and opportunities for meshless methods, but, at the same time, possessing advantages of mesh methods. First of the meshless method of new generation is appeared the Natural Element Method (NEM) [6]. Feature of NEM is that for stationary problems it is usual (classical) Galerkin's method, that is mesh method. For non-stationary problems in which is applied the approach of Lagrangian to the description of investigated process, on each step on time on the position of points found is under construction on the previous step the new grid determining new structure of neighbours for each central point of area. On again constructed grid the approximated system of the equations again is solved Galerkin's method. By virtue of NEM keeps some advantages of classical Galerkin's method, namely simplicity of functions of the form in ranges of definition, a continuity between elements, ease of introduction of boundary conditions. Thus it has all advantages meshless methods as functions of the form of natural element method are depended only on position of central points.

As the natural element method is represent a version of Galerkin's method, then for formation of discrete system of the equations the method weighed is used in which a set of the weight functions are conterminous with basic. Integrals undertake on elements of the expanded Delone triangulation [4]. Set of natural neighbours for each node, and also node of free boundary on a new time step are defined with the help of sweep line method and  $\alpha$ - shape method. For approximation of unknown functions are used functions of form Sibson and Nonsibson [7, 2]. The received system of the linear algebraic equations after introduction of boundary conditions is solved by conjugate gradients method.

In the present work is considered updating of natural element method for viscous incompressible fluid flow. For construction approximate functions is used Watson's algorithm. The decision of a multivariate problem on rhythmic steps is reduced to the decision of the separate equations by a conjugate gradients method. Results of the decision of dam break problem and definition of hydrodynamical loadings on walls of pool problem are

reduce. Comparison of results of calculations with experimental data allows to draw a conclusion on efficiency of a considered method.

### 1. Statement of a problem and algorithm of the decision

Let in the settlement area  $D$  limited to a free surface  $\Gamma_0$  and firm boundary  $\Gamma_1$ ,  $\Gamma_2$  and  $\Gamma_3$ , there is the movement of a viscous incompressible liquid which described by Navier-Stokes system:

$$\frac{Du_i}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\mu}{\rho} \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} \right) + f_i, \quad \frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

There  $x(t) = x_i(t) = (x_1(t), x_2(t)), i = 1, 2$ , By the accepted rule on a repeating index summation is carried out. In system (1) required functions are pressure  $p(x, t)$  and a vector of speed. Parameters will be density  $\rho$ , factor of dynamic viscosity  $\mu$  and a vector of mass forces  $f_i = (f_1, f_2)$ .

On a free surface  $\Gamma_0$  the dynamic condition  $p(x, t) = p_{atm}$  satisfies; as a liquid viscous on firm walls  $\Gamma_1$ ,  $\Gamma_2$  also  $\Gamma_3$  the condition of sticking  $u_i = 0$  satisfies.

**Sampling.** For integration of system of the equations (1) in a method of natural element method of splitting (a method of rhythmic steps), offered in the report [3] for integration of system Navier-Stokes is used. The essence of this method is consist in splitting physical process into two: diffusion and the contribution of pressure. At the first stage in the equation of movement members therefore the fictitious variable  $u^*(x, t)$  is allocated are taken into account only diffusion and expressions for predictor and the proof-reader of velocity enter the name.

$$u_i^* = u_i^n + f_i \Delta t + \frac{\mu}{\rho} \frac{\partial}{\partial x_j} \left( \frac{\partial u_i^{*+1/2}}{\partial x_j} \right) \Delta t, \quad (2)$$

$$u_i^{n+1} = u_i^* - \frac{\Delta t}{\rho} \frac{\partial}{\partial x_i} p^{n+1}. \quad (3)$$

This  $[u(x, t)]^{*+1/2} = 0.5(u^{n+1}(x, t) + u^*(x, t))$ ,  $\Delta t$  is a step of time.

At the second stage Poisson equation on pressure is solved:

$$\frac{\rho}{\Delta t} \frac{\partial u_i^*}{\partial x_i} = \frac{\partial}{\partial x_j} \frac{\partial p^{n+1}}{\partial x_j}. \quad (4)$$

In conformity with idea of a method of splitting the algorithm of movement on time of a method of natural neighbours will consist of the following steps.

- I) Definition of border of area and construction интерполяционных functions;
- II) calculation предиктора speeds  $u^*(x, t)$  from system (1);
- III) the decision of equation Пуассона (4) for definition of pressure  $p^{n+1}(x, t)$ ;
- IV) calculation of value of speed  $u^{n+1}(x, t)$  from the equation (3) in view of the found pressure;
- V) Calculation of position of units on  $n + 1$  an ohm a time step:  $x^{n+1} = x^n + u^{n+1} \Delta t$ , further on step I.

**Stabilization of a incompressible condition.** One of the main difficulties of numerical modeling non-stationary Navier-Stokes equations is stabilized incompressible condition. In the method of splitting described above on spatial variables the incompressible condition is submitted by Poisson equation for pressure (4). For elimination nonphysical oscillations of pressure it is used Finite Increment Calculus [5].

Stability of the decision of Navier-Stokes system by methods based on Galerkin's method, is provided with a choice of certainly - element spaces for speed and pressure: degrees interpolation polynoms should satisfy a component of a vector of speed and pressure to LBB conditions. In the given work for approximation of function of pressure linear basic functions (function of the form of expanded interpolation Laplass), for

approximation a component of a vector of speed - square-law basic functions (Sibson function of form) were used. Construction of such General Natural Element Method is result in satisfaction of conditions LBB for joint approximation that are guarantees generate decisions.

## 2. The decision of a dam break problem

The problem about destruction of a dam is considered at presence of a layer of a liquid on the basis. The settlement area will consist of pool with an equal bottom and the firm impenetrable walls, filled with a homogeneous viscous incompressible liquid and time is divided at the initial moment the thin impenetrable partition creating difference of a level of a liquid (fig. 1). The partition starts to move in regular intervals upwards with the set speed, the column of a liquid formed at it with a zero initial vector of speed starts break by gravity.

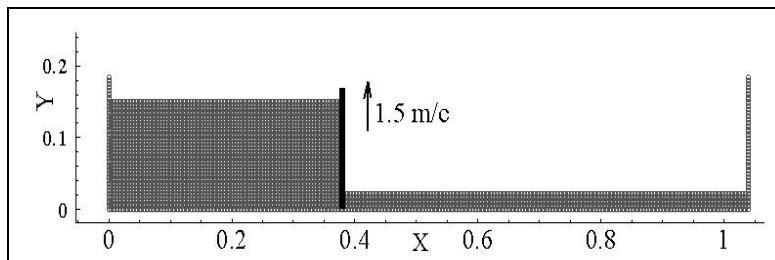
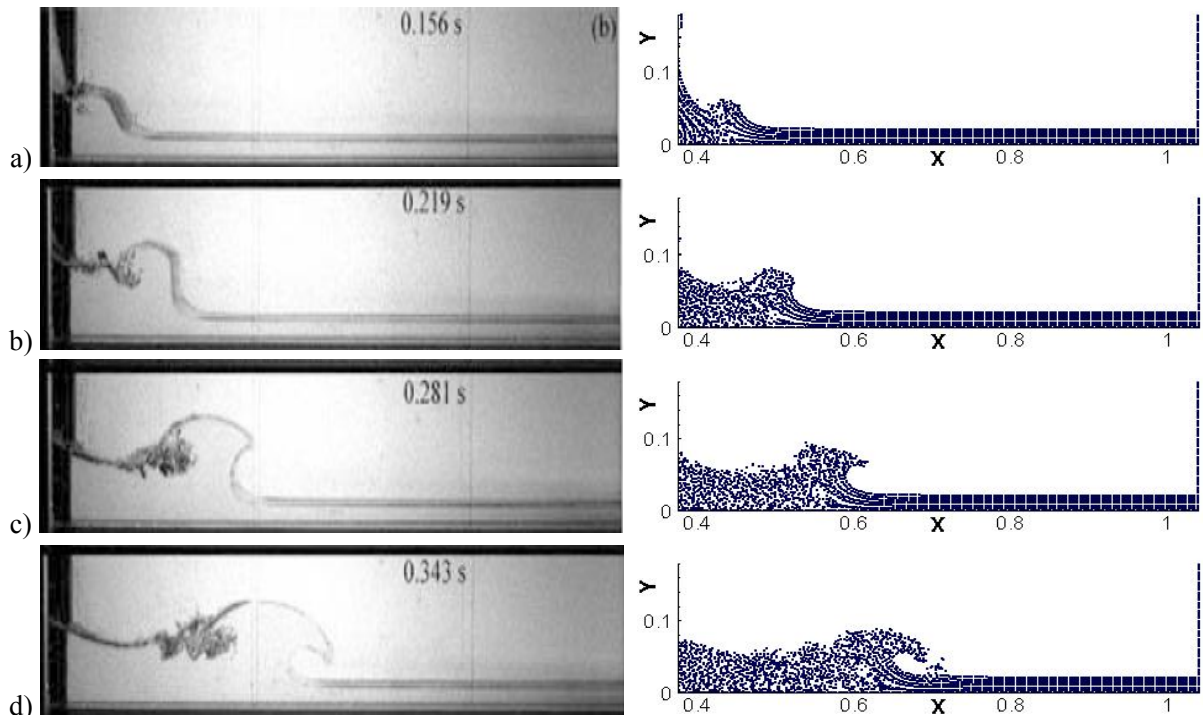


Fig. 1. Data of area

On fig. 2 is resulted the comparison of calculations of a dam break problem by the generalized natural element method of with experiment. The data of experiment have been taken from work [1].



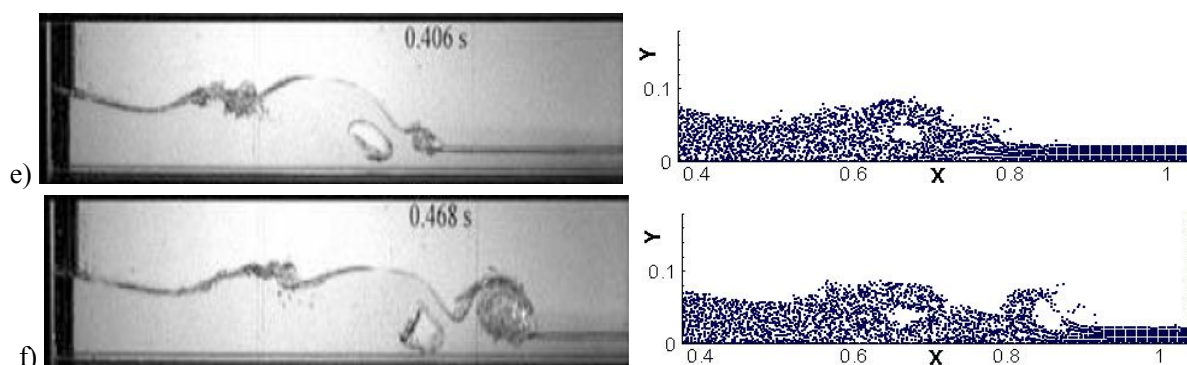


Fig. 2. a)  $t = 0.156 c$ ; b)  $t = 0.219 c$ ; c)  $t = 0.281 c$ ; d)  $t = 0.343 c$ ; e)  $t = 0.406 c$ ; f)  $t = 0.468 c$

During calculation distribution of a field of pressure which allows to define values of hydrodynamical loadings on vertical walls of area has been received. For various values of parameter  $h$  (height of a layer of a liquid at the basis) loadings on the right vertical wall of pool were calculated.

### The conclusion

The purpose of the given work - research of opportunities general natural element method for the decision problems of viscous incompressible liquid dynamics with the free surface, accompanying with strong deformations of settlement area. The given method constructed on variational Galerkin's principle, enables to receive a picture of pressure on each time layer and to define hydrodynamical loadings on firm walls of area, that favourably distinguishes it from known meshless methods. In this work is described the circuit of splitting of system of Navier-Stokes equations and the equations of indissolubility is considered, and also the algorithm of natural element method. Comparison of the numerical results received by natural element method, with known analytical decisions and experimental data is resulted. Results of numerical experiments by calculation of a bidimensional problem with deformations are submitted to a viscous incompressible liquid are determined and also are determined values of hydrodynamical loadings on firm walls of area.

### Bibliography

1. Alejandro J. S. Crespo Effect of wet bottom on dam break evolution // SPH European research interest community SIG. 2007. Vol. 6. P. 3.
2. Belikov, V.V. Non-Sibsonian interpolation on arbitrary system of points in Euclidian space and adaptive isolines generation / V.V. Belikov, A.Yu. Semenov // Applied numerical mathematics. - 2000. - Vol. 32. - P. 371-387.
3. Chorin, A. Numerical solution of the Navier-Stokes equations / A. Chorin // Math. comp, 1968. -Vol. 22. - P. 745-762.
4. Farin, G. Surfaces over Dirichlet tessellations / G. Farin // Computer Aided Geometric Design. - 1990. - Vol.7. - P. 281-292.
5. Onate, E. A stabilized finite element method for incompressible viscous flows using a finite increment calculus formulation / E. Onate // Comput. meth. appl. mech. eng, 2000. - Vol. 182, № 1-2. - P. 355-370.
6. Sukumar, N. The natural element method in solid mechanics / N. Sukumar, B. Moran, T. Belytschko // Int. J. Num. Methods Eng., 1998. - Vol.43, № 5. - P. 839-887.
7. Sibson, R. A brief description a natural neighbor interpolation / R. Sibson // Interpret multivariate data, 1981. - Chichester: John Wiley. - P. 21-36.