Interference Drag of Multiple Pressure Cushions

Ronald W. Yeung*, Hui Wan, and Jae-Moon Lew*

Department of Mechanical Engineering
University of California at Berkeley, Berkeley, CA 94720-1740, USA

*Correspondence author. E-mails: rwyeung@berkeley.edu, wanh@berkeley.edu, & jmlew@cnu.ac.kr

1 Background

Faster speed, yet lower power consumption, has often been the design objective of high-performance marine vehicles such as hovercrafts, Surface-Effect Ships (SES), among others. Lower power consumption also means less carbon-dioxide emission, an issue of great environmental concern. The concept of a multi-hull system offers favorable possibility of powering reduction in steady motion. Configuration arrangement of component hulls is therefore an important design issue to address.

The problems of steady forward-motion of multi-hulls and SES (hulls with a pressure cushion) were analyzed in Yeung et al. [1], and Yeung & Wan [2], respectively. Therein, linearized theory was used to obtain the interference wave resistance, which can be either positive or negative, increasing or reducing the powering for a given speed. Results for a single pressure cushion are quite well known (see, e.g., Wehausen & Laitone [3], Newman & Poole [4], and Doctors & Sharma [5]). The possibility of shaping the pressure function within a cushion was considered in the interesting work of Tuck et al. [6]. However, the effects of combining multiple numbers of cushions, perhaps even of dis-similar shapes, have yet to be thoroughly explored. This paper addresses the multiple pressure-cushion problem in the same vein as [1] & [2], with the aim of obtaining the necessary interference expressions for rapid evaluation of the behavior of a pressure collection. Given that there have been reports [7] on the use of multiple cushions to successfully improve the rides and maneuverability of SES and other cushioned crafts, developing a methodology to assess the powering performance of multi-cushions is desirable.

2 Resistance of a Translating Pressure Cushion

Within the framework of linear theory, the generalized steady wave resistance problem can be summarized as finding a velocity potential $\phi(x, y, z)$ that satisfies Laplace’s equation, but is subject to the free-surface boundary condition:

$$k_0\phi_x(x, y, 0) + \phi_{xx}(x, y, 0) = P_x(x, y)/\rho U$$  \hspace{1cm} (1)

where $k_0 = g/U^2$ and $U$ is the forward speed in direction $x$ (Fig. 1) and $\rho$ the water density. Here, $P(x, y)$ is the applied (cushion) pressure, which vanishes except in the planform regions $S_P$. Conditions of decaying disturbances as $z \to \infty$ and the absence of upstream waves ($x \to \infty$) are also to be observed. We note also that the linearized fluid pressure $p$ and the longitudinal free-surface slope $\zeta$ are given by:

$$\rho U\phi_x + P_x - \rho g\zeta,$$  \hspace{1cm} (2)

$$\frac{\partial P}{\partial x} = -\rho U\phi_x + P_x,$$  \hspace{1cm} (3)

The velocity potential $\phi_P(x, y, z)$ can be given in terms of derivative of the Green function $G$ as:

$$\phi_P = \frac{U}{4\pi \rho g} \int_{S_P} P(\xi, \eta) G(x - \xi; y - \eta, z, 0) d\xi d\eta,$$  \hspace{1cm} (4)

after performing an integration by part in $\xi$. The Green function $G$ is given in [3]:

$$G(x - \xi; y - \eta; z, \zeta) = \frac{1}{\pi r^2} + \frac{1}{\pi} \int_0^{\pi/2} \frac{d\theta}{\sec^2 \theta} \cos[k(x - \xi) \cos \theta] \cos[k(y - \eta) \sin \theta]$$

$$+ 4k_0 \int_0^{\pi/2} \frac{d\theta}{\sec^2 \theta} \cos \left[ k_0(x - \zeta) \sec^2 \theta \right] \cos[k_0(x - \xi) \sec \theta] \cos[k_0(y - \eta) \sin \theta] sec^2 \theta$$

$$\equiv G_L + G_w.$$

(5)
where $G_L$ and $G_w$ denote the terms that are symmetric (the first three) and asymmetric with respect to $(x - \xi)$, respectively.

The wave resistance induced by a moving cushion is given by the integral of product of the pressure $P$ and free-surface slope Eq. (3): $-\int_{S_p} P(x, y) \zeta d\eta$ and can be simplified to (with the change of variable $\lambda = \sec \theta$):

$$R_{wp} = \frac{\pi \rho U^2}{\lambda} \int_{1}^{\infty} \frac{d\lambda}{\lambda^4 \sqrt{\lambda^2 - 1}} |A_P(\lambda)|^2, \quad (6)$$

where $A_P(\lambda)$, the complex wave-making amplitude (or the Kochin) function, is given by:

$$A_P(\lambda) = \frac{k_0 \lambda^4}{\pi \rho U^2} \int_{S_p} P(\xi, \eta) e^{ik_0 \lambda(\xi + \sqrt{\lambda^2 - 1} \eta)} d\xi d\eta \quad (7)$$

In arriving at Eq. (6), we note that there was no contribution from $G_L$. Further, from Eqs. (6-7), we observe that $R_{wp}$ will be decreased by 75% when pressure $P$ is reduced by 50%. So, smaller $P$ is favored in terms of reducing wave resistance. However this will increase the size of the cushion for a fixed displacement.

A pressure cushion profile of peak value $P_m$, that is infinitely differentiable in the horizontal plane [4] is shown in Fig. 2. This hyperbolic tangent form with the tapering parameters $\alpha$ and $\beta$, in the longitudinal and transverse directions respectively, leads to a closed form expression for (7) (see [2]).

For a confirmation of our computed results with [4], the wave resistance experienced by the pressure cushion is shown in Fig. 3 as a function of the Froude number ($F_n^{-2}$), with the beam-to-length ratio, $B_p/L_p$, of the cushion as a parameter. Here, the non-dimensionalized resistance coefficient is defined by

$$C_{wp} = \frac{R_{wp}}{2 \pi \rho c B_p / \rho g} \propto \frac{R_{wp}}{2 \Delta (h/L_p)}, \quad (8)$$

where $\Delta$ is the displacement (or "lift") due to the cushion, and $h/L_p$ is the (hydrostatic) head of $P_m$ to cushion length $L_p$ ratio. The plot provides the interesting observation: The wave drag to displacement ratio is proportional to the head-to-length ratio times a function that depends only on $B_p/L_p$ and $F_n$. In Fig. 3, for a fixed $h/L_p$, a wide cushion always yields higher resistance. The highly oscillatory behavior is related to the interference of the waves generated by the bow and stern of the cushion.

3 Dual Cushions with Separation and Stagger

In the case of two pressure cushions with separation and stagger, as defined in Fig. 1, the total wave resistance $R_{wt}$ on the two pressure cushions is not only the sum of the resistance due to pressure $P_1(x, y)$ (i.e., $R_{wp1}$) and pressure $P_2$ ($R_{wp2}$) individually, but also of an interference term $R_{wp1 wp2}$, which cannot be ignored. This term accounts for the effect of pressure 1 on pressure 2 ($R_{wp1 wp2}$) as well as the effect of pressure 2 on pressure 1 ($R_{wp1 wp2}$), or effectively, the superposition of the wave-interference effects of each of the surface distribution in the field. Following [1], we can establish:

$$R_{wt} = R_{wp1} + R_{wp2} + R_{wp1 wp2}$$

$$= R_{wp1} + R_{wp2} + R_{wp1 wp2} + R_{wp2 wp1} \quad (9)$$

where $R_{wp1}$ and $R_{wp2}$ are each given by the equivalents of the Michell formula [8], or Eq. (6-7) here.

3.1 The Interference Resistance $R_{wp1 wp2}$

Consider the two local frames of reference, $O_1 x_1 y_1 z_1$ and $O_2 x_2 y_2 z_2$ in Fig. 1. Using Eqs. (2-4), we can write the expression of the interference resistances $R_{wp1 wp2}$ (pressure 2 acting on the wave slope at cushion 2 generated by cushion 1) and $R_{wp2 wp1}$ (pressure 1 acting...
on the wave slope at cushion 1 generated by cushion 2) as:

\[
R_{wP_1-P_2} = \frac{U^2}{4\pi\rho g^2} \int_{S_p} \int P_2(x_2, y_2) dx_2 dy_2 \int_{S_p} P_1(\xi_1, \eta_1) \frac{dx_1}{\eta_1} \frac{dy_1}{\eta_1}
\]

\[
G_{x_2x_2z_2}(x_2 + st - \xi_1; y_2 + sp - \eta_1; z_2, 0) d\xi_1 d\eta_1
\]  

(10)

\[
R_{wP_2-P_1} = \frac{U^2}{4\pi\rho g^2} \int_{S_p} \int P_1(x_1, y_1) dx_1 dy_1 \int_{S_p} P_2(\xi_2, \eta_2) \frac{dx_2}{\eta_2} \frac{dy_2}{\eta_2}
\]

\[
G_{x_1x_1z_1}(x_1 + st - \xi_2; y_1 + sp - \eta_2; z_1, 0) d\xi_2 d\eta_2
\]  

(11)

Then combining Eqs. (10) and (11), and recalling that \(x_2 = x_1 - st\), \(y_2 = y_1 + sp\), and \(z_2 = z_1\), we can show that the summed resistance on the two pressure cushions can be written as:

\[
R_{wP_1-P_2} = \frac{U^2}{4\pi\rho g^2} \int_{S_p} \int P_1(\xi_1, \eta_1) \frac{dx_1}{\eta_1} \frac{dy_1}{\eta_1} \int_{S_p} P_2(\xi_2, \eta_2) \frac{dx_2}{\eta_2} \frac{dy_2}{\eta_2}
\]

\[
G_{x_2x_2z_2}(x_1 - st - \xi_1; y_1 - sp - \eta_1; z_1, 0) d\xi_1 d\eta_1
\]  

(12)

Of interest is that only \(G_{w}\) survives in this summation. Eq. (12) is still unwieldy. However, if both cushions are symmetric about their own axis, \(P(x, -y) = P(x, y)\), the result simplifies greatly in a manner similar to the hull-to-hull interference problem of [1]. Under this assumption, \(A_P\) in Eq. (7) can be written as:

\[
A_P(\lambda) = \frac{k_0 \lambda^4}{\pi \rho U^2} \int_{S_p} P(\xi, \eta) e^{ik_0 \lambda \eta} \cos(k_0 \lambda \sqrt{\lambda^2 - 1} \eta) d\xi d\eta
\]

(13)

and the interference resistance is given by:

\[
R_{wP_1-P_2} = 2\pi \rho U^2 \int_{1}^{\infty} \frac{d\lambda}{\lambda^4 \sqrt{\lambda^2 - 1}} \cos(k_0 sp \lambda \sqrt{\lambda^2 - 1})
\]

\[
\times \{\Re(A_P \bar{A}_P) \cos(k_0 st \lambda) + \Im(A_P \bar{A}_P) \sin(k_0 st \lambda)\}
\]

(14)

Here, \(\Re\) and \(\Im\) denote real and imaginary parts, respectively. Similarly, if the pressure cushions have symmetry about the y axis, \(P(x, y) = P(-x, y)\), i.e. fore-aft symmetry, then Eq. (7) can be written as:

\[
A_P(\lambda) = \frac{k_0 \lambda^4}{\pi \rho U^2} \int_{S_p} P(\xi, \eta) e^{ik_0 \lambda \sqrt{\lambda^2 - 1} \xi} \cos(k_0 \lambda \sqrt{\lambda^2 - 1} \eta) d\xi d\eta
\]

(15)

and the interference resistance will be:

\[
R_{wP_1-P_2} = 2\pi \rho U^2 \int_{1}^{\infty} \frac{d\lambda}{\lambda^4 \sqrt{\lambda^2 - 1}} \cos(k_0 st \lambda)
\]

\[
\times \{\Re(A_P \bar{A}_P) \cos(k_0 \lambda \sqrt{\lambda^2 - 1} sp) + \Im(A_P \bar{A}_P) \sin(k_0 \lambda \sqrt{\lambda^2 - 1} sp)\}
\]

(16)

Eqs. (14) and (16) show explicitly how the stagger and separation between the pressure cushions can influence the total wave resistance. These new expressions can be computed concurrently with the monopressure resistances \(R_{wP_j} j = 1, 2\), given by Eq. (6).

4 Results and Discussion

Restricting the investigation to dual cushions in this paper, we show some sample results of having first dual cushions in parallel, and then in tandem, configurations. For \(B_P/L_P = 0.5\), we compare the performance of the dual cushions, each of peak pressure \(P_o\), against a mono-cushion of the same displacement and geometry. The mono-cushion resistance \(R_o\), therefore, has a pressure of \(2P_o\), applied over the same “footprint”. Figs. 4 and 5 show the interference and total wave resistance, respectively, relative to \(R_o\) for dual cushions in a parallel configuration. In these figures, the surface functions approach unity at \(sp = 0\), when the two cushions overlay. Then both functions drop off in an oscillatory manner in both directions. Significant interference drag occurs when \(sp/B_P\) is ~ unity and \(F_n\) is below the first resistance hollow of the mono-cushion. Note that for large \(F_n\) or \(sp\), the dual-cushion resistance approaches the expected value of 50% of that of the mono-cushion.

To obtain the actual dual-cushion \(C_{wT}\), one should multiply the \(R_T/R_o\) ratio by \(C_o\), the mono-cushion resistance coefficient defined by Eq. (8), this latter function is plotted as a trace against \(F_n\) for reference. The corresponding results of having the dual cushions in tandem with \(st\) being varied are shown in Figs. 6 and 7. The oscillatory patterns are more complex. The lower \(F_n\) region shows clearly the interference effects of transverse waves. Besides that, a valley of low total drag occurs for a combination of \(F_n\) and \(st/L_p\). This valley extends to larger values of \(st/L_p\) (partly visible).

The effects of varying both stagger and separation are shown in Figs. 8 and 9 for \(F_n = 0.42\), which is at the first hollow, and for a higher Froude number, \(F_n = 1\). Here, \(\lambda_o\) is the maximum (transverse) wavelength of the Kelvin wave system. These plots parallel the so-called Weinblum configurations of di-hulls. The behavior at the two speeds are drastically different, but \(R_T/R_o\) at 30% is achievable for a wide range of \(sp-st\) combinations. These and other complex features will be further discussed in the Workshop.

REFERENCES

Figure 4: $R_{\text{interf}}/R_0$ for dual cushions vs. $F_n$ and $sp$, for $st = 0$.

Figure 5: $R_{T}/R_0$ vs. $F_n$ and $sp$, for $st = 0$, with $C_0(F_n)$ shown.

Figure 6: $R_{\text{interf}}/R_0$ vs. $F_n$ and $st$ for dual cushions in tandem ($sp = 0$).

Figure 7: $R_{T}/R_0$ vs. $F_n$ and $st$, for dual cushions in tandem ($sp = 0$), with $C_0(F_n)$ shown.

Figure 8: Dual-cushion $R_{T}/R_0$ vs. $st/\lambda_o$ and $sp/\lambda_o$, at $F_n = 0.42$

Figure 9: Dual-cushion $R_{T}/R_0$ vs. $st/\lambda_o$ and $sp/\lambda_o$, at $F_n = 1.0$