Gap Resonances in Focused Wave Groups

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Introduction

We are concerned with the free surface behaviour between two closely spaced hulls, such as a floating LNG barge and a shuttle LNG tanker. Linear diffraction theory predicts that when regular waves are incident on such a configuration, very large resonant responses of the free surface may occur at certain frequencies. The phenomenon is similar to the resonances which may be excited in a moonpool (e.g. Molin, 2001). We refer to the present behaviour as "gap resonances". Model tests have shown that the resonant responses in regular waves can indeed be considerably larger than the incident waves in some cases, though they may also be rather lower than predicted by linear potential theory. The reduced amplitudes measured in the tests are due, at least in part, to viscous effects such as shear on the side walls and separation at the corners, but the influence of these at full scale is not clear. Another consideration is that, in general, real waves at sea are not regular, and the build-up of resonant motions will depend on the nature of the incident wave packet. It is this aspect that we investigate here.

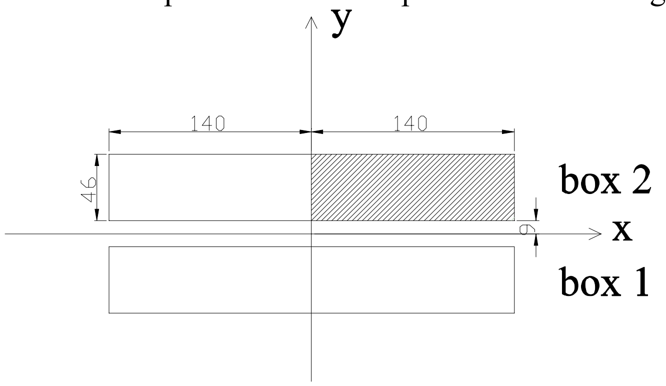

Figure 1 Sketch of geometry

Diffraction analysis in regular waves

To characterise the problem, we consider two fixed rectangular boxes having relevant dimensions (each of length 280m, breadth 46m and draft 16.5m, see figure 1). The width of the gap between the hulls is 18m, corresponding to a case investigated in Sun et al (2008). We have used the quadratic boundary element program DIFFRACT to evaluate the free surface elevations in the vicinity of the two hulls and within the gap, for a range of wave frequencies in both head and beam seas. The body surfaces are represented by a mesh over one quadrant, having 1325 nodes (shown shaded in the figure), as we may use two planes of geometric symmetry in solving this problem. (The hydrodynamic behaviour of course has just one plane of symmetry). The program implements a scheme for eliminating the effects of irregular frequencies, which is essential in the analysis of problems such as this. Here we are particularly interested in the elevation in beam seas at the mid-length position on the inner side of the downwave hull. The resulting frequency response function $F(\omega)$ (real and imaginary parts and modulus) is shown in figure 2 (together with a set of fitted results discussed below). Over the range plotted, several peaks are observed in the plot of modulus, with corresponding rapid changes of phase of the complex amplitude, at the frequencies given in the first and third rows of table 1. The four peaks above 1.3 rad/s are seen to be high, and very narrow.

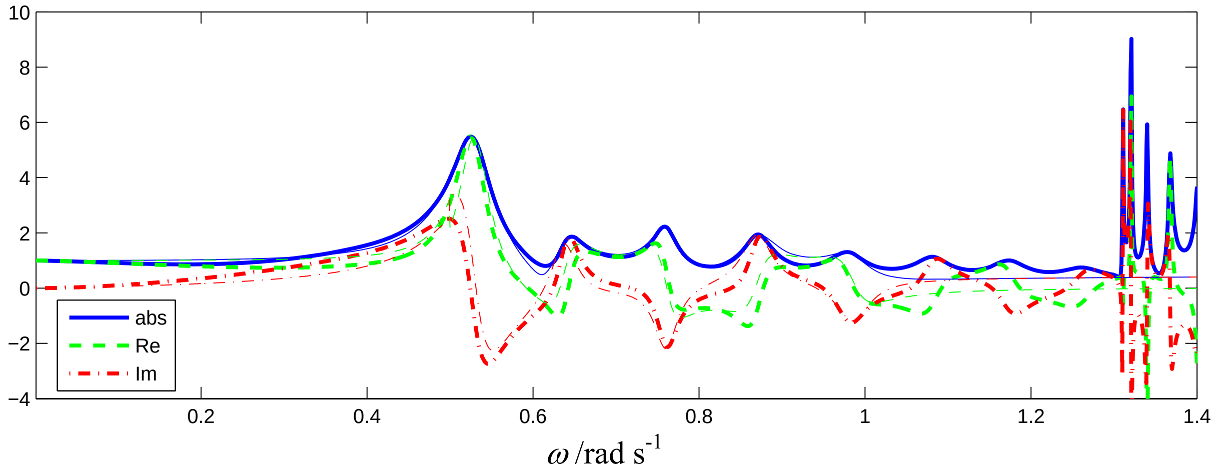

Figure 2 Frequency response function for elevation in gap: ▬▬▬ DIFFRACT; —— fitted

Mode m	1	3	5	7	9	11	13	15
DIFFRACT	0.525	0.647	0.758	0.871	0.978	1.081	1.173	1.261
Theory n=0 modes	0.516	0.651	0.774	0.891	1.001	1.103	1.197	1.285
DIFFRACT	1.311	1.321	1.340	1.368				
Theory n=1 modes	1.313	1.323	1.343	1.371	1.407	1.449	1.494	1.542

Table 1 Frequencies of gap resonances, computed from DIFFRACT and simple theory

It is relatively easy to obtain an analytical solution of a simpler version of this problem (i.e. resonances in a gap of width B, between two plates of length L, thickness H, and of semi-infinite width), based on the moonpool analysis of Molin (2001). The principal difference is to apply a Dirichlet condition at the ends of the gap instead of the Neumann condition required on all four walls of the moonpool. The matching condition at the bottom of the gap (H below the free surface) remains the same. The lowest resonances relevant to the present problem have odd integer numbers (m) of half sine waves along the length, and are constant across the gap (n=0). The next set involves the same longitudinal variations, and one half wave (n=1) transversely across the gap. The frequencies ω_{mn} associated with these modes are given in the second row of table 1, along with the corresponding modal numbers m, n. Not only are these seen to be very close to the frequencies of peak responses as predicted by the full numerical diffraction analysis; it is also found that the free surface elevations predicted by DIFFRACT are very close to the sinusoidal modes of the analytical solution for both the n = 0 transverse modes and the half sinusoidal n = 1 modes (with minor discrepancies near the ends of the gap, as expected). The qualitative difference between n = 0 and n = 1 modes is evident in the frequency response function. It would appear that the analytical model provides an excellent simple tool for estimating the frequencies of peak responses in regular waves.

In order to investigate possible viscous influences on responses of practical vessels, it is convenient to find a parametric fit to the frequency response function $F(\omega)$, and then to adjust the parameters which correspond to damping. We can do this using the idea that the resonant responses are linked to scattering frequencies at complex wavenumbers, k_p (Eatock Taylor & Meylan 2007); at neighbouring (real) wavenumbers k the response is proportional to $1/(k - k_p)$. Considering deep water waves with $k = \omega^2/g$, we could therefore seek the response in the form

$$F(\omega) = \sum_p \frac{\hat{a}_p}{\omega^2 - \omega_p^2 - i\mu_p}, \qquad (1)$$

where ω_p corresponds to the (real) resonant frequencies ω_{mn} obtained above, and μ_p is some measure of dissipation. In the potential flow analysis, μ_p is associated with energy escaping from the gap in the form of waves radiated to infinity. Rather than using equation (1), however, it is more convenient to separate out the low frequency behaviour, writing instead:

$$F(\omega) = F(0) + \sum_p \frac{\omega^2 \hat{a}_p}{\omega^2 - \omega_p^2 - \mathrm{i}\mu_p}. \qquad (2)$$

In figure 2 the results obtained using equation (2) to fit the frequency response function over the range $0 < \omega < 1.0$ rad/s are compared with $F(\omega)$ evaluated by the diffraction analysis. A first approximation to the values of μ_p may be obtained from the half power bandwidth associated with each peak. These, however, then need to be adjusted because the observed bandwidth around some of the peaks is influenced by adjacent peaks. Special techniques can be used to improve the fit (e.g. using the Matlab® System Identification Toolbox).

Behaviour in NewWave focused groups

According to the standard NewWave theory (Tromans et al. 1991), we may write the response time history, $\eta(t)$, in the neighbourhood of a large wave crest in a random seastate characterised by a spectral density function $S(\omega)$, in the form

$$\eta(t) = \int_{-\infty}^{\infty} F(\omega) S(\omega) e^{i\omega t} d\omega. \qquad (3)$$

Taking $F(\omega)$ as the complex frequency response function given above, we may use this to obtain the time history of free surface elevation in the gap when a beam seas wave group encounters the closely spaced pair of hulls. If $F(\omega)$ is taken as unity, equation (1) also gives the time history of the incident wave.

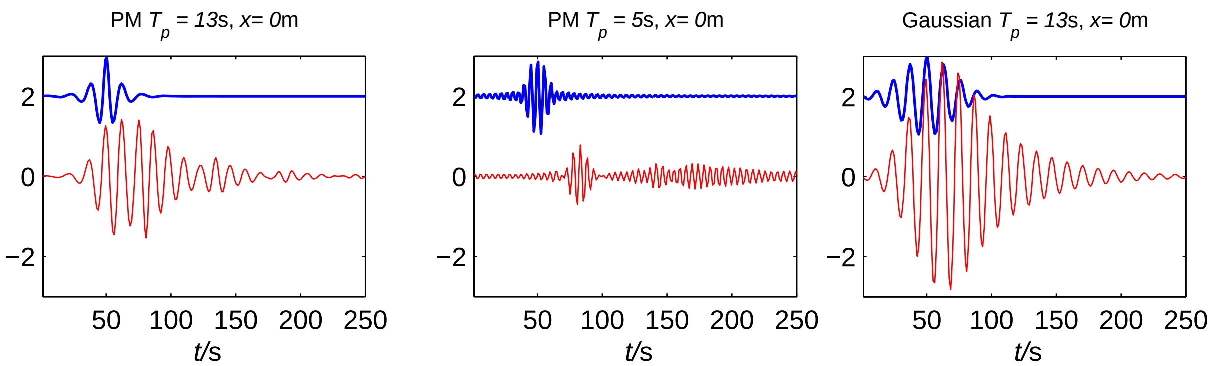

Figure 3 Time histories of elevation at $x = 0$ in different spectra.

Figure 3 compares time histories of the incident wave group, and the resulting response, for Pierson Moskowitz (PM) and Gaussian spectra with different peak periods. For clarity in these and subsequent figures the incident wave is shown shifted up by 2 units. The significant wave height is immaterial, because the incident group is in each case normalised to a unit peak elevation. The variance of the Gaussian function is chosen to fit the behaviour around the peak of a Jonswap spectrum. The discretised Fourier integral is evaluated using the FFT after padding with zeros to avoid aliasing. One notes the expected feature that a broader incident wave group is associated with the narrower banded Gaussian spectrum, though the behaviour of the response in the gap at $x = 0$ (see figure 1) is more complex. Figure 4 shows the response at different positions along the gap, in the same spectrum. These and other cases indicate significant magnification in the more extreme conditions (high T_p), and other effects, such as a form of beating at lower values of T_p (which may be linked to the disturbance propagating along the gap as in a waveguide). This is being investigated in further detail.

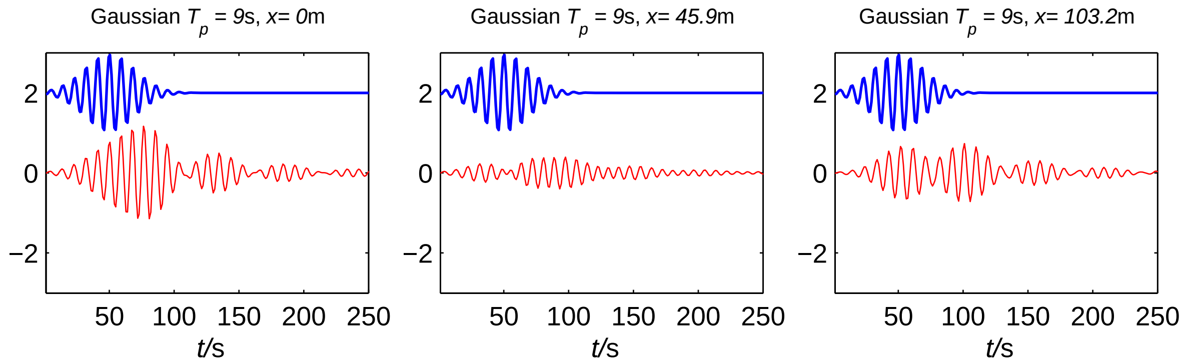

Figure 4 Elevation time histories in the same Gaussian spectrum at different positions along the gap

The above results correspond to the potential flow model. One can investigate the effect of additional damping such as may be induced by viscosity, simply by changing the values of μ_p in the fitted model of the frequency response function. Calculations using this model for the Gaussian spectrum with $T_p = 13$s are shown in figure 5, corresponding to the point $x = 0$. The first subplot shows the wave time history for the fitted function in figure 2, and therefore may be compared with the right hand plot in figure 3 based on the function evaluated by DIFFRACT. These are very similar, partly because the influence from responses above 1 rad/s (where we have not attempted to fit the frequency response function) is negligible for spectra with high peak periods. The other two subplots in figure 5 show the effects of doubling and quadrupling respectively all the damping coefficients μ_p in the fitted function. The magnifications of the peak incident elevation in the group are 2.75, 2.15 and 1.60 respectively for the reference case and the two cases of increased damping.

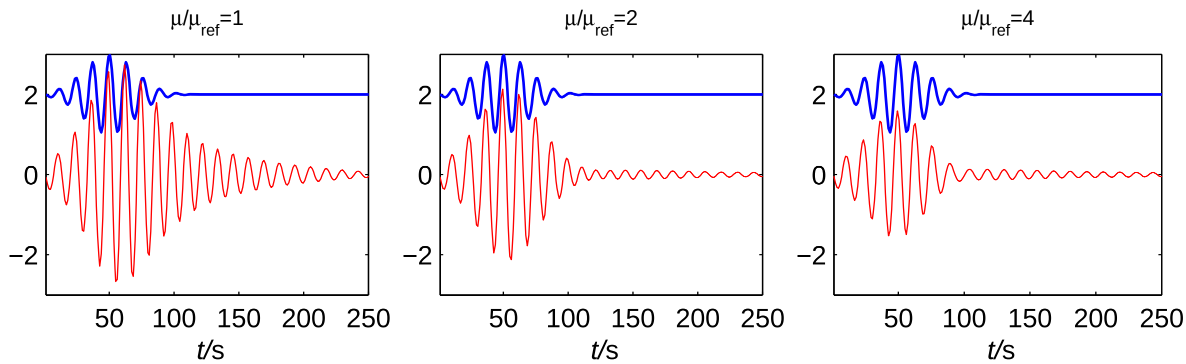

Figure 5 Influence of damping on time histories of elevation at $x = 0$ in the same Gaussian spectrum

Acknowledgments

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