MODELLING WAVE-COASTAL STRUCTURE INTERACTIONS USING A CARTESIAN CUT CELL METHOD

J Zang[1], D Ning, Q Liang[2], P H Taylor, A G L Borthwick and R Eatock Taylor

Department of Engineering Science, University of Oxford, Oxford, UK
[1] Now at Department of Architecture & Civil Engineering, University of Bath, Bath, UK
[2] Now at School of Civil Engineering & Geoscience, Newcastle University, Newcastle upon Tyne, UK

(Contact e-mail: **J.Zang@bath.ac.uk**)

INTRODUCTION

High quality numerical schemes now play a key role in design and re-assessment in coastal and offshore engineering. One of the aims of our work has been to develop simple, but accurate and easy adaptable methods for a wide range of applications in arbitrarily complicated geometry. This paper presents a Cartesian cut cell method and demonstrates its ability to accurately model curved coastal structures for various problems using both the non-linear shallow water equations (NSWE) and a Boussinesq equation set. In combination with Godunov-type shock capturing schemes, the numerical solvers are able to provide high-resolved solutions for a wide variety of problems, including steep-fronted shallow flows and strongly non-linear waves, including breaking wave interactions with curved coastal structures.

The Cartesian cut-cell technique (Yang *et al.* 1997) is implemented for body fitting using line segments. We updated the method by using ghost-cells to overcome the small time step restriction caused by very small cut cells. Our recent paper (Liang *et al.* (2006)) discusses a solution method of the non-linear shallow water equations on adaptive cut cell quadtree grids for the first time. For the problem of steady potential flow passing a circular cylinder, the Cartesian cut cells have a great advantage over the conventional staircase approximation of the curved boundary. The numerical results with Cartesian cut cells show good agreement with the analytical solution. In contrast, the usual type of staircase approximation for the boundary associated with Cartesian grids produced a spurious rotational wake downstream and the numerical velocity at the shoulder point is reduced to zero.

In this paper, two problems are presented for waves impacting a vertical bottom mounted circular cylinder, a typical configuration for an offshore wind turbine foundation. The first case is a shock-like bore (broken wave) interaction, and the second one a solitary wave hitting the cylinder.

CARTESIAN CUT CELL METHOD

For Cartesian cut-cell grids, input geometries (solid bodies) are cut out of the background grid, to approximate the boundary with piece-wise linear segments. The Cartesian cut-cell grids remain unchanged for the solid structure through the entire simulation, once they are generated at the beginning of the calculation. The governing equations are solved directly in Cartesian coordinates. A cut cell can have 3-5 faces. According to the angle (φ) formed by the cut edge and the x-axis: each cut cell can be categorised into one of four types: $\varphi \in (0^o, 90^o)$; $\varphi \in (90^o, 180^o)$; $\varphi \in (180^o, 270^o)$; and $\varphi \in (270^o, 360^o)$. Each type has four sub-types. The four sub-types for the case with $\varphi \in (0^o, 90^o)$ are illustrated in Figure 1; the sub-types for other cases can be obtained by rotation.

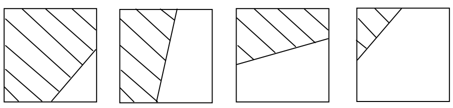

Figure 1. Grid generation: sub-types of cut cell for case with $\varphi \in (0^o, 90^o)$.

After all the cut cells have been produced, a boundary identification technique is performed to identify which cells contain fluid, are solid or are on the interface as cut-cells. As an example of grid generation, a typical quadtree grid with cut cells obtained for a single petal of a flower-shaped geometry is shown in Figure 2.

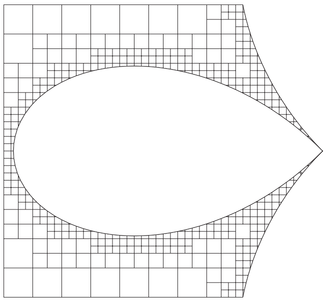

Figure 2. Cut cell quadtree grid generated about a petal.

APPLICATIONS IN COASTAL ENGINEERING

1.Shock-like Bore Interaction with Circular Cylinder

The calculation is carried out in a 5 m × 5 m domain, with a horizontal, frictionless bed. At the centre is placed a circular surface-piercing cylinder of diameter 1 m. The domain has open inlet and outlet boundaries at its west and east ends. The north and south lateral boundaries are open. A shock-like bore (broken wave) propagates from the western (left) boundary. Initially, the water is at rest, and has a still water depth equal to 1 m. The flow states before the shock are: h_R = 1.0 m; u_R = 0 m/s; and v_R = 0 m/s. The Froude number of the incident shock is set to 2.81. At t = 0 the shock hits the left edge of the rigid surface of the circular cylinder. The flow is assumed to be inviscid. The two-dimensional non-linear shallow water equations derived by integrating the Reynolds equations over the flow depth are used for the numerical simulation. In matrix form, the hyperbolic conservation law formed by the equation set can be written as

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} + \frac{\partial \mathbf{g}}{\partial y} = \mathbf{s}, \qquad (1)$$

where **u**, **f**, **g** and **s** are vectors representing conserved variables, fluxes in the x and y-directions, and source terms, respectively. Neglecting the viscous fluxes, surface and bed stresses and the Coriolis effects, the vectors can be written as

$$\mathbf{u} = \begin{bmatrix} \zeta \\ uh \\ vh \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} uh \\ u^2h + g(\zeta^2 + 2\zeta h_s)/2 \\ uvh \end{bmatrix},$$
$$\mathbf{g} = \begin{bmatrix} vh \\ uvh \\ v^2h + g(\zeta^2 + 2\zeta h_s)/2 \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} 0 \\ -g\zeta S_{ox} \\ -g\zeta S_{oy} \end{bmatrix} \qquad (2)$$

Here, ζ is the water elevation above the still water level datum; h $(= h_s + \zeta)$ is the total depth, where h_s is the still water depth; u and v are depth-averaged velocity components in the two Cartesian directions; g is the acceleration due to gravity; S_{ox} $(= -\partial h_s/\partial x)$ and S_{oy} $(= -\partial h_s/\partial y)$ are bed slopes in the x and y-directions, respectively. These equations are written in deviatoric form, obtained using flux-balancing. These equations also apply to shallow flow hydrodynamics in domains with varying bed topography. Here, the NSWE equations are solved numerically using a Godunov-type finite volume method on Cartesian cut-cell grids. An HLLC approximate Riemann solver is used to evaluate the interface fluxes. Second-order accuracy in time and space is achieved using the MUSCL-Hancock predictor-corrector method. The main advantage of this approach is that not only is the proposed scheme conservative, but also no special algorithm is required to simulate discontinuous or close to discontinuous flows. Figure 3 presents water depth contours and the adapted grid at t = 0.3s, and Figure 4 shows a 3-D view of the surface profile. These numerical results show the dominant physical phenomena with accurate capture of the sharp shock front using the adaptive grid.

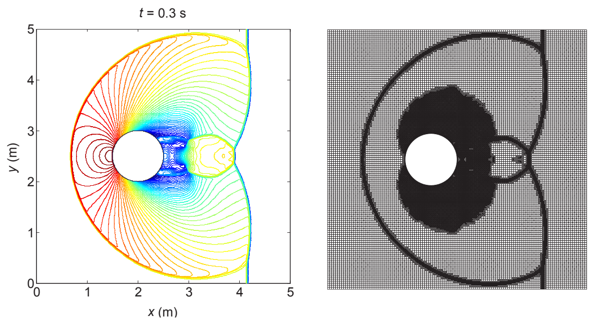

Figure 3. Depth contour and adaptive grid at t = 0.3s.

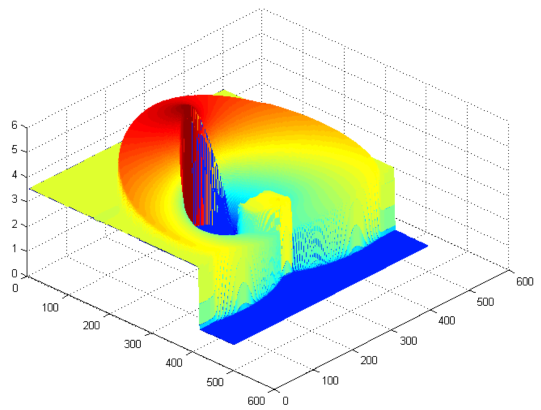

Figure 4. 3-D surface profile at t = 0.3s.

2. Solitary Wave Interaction with Circular Cylinder

Boussinesq (1872) was the first to derive shallow water wave models containing representations of both of non-linearity and dispersion. Over the last 50 years many different versions of Boussinesq-type equations have appeared, aiming at practical problems in coastal engineering. Herein the version of the two-dimensional Boussinesq-type equations due to Madsen and Sørenson (1992) is used. These equations are written as:

$$\frac{\partial \mathbf{q}}{\partial t}+\frac{\partial \mathbf{f}}{\partial x}+\frac{\partial \mathbf{g}}{\partial y}=\mathbf{h} \qquad (3)$$

where **q**, **f**, **g** and **h** are vectors representing conserved variables, x-direction fluxes, y-direction fluxes and source terms, respectively. These vectors are given by

$$\mathbf{q}=\begin{bmatrix}\zeta\\ P\\ Q\end{bmatrix}=\begin{bmatrix}\zeta\\ ud\\ vd\end{bmatrix} \qquad \mathbf{f}=\begin{bmatrix}P\\ P^2/d+g(\zeta^2+2\zeta h)/2\\ PQ/d\end{bmatrix}$$

$$\mathbf{g}=\begin{bmatrix}Q\\ PQ/d\\ Q^2/d+g(\zeta^2+2\zeta h)/2\end{bmatrix} \qquad \mathbf{h}=\begin{bmatrix}0\\ g\zeta h_x-\tau_{bx}/\rho+\psi_x\\ g\zeta h_y-\tau_{by}/\rho+\psi_y\end{bmatrix} \qquad (4)$$

where u and v are the depth-averaged velocity components in x- and y-directions; d, h and ζ are the total water depth, still water depth and free surface elevation ($d = h + \zeta$), respectively; τ_{bx} and τ_{by} are the bed friction stresses; h_x and h_y are the partial derivatives of the mean water depth in the two Cartesian directions; ψ_x and ψ_y are the Boussinesq dispersive terms, given by

$$\psi_x=\left(B+\frac{1}{3}\right)h^2\left(P_{xxt}+Q_{xyt}\right)+Bgh^3\left(\zeta_{xxx}+\zeta_{xyy}\right)$$
$$+hh_x\left(\frac{1}{3}P_{xt}+\frac{1}{6}Q_{yt}+2Bgh\zeta_{xx}+Bgh\zeta_{yy}\right)+hh_y\left(\frac{1}{6}Q_{xt}+Bgh\zeta_{xy}\right) \qquad (5)$$

$$\psi_y=\left(B+\frac{1}{3}\right)h^2\left(Q_{yyt}+P_{xyt}\right)+Bgh^3\left(\zeta_{yyy}+\zeta_{xxy}\right)$$
$$+hh_y\left(\frac{1}{3}Q_{yt}+\frac{1}{6}P_{xt}+2Bgh\zeta_{yy}+Bgh\zeta_{xx}\right)+hh_x\left(\frac{1}{6}P_{yt}+Bgh\zeta_{xy}\right)$$

The coefficient B is set to 1/15 in accordance with to Madsen and Sørenson (1992). The only difference between the current Boussinesq equations (equations 3, 4, and 5) and the nonlinear shallow water equations (equations 1 and 2) is the presence of dispersive terms in the source vector of the Boussinesq equations. This implies that the current Boussinesq equations may be solved similarly to the non-linear shallow water equations if the dispersive terms are properly accounted for (Weston, 2004).

In the present work, the Boussinesq equations are used to model wave dynamics in the entire domain except in cut-cells containing fluid at the curved boundary where a cut-cell based shallow water equation solver is used locally. Extensive tests show that this treatment of the boundaries is robust and the global accuracy of the solution is not affected. Because the similarity between the shallow water equations and the Boussinesq equations, this is easy to implement numerically by including a switch controlling whether or not the dispersive terms ψ_x and ψ_y are included.

Figure 5 presents spatial profiles of the free surface along the centre-line for a solitary wave interaction with a circular cylinder at various times. In this test case, the water depth is h = 4cm, the input wave height is H = 1.6cm and the cylinder radius is a = 6.35cm. General views of the free-surface at three typical times t = 0.8s, t = 1.28s, and t = 1.68s are given in Figure 6, showing the wave-structure interaction. Additional details on wave run-up with various degrees of input wave non-linearity are given in Figure 7, for the case when the undisturbed water depth is equal to the cylinder radius h/a = 1.0,

These results demonstrate that our new numerical implementation of a well-known Boussinesq model is robust, and able to simulate strongly non-linear waves propagating and interacting with curved coastal structures.

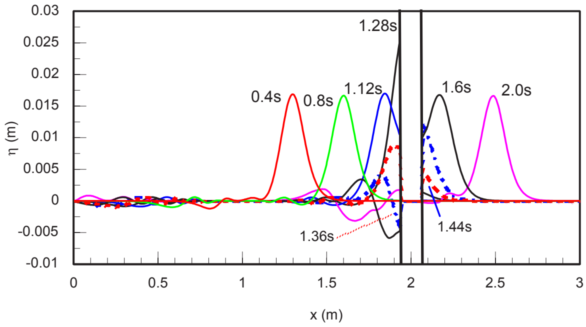

Figure 5. Spatial profiles of free surface along the centre-line at various times

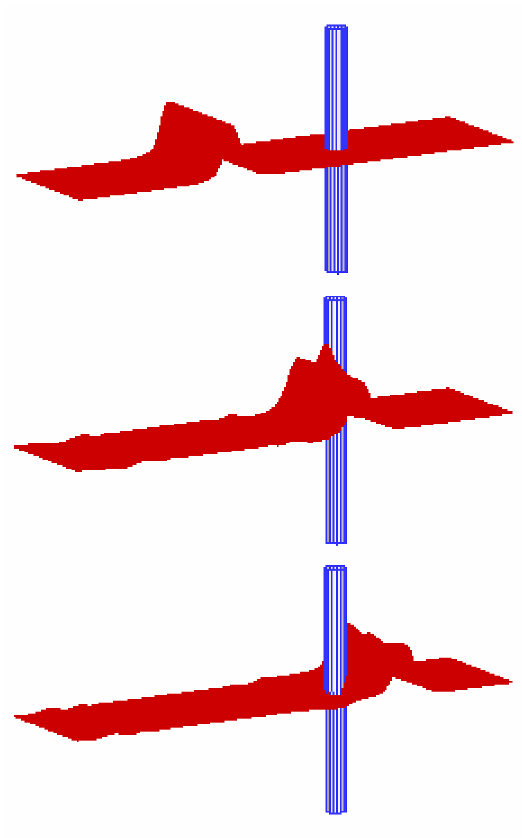

Figure 6. 3-D free surface profiles at three typical times $t = 0.8$s, $t = 1.28$s, and $t = 1.68$s.

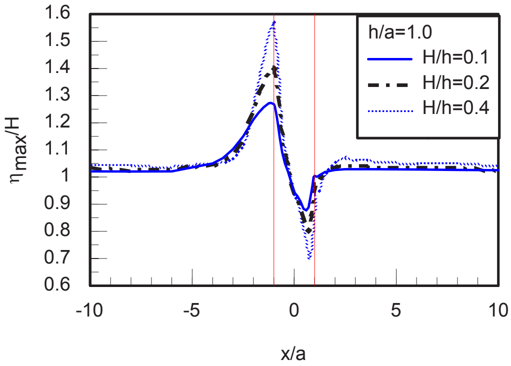

Figure 7. Wave run-up with different non-linearity

CONCLUSIONS

This paper presents a new method for the numerical solution of Boussinesq equation sets based on efficient mesh discretisation using Cartesian cut-cells. The approach is applicable to a wide range of problems arising in coastal engineering involving complex geometries. Based locally on the non-linear shallow water equations (NSWE) with shock capturing, the extra dispersive components in the Boussinesq equations are treated as simple source terms. The results are shown to be accurate, and the model is robust and easy to apply. Further examples of wave-structure interaction including comparisons to experiments on model offshore wind turbine foundations are ongoing.

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