Investigation of freak waves in large scale 3D Higher-Order Spectral simulations

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Introduction

In open ocean, floating bodies may encounter so-called freak or rogue waves. Such events constitute a major problem for both structure integrity and human safety. Thus, the knowledge of these extreme sea-states and the capability of models to reproduce it precisely are therefore crucial. In this paper, we report on recent results obtained on freak waves formation and particularly in 3D simulations. A time accurate fully nonlinear potential flow model has been developed to simulate the propagation of gravity waves in 2D or 3D. This model relies on a periodic Higher-Order Spectral (HOS) technique based on the original HOS model of West *et al.* [7] and Dommermuth and Yue [3]. We propose 3D long-time simulations of typical North Sea wave fields in which freak waves are detected. Then, an analysis of the 3D shape of these extreme events is proposed and the influence of the directionality is discussed.

Description of numerical model & simulations

We use a HOS model which is an enhanced version of the order-consistent scheme initially proposed by West et al. [7] and extensively validated. Specific attention has been paid to the aliasing phenomenon as well as the numerical efficiency (see e.g. Bonnefoy [1]). This allows to simulate hundreds of spectral peak periods on a large ocean domain with a typical single processor computer.

We work on a periodic unbounded domain representing a part of the ocean with infinite depth (extension to finite depth is direct). Using the potential flow theory and following Zakharov [8], the fully-nonlinear free surface boundary conditions can be written in terms of surface quantities, namely the single-valued free surface elevation η and the surface velocity potential $\phi^s(\mathbf{x},t) = \phi(\mathbf{x},\eta,t)$ (ϕ is the velocity potential):

$$\frac{\partial \phi^s}{\partial t} = -g\eta - \frac{1}{2}|\nabla\phi^s|^2 + \frac{1}{2}\left(1+|\nabla\eta|^2\right)\left(\frac{\partial\phi}{\partial z}\right)^2 \qquad \text{on } z=\eta(\mathbf{x},t) \tag{1}$$

$$\frac{\partial \eta}{\partial t} = \left(1+|\nabla\eta|^2\right)\frac{\partial\phi}{\partial z} - \nabla\phi^s\nabla\eta \qquad \text{on } z=\eta(\mathbf{x},t) \tag{2}$$

In this way, the only remaining unknown is the veretical velocity $\frac{\partial\phi}{\partial z}$. This quantity will be evaluated thanks to the order-consistent HOS scheme of West et al. [7]. The HOS formulation allows us to use a very efficient FFT-based solution scheme with numerical cost growing as $N log_2 N$, N being the number of modes. Then, the two surface quantities are time marched using an efficient 4^{th} order Runge-Kutta scheme with an adaptative step-size control associated to an acceleration procedure. This acceleration is based on an analytical integration of the linear part of the equations (see e.g. Fructus et al. [4]). Nonlinear products appearing in free surface boundary conditions (Eqs. (1) & (2)) as well as in the HOS procedure are carefully dealisaed (see e.g. Bonnefoy [1]).

In this paper we present long-time evolutions of 3D wave fields that are analysed to detect freak events. An important point is the initialization of the wave field. Indeed, we will just let a specified initial solution evolve. Dommermuth [2] indicates that the definition of adequate initial solution to start fully nonlinear computations is not an easy task and can lead to numerical instabilities. However, the solution proposed in [2] is based on a simple linear initialization followed by a transition period between linear and fully nonlinear conditions. We have applied a similar relaxation period, over a duration $T_a = 10T_p$, T_p being the spectrum peak period. For more details, see Dommermuth [2].

Initial conditions are similar to those of Tanaka [6]. A classical directional spectrum $\Phi(\omega, \theta)$ is defined:

$$\Phi(\omega, \theta) = \psi(\omega) \times G(\theta)$$

In this study, we use a JONSWAP spectrum:

$$\psi(\omega) = \alpha g^2 \omega^{-5} \exp\left(-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right)\gamma^{\exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]}$$

with α being the Phillips constant and ω_p the angular frequency at the peak of the spectrum.

$$\alpha = 3.279E, \quad \gamma = 3.3, \quad \sigma = \begin{cases} 0.07 \ (\omega < 1), \\ 0.09 \ (\omega \geq 1) \end{cases}$$

E is the dimensionless energy density of the wavefield. The significant wave height could be estimated by $H_s \approx 4\sqrt{E}$ and the directionality is defined as follows:

$$G(\theta) = \begin{cases} A_n \cos^n \theta, & |\theta| \leq \frac{\pi}{2} \\ 0, & |\theta| > \frac{\pi}{2} \end{cases} \quad \text{with} \quad A_n = \begin{cases} \frac{(2^p!)^2}{\pi(2p)!}, & \text{if } n = 2p \\ \frac{(2p+1)!}{2(2^p p!)^2}, & \text{if } n = 2p+1 \end{cases} \tag{3}$$

Then, initial free surface elevation η and surface velocity potential ϕ^s are computed from this directional spectrum definition. The mean direction is along the x-axis, the y-axis defining the transverse direction. The numerical conditions chosen in the following simulations are:

- Wave field characterized by $E = 0.005$ i.e. $\alpha = 0.016, H_s = 0.28$ in non-dimensional quantities (with respect to g and ω_p),
- Domain length: $L_x = 42\lambda_p \times L_y = 42\lambda_p$ (λ_p is the peak wavelength),
- Number of modes used: $N_x = 1024 \times N_y = 512$, HOS order $M = 5$,
- Dimensional quantities give, if we fix $T_p = 9.5s$ (typical in North Sea): $H_s = 6.2m$ and $\lambda_p = 140m$. Dimensional domain area: $5740m \times 5740m$.

Long-time simulations of such 3D wave fields are performed. During these computations, extreme events naturally appear within the wavefield which is a typical sea-state that could be encountered in North Sea. These are not forced freak waves which can be generated through directional focusing or development of Benjamin-Feir instability. An example of freak event is shown in Fig. 1 with a closer view made in Fig. 2. In the present paper, freak waves are defined as waves with heights exceeding the significant wave height H_s by a factor in 2.2 (see e.g. several papers in the recent Rogue Waves Workshop (2005)). We choose to define the wave height as the height of waves taken along the mean direction of propagation (x-axis). We perform zero up-and-down crossing analyses on each mesh line along the x-axis. The freak waves are thus detected by applying the criterion defined above.

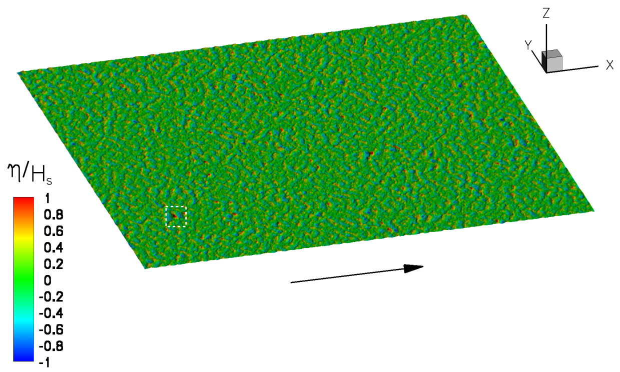

Figure 1: 3D surface elevation of the extreme event at $t = 26.5T_p$

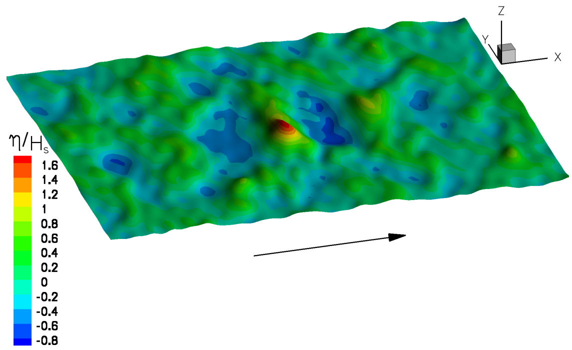

Figure 2: Zoom on the 3D extreme event $t = 26.5T_p$, $H_{max} = 2.44H_s$

Influence of directionality

In this section we investigate the influence of directionality on freak waves formation. Indeed, it appears that this parameter plays a key role in the occurence of these events. We choose three different values of n (c.f. definition of directionality Eq. (3)). First, simulations were performed with $n = 2$ following Tanaka [6], then we choose more 'realistics' choices of directionality: $n = 30$ & 90. I remind here that when n rises the directional spreading of the wave field decreases. These are long-time simulations over 250 wave spectrum peak periods.

The occurence and shapes of the freak events are recorded all along the simulations. The extreme interesting events are detected as explained in the previous section and a transverse analysis is also made. Namely, once we detect the position (x, y) of a freak event, we perform a zero-crossing analysis in the transverse direction. In this way, we are able to determine the length as well as the width of the freak events. This is plotted in the next figures for the 3 different values of the directionality parameter n. Each rectangular box represents a freak wave with its own measured dimension (L_x, L_y) being the spatial extent of the extreme event.

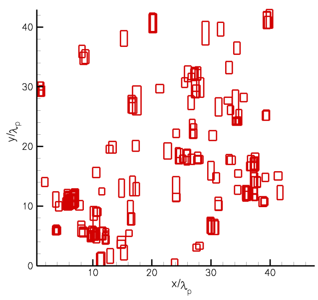
Figure 3: Freak events in the computational domain. $n = 2$

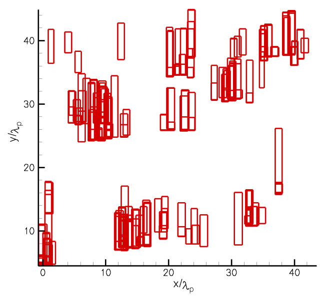
Figure 4: Freak events in the computational domain. $n = 30$

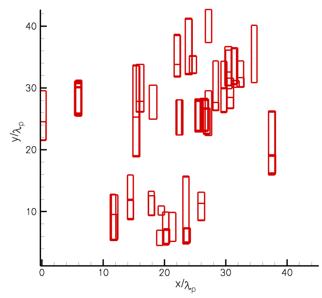
Figure 5: Freak events in the computational domain. $n = 90$

Fig. 3 gives some information about the freak events with such a large directional spreading ($n = 2$). The corresponding extreme events seem to have a rather short transverse extent. Another interesting feature of this figure is that it allows us to observe that the appearing freak waves are either isolated events (see e.g. $(x/\lambda_p, y/\lambda_p) = (16, 38)$) or part of a group of extreme events that remain coherent for several periods in a row (see e.g. $(x/\lambda_p, y/\lambda_p) = (6, 10)$). For this broad directional spectrum, the coherent group can last up to 15 peak periods.

Then, when comparing with Fig. 4 ($n = 30$), we observe, as expected, that the mean transverse wavelength is enlarged. This tendency is confirmed with Fig. 5 ($n = 90$). With these latter 2 more realistic choices of directionality parameter n, we observe the formation of the commonly described 'wall of water' (see e.g. review of observed freak waves by Kharif and Pelinovsky [5]). Isolated and groups of freak waves also appear, however the extreme events remain rather localized in space and time (the longest life time of such event does not exceed 5 peak periods of propagation for $n = 90$).

	Number of freak events	Mean of H_{max}/H_s	Mean transverse wavelength L_y
$n = 2$	2350	2.13	2.1 λ_p
$n = 30$	756	2.05	4.3 λ_p
$n = 90$	443	2.0	5.8 λ_p

Table 1: Influence of directionality parameter n

An important observation is that the number of detected freak waves grows with the spreading of the wave field. Table 1 reports the number of freak events as well as the mean of H_{max}/H_s and the mean transverse wavelength L_y for each simulation. The parameter H_{max}/H_s represents a good indication on the encountered global wave field. It is evaluated at each time step on the whole fluid domain (a very large domain is computed explaining the high value of the parameter) and the mean along the whole time of simulation is calculated. This mean and the number of observed extreme events both decreases when the directionality parameter increases. The occurence of extreme events is thus closely linked to the directionality as well as the transverse extent of the extreme wave. Further analysis will be done once the parallelization of the model is achieved. It will made accessible statistical analyses on 3D extreme waves, allowing computations on larger 3D domain over longer times of simulation.

Conclusion

In this paper we have pointed out the ability of our model to modelize natural occurence of freak wave events. The efficiency of our HOS model allows us to perform 3D long-time computations on large domains (here typically simulations lasts for 250 wave peak periods with more than 30km^2 of ocean computed). An original analysis of the 3D shape of the freak waves is presented and particularly the influence of the directionality is presented. This typical 3D parameter seem to play a key role in the occurence of the freak waves. The results obtained are really encouraging for the pursuit of investigations in this domain with our HOS model. More systematic studies over repeated long-time simulations are in particular required to obtain stochastically significant results. This will be possible once the parallelization of the code is achieved.

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